AQA Maths Mechanics 3

Past Paper Pack

2006-2015

General Certificate of Education
June 2006
Advanced Level Examination

MATHEMATICS **MM03**
Unit Mechanics 3

Wednesday 21 June 2006 1.30 pm to 3.00 pm

For this paper you must have:
- an 8-page answer book
- the **blue** AQA booklet of formulae and statistical tables

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MM03.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Take $g = 9.8\,\text{m}\,\text{s}^{-2}$, unless stated otherwise.

Information
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 The time T taken for a simple pendulum to make a single small oscillation is thought to depend only on its length l, its mass m and the acceleration due to gravity g.

By using dimensional analysis:

(a) show that T does **not** depend on m; *(3 marks)*

(b) express T in terms of l, g and k, where k is a dimensionless constant. *(4 marks)*

2 Three smooth spheres A, B and C of equal radii and masses m, m and $2m$ respectively lie at rest on a smooth horizontal table. The centres of the spheres lie in a straight line with B between A and C. The coefficient of restitution between any two spheres is e.

The sphere A is projected directly towards B with speed u and collides with B.

(a) Find, in terms of u and e, the speed of B immediately after the impact between A and B. *(5 marks)*

(b) The sphere B subsequently collides with C. The speed of C immediately after this collision is $\frac{3}{8}u$. Find the value of e. *(7 marks)*

3 A ball of mass 0.45 kg is travelling horizontally with speed $15\,\mathrm{m\,s^{-1}}$ when it strikes a fixed vertical bat directly and rebounds from it. The ball stays in contact with the bat for 0.1 seconds.

At time t seconds after first coming into contact with the bat, the force exerted on the ball by the bat is $1.4 \times 10^5(t^2 - 10t^3)$ newtons, where $0 \leqslant t \leqslant 0.1$.

In this simple model, ignore the weight of the ball and model the ball as a particle.

(a) Show that the magnitude of the impulse exerted by the bat on the ball is 11.7 N s, correct to three significant figures. *(4 marks)*

(b) Find, to two significant figures, the speed of the ball immediately after the impact. *(4 marks)*

(c) Give a reason why the speed of the ball immediately after the impact is different from the speed of the ball immediately before the impact. *(1 mark)*

4 The unit vectors $\mathbf{i}$ and $\mathbf{j}$ are directed due east and due north respectively.

Two cyclists, Aazar and Ben, are cycling on straight horizontal roads with constant velocities of $(6\mathbf{i} + 12\mathbf{j})\,\mathrm{km\,h^{-1}}$ and $(12\mathbf{i} - 8\mathbf{j})\,\mathrm{km\,h^{-1}}$ respectively. Initially, Aazar and Ben have position vectors $(5\mathbf{i} - \mathbf{j})$ km and $(18\mathbf{i} + 5\mathbf{j})$ km respectively, relative to a fixed origin.

(a) Find, as a vector in terms of $\mathbf{i}$ and $\mathbf{j}$, the velocity of Ben relative to Aazar. *(2 marks)*

(b) The position vector of Ben relative to Aazar at time t hours after they start is $\mathbf{r}$ km.

Show that

$$\mathbf{r} = (13 + 6t)\mathbf{i} + (6 - 20t)\mathbf{j}$$ *(4 marks)*

(c) Find the value of t when Aazar and Ben are closest together. *(6 marks)*

(d) Find the closest distance between Aazar and Ben. *(2 marks)*

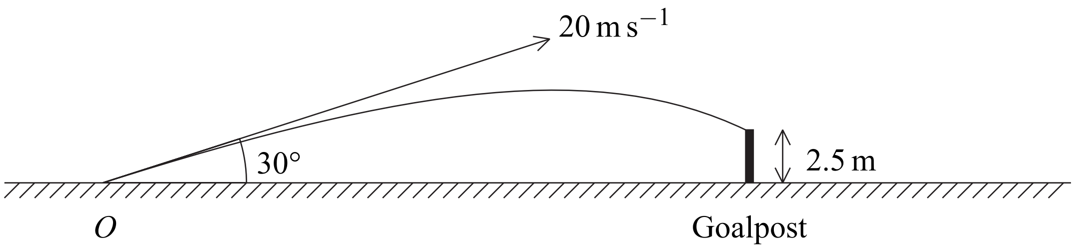
5 A football is kicked from a point O on a horizontal football ground with a velocity of $20\,\mathrm{m\,s^{-1}}$ at an angle of elevation of 30°. During the motion, the horizontal and upward vertical displacements of the football from O are x metres and y metres respectively.

(a) Show that x and y satisfy the equation

$$y = x \tan 30° - \frac{gx^2}{800 \cos^2 30°}$$ *(6 marks)*

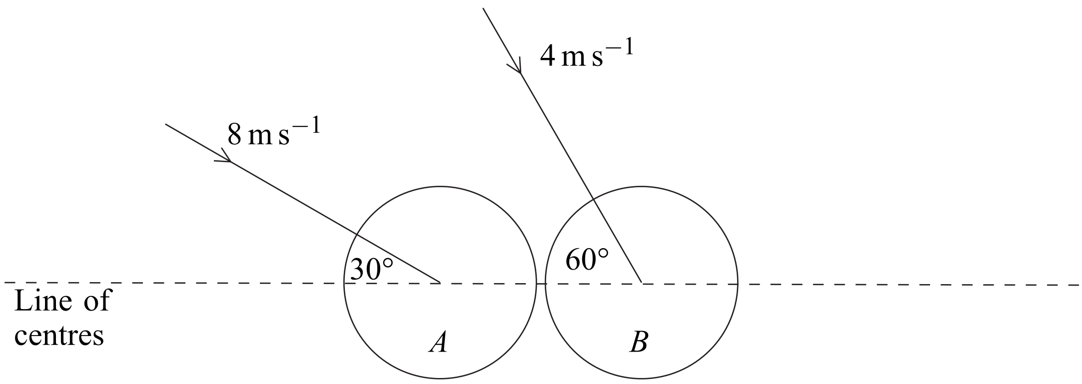
(b) On its downward flight the ball hits the horizontal crossbar of the goal at a point which is 2.5 m above the ground. Using the equation given in part (a), find the horizontal distance from O to the goal.

(4 marks)



(c) State **two** modelling assumptions that you have made. *(2 marks)*

6 Two smooth billiard balls A and B, of identical size and equal mass, move towards each other on a horizontal surface and collide. Just before the collision, A has velocity $8\,\mathrm{m\,s^{-1}}$ in a direction inclined at 30° to the line of centres of the balls, and B has velocity $4\,\mathrm{m\,s^{-1}}$ in a direction inclined at 60° to the line of centres, as shown in the diagram.



The coefficient of restitution between the balls is $\frac{1}{2}$.

(a) Find the speed of B immediately after the collision. *(9 marks)*

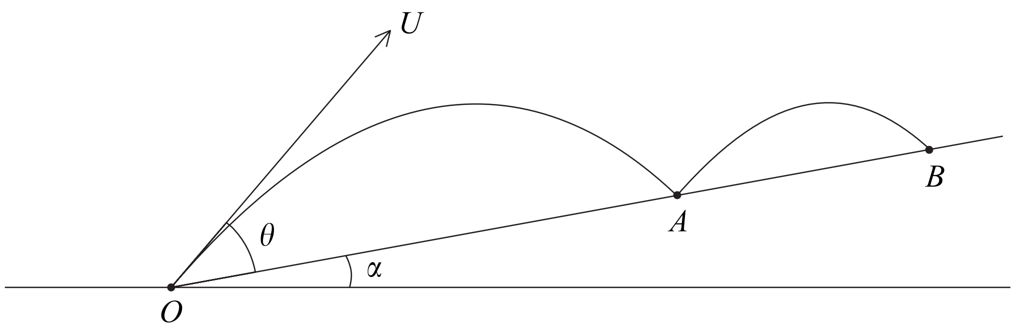
(b) Find the angle between the velocity of B and the line of centres of the balls immediately after the collision. *(2 marks)*

7 A projectile is fired from a point O on the slope of a hill which is inclined at an angle α to the horizontal. The projectile is fired up the hill with velocity U at an angle θ above the hill and first strikes it at a point A. The projectile is modelled as a particle and the hill is modelled as a plane with OA as a line of greatest slope.

(a) (i) Find, in terms of U, g, α and θ, the time taken by the projectile to travel from O to A. *(3 marks)*

(ii) Hence, or otherwise, show that the magnitude of the component of the velocity of the projectile perpendicular to the hill, when it strikes the hill at the point A, is the same as it was initially at O. *(3 marks)*

(b) The projectile rebounds and strikes the hill again at a point B. The hill is smooth and the coefficient of restitution between the projectile and the hill is e.



Find the ratio of the time of flight from O to A to the time of flight from A to B. Give your answer in its simplest form. *(4 marks)*

END OF QUESTIONS

General Certificate of Education
June 2007
Advanced Level Examination

MATHEMATICS **MM03**
Unit Mechanics 3

Monday 11 June 2007 1.30 pm to 3.00 pm

For this paper you must have:
- an 8-page answer book
- the **blue** AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
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- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Take $g = 9.8 \text{ m s}^{-2}$, unless stated otherwise.

Information
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 The magnitude of the gravitational force, F, between two planets of masses m_1 and m_2 with centres at a distance x apart is given by

$$F = \frac{Gm_1m_2}{x^2}$$

where G is a constant.

(a) By using dimensional analysis, find the dimensions of G. *(3 marks)*

(b) The lifetime, t, of a planet is thought to depend on its mass, m, its initial radius, R, the constant G and a dimensionless constant, k, so that

$$t = km^{\alpha}R^{\beta}G^{\gamma}$$

where α, β and γ are constants.

Find the values of α, β and γ. *(5 marks)*

2 The unit vectors **i**, **j** and **k** are directed due east, due north and vertically upwards respectively.

Two helicopters, A and B, are flying with constant velocities of $(20\mathbf{i} - 10\mathbf{j} + 20\mathbf{k})\,\text{m s}^{-1}$ and $(30\mathbf{i} + 10\mathbf{j} + 10\mathbf{k})\,\text{m s}^{-1}$ respectively. At noon, the position vectors of A and B relative to a fixed origin, O, are $(8000\mathbf{i} + 1500\mathbf{j} + 3000\mathbf{k})\,\text{m}$ and $(2000\mathbf{i} + 500\mathbf{j} + 1000\mathbf{k})\,\text{m}$ respectively.

(a) Write down the velocity of A relative to B. *(2 marks)*

(b) Find the position vector of A relative to B at time t seconds after noon. *(3 marks)*

(c) Find the value of t when A and B are closest together. *(5 marks)*

3 A particle P, of mass 2 kg, is initially at rest at a point O on a smooth horizontal surface. The particle moves along a straight line, OA, under the action of a horizontal force. When the force has been acting for t seconds, it has magnitude $(4t + 5)$ N.

(a) Find the magnitude of the impulse exerted by the force on P between the times $t = 0$ and $t = 3$. *(3 marks)*

(b) Find the speed of P when $t = 3$. *(2 marks)*

(c) The speed of P at A is $37.5\,\text{m s}^{-1}$. Find the time taken for the particle to reach A. *(4 marks)*

4 Two small smooth spheres, A and B, of equal radii have masses 0.3 kg and 0.2 kg respectively. They are moving on a smooth horizontal surface directly towards each other with speeds $3\,\text{m}\,\text{s}^{-1}$ and $2\,\text{m}\,\text{s}^{-1}$ respectively when they collide. The coefficient of restitution between A and B is 0.8 .

(a) Find the speeds of A and B immediately after the collision. *(6 marks)*

(b) Subsequently, B collides with a fixed smooth vertical wall which is at right angles to the path of the sphere. The coefficient of restitution between B and the wall is 0.7 .

Show that B will collide again with A. *(3 marks)*

5 A ball is projected with speed $u\,\text{m}\,\text{s}^{-1}$ at an angle of elevation α above the horizontal so as to hit a point P on a wall. The ball travels in a vertical plane through the point of projection. During the motion, the horizontal and upward vertical displacements of the ball from the point of projection are x metres and y metres respectively.

(a) Show that, during the flight, the equation of the trajectory of the ball is given by

$$y = x\tan\alpha - \frac{gx^2}{2u^2}(1 + \tan^2\alpha)$$ *(6 marks)*

(b) The ball is projected from a point 1 metre vertically below and R metres horizontally from the point P.

(i) By taking $g = 10\,\text{m}\,\text{s}^{-2}$, show that R satisfies the equation

$$5R^2\tan^2\alpha - u^2R\tan\alpha + 5R^2 + u^2 = 0$$ *(2 marks)*

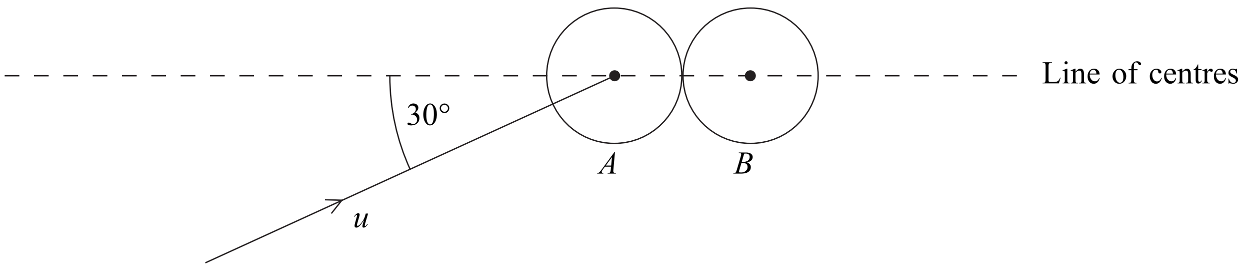
(ii) Hence, given that u and R are constants, show that, for $\tan\alpha$ to have real values, R must satisfy the inequality

$$R^2 \leqslant \frac{u^2(u^2 - 20)}{100}$$ *(2 marks)*

(iii) Given that $R = 5$, determine the minimum possible speed of projection. *(3 marks)*

6 A smooth spherical ball, A, is moving with speed u in a straight line on a smooth horizontal table when it hits an identical ball, B, which is at rest on the table.

Just before the collision, the direction of motion of A makes an angle of 30° with the line of the centres of the two balls, as shown in the diagram.



The coefficient of restitution between A and B is e.

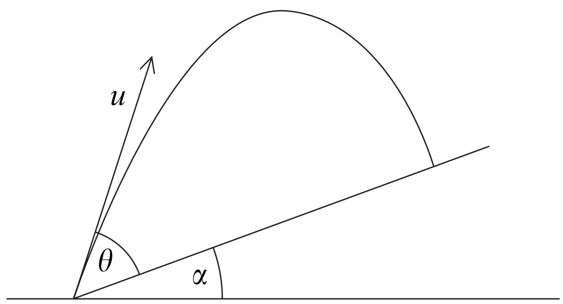
(a) Given that $\cos 30° = \frac{\sqrt{3}}{2}$, show that the speed of B immediately after the collision is

$$\frac{\sqrt{3}}{4}u(1+e)$$ *(5 marks)*

(b) Find, in terms of u and e, the components of the velocity of A, parallel and perpendicular to the line of centres, immediately after the collision. *(3 marks)*

(c) Given that $e = \frac{2}{3}$, find the angle that the velocity of A makes with the line of centres immediately after the collision. Give your answer to the nearest degree. *(3 marks)*

7 A particle is projected from a point on a plane which is inclined at an angle α to the horizontal. The particle is projected up the plane with velocity u at an angle θ above the plane. The motion of the particle is in a vertical plane containing a line of greatest slope of the inclined plane.



(a) Using the identity $\cos(A+B) = \cos A \cos B - \sin A \sin B$, show that the range up the plane is

$$\frac{2u^2 \sin\theta \cos(\theta+\alpha)}{g\cos^2\alpha}$$ *(8 marks)*

(b) Hence, using the identity $2\sin A \cos B = \sin(A+B) + \sin(A-B)$, show that, as θ varies, the range up the plane is a maximum when $\theta = \dfrac{\pi}{4} - \dfrac{\alpha}{2}$. *(3 marks)*

(c) Given that the particle strikes the plane at right angles, show that

$$2\tan\theta = \cot\alpha$$ *(4 marks)*

END OF QUESTIONS

General Certificate of Education
June 2008
Advanced Level Examination

MATHEMATICS **MM03**
Unit Mechanics 3

Friday 23 May 2008 9.00 am to 10.30 am

For this paper you must have:
- a 12-page answer book
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MM03.
- Answer **all** questions.
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- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Take $g = 9.8\ \text{m s}^{-2}$, unless stated otherwise.

Information
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 The speed, $v \text{ m s}^{-1}$, of a wave travelling along the surface of a sea is believed to depend on

the depth of the sea, d m,
the density of the water, $\rho \text{ kg m}^{-3}$,
the acceleration due to gravity, g, and
a dimensionless constant, k

so that

$$v = kd^{\alpha}\rho^{\beta}g^{\gamma}$$

where α, β and γ are constants.

By using dimensional analysis, show that $\beta = 0$ and find the values of α and γ. *(6 marks)*

2 The unit vectors **i** and **j** are directed due east and due north respectively.

Two runners, Albina and Brian, are running on level parkland with constant velocities of $(5\mathbf{i} - \mathbf{j}) \text{ m s}^{-1}$ and $(3\mathbf{i} + 4\mathbf{j}) \text{ m s}^{-1}$ respectively. Initially, the position vectors of Albina and Brian are $(-60\mathbf{i} + 160\mathbf{j})$ m and $(40\mathbf{i} - 90\mathbf{j})$ m respectively, relative to a fixed origin in the parkland.

(a) Write down the velocity of Brian relative to Albina. *(2 marks)*

(b) Find the position vector of Brian relative to Albina t seconds after they leave their initial positions. *(3 marks)*

(c) Hence determine whether Albina and Brian will collide if they continue running with the same velocities. *(3 marks)*

3 A particle of mass 0.2 kg lies at rest on a smooth horizontal table. A horizontal force of magnitude F newtons acts on the particle in a constant direction for 0.1 seconds. At time t seconds,

$$F = 5 \times 10^3 t^2, \quad 0 \leqslant t \leqslant 0.1$$

Find the value of t when the speed of the particle is 2 m s^{-1}. *(4 marks)*

4 Two smooth spheres, A and B, have equal radii and masses m and $2m$ respectively. The spheres are moving on a smooth horizontal plane. The sphere A has velocity $(4\mathbf{i} + 3\mathbf{j})$ when it collides with the sphere B which has velocity $(-2\mathbf{i} + 2\mathbf{j})$. After the collision, the velocity of B is $(\mathbf{i} + \mathbf{j})$.

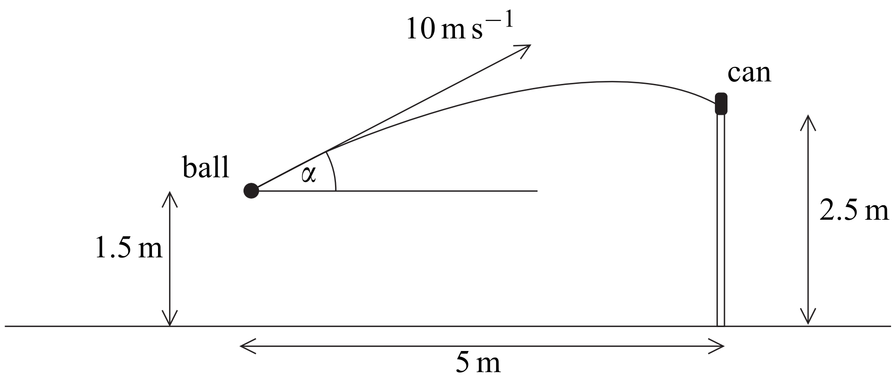
(a) Find the velocity of A immediately after the collision. *(3 marks)*

(b) Find the angle between the velocities of A and B immediately after the collision. *(3 marks)*

(c) Find the impulse exerted by B on A. *(3 marks)*

(d) State, as a vector, the direction of the line of centres of A and B when they collide. *(1 mark)*

5 A boy throws a small ball from a height of 1.5 m above horizontal ground with initial velocity $10\,\text{m s}^{-1}$ at an angle α above the horizontal. The ball hits a small can placed on a vertical wall of height 2.5 m, which is at a horizontal distance of 5 m from the initial position of the ball, as shown in the diagram.



(a) Show that α satisfies the equation

$$49\tan^2\alpha - 200\tan\alpha + 89 = 0$$ *(7 marks)*

(b) Find the **two** possible values of α, giving your answers to the nearest 0.1°. *(3 marks)*

(c) (i) To knock the can off the wall, the horizontal component of the velocity of the ball must be greater than $8\,\text{m s}^{-1}$.

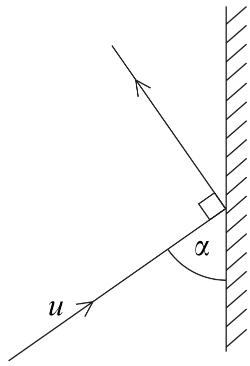
Show that, for one of the possible values of α found in part (b), the can will be knocked off the wall, and for the other, it will **not** be knocked off the wall. *(3 marks)*

(ii) Given that the can is knocked off the wall, find the direction in which the ball is moving as it hits the can. *(4 marks)*

Turn over ▶

6 A small smooth ball of mass m, moving on a smooth horizontal surface, hits a smooth vertical wall and rebounds. The coefficient of restitution between the wall and the ball is $\frac{3}{4}$.

Immediately before the collision, the ball has velocity u and the angle between the ball's direction of motion and the wall is α. The ball's direction of motion immediately after the collision is at right angles to its direction of motion before the collision, as shown in the diagram.



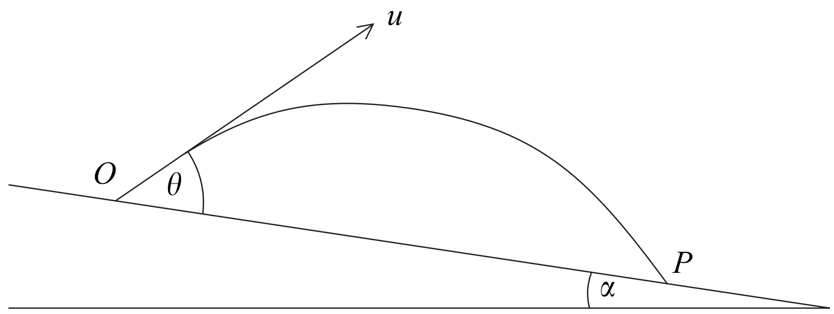
(a) Show that $\tan\alpha = \dfrac{2}{\sqrt{3}}$. *(5 marks)*

(b) Find, in terms of u, the speed of the ball immediately after the collision. *(2 marks)*

(c) The force exerted on the ball by the wall acts for 0.1 seconds.

Given that $m = 0.2\,\text{kg}$ and $u = 4\,\text{m}\,\text{s}^{-1}$, find the average force exerted by the wall on the ball. *(6 marks)*

7 A projectile is fired with speed u from a point O on a plane which is inclined at an angle α to the horizontal. The projectile is fired at an angle θ to the inclined plane and moves in a vertical plane through a line of greatest slope of the inclined plane. The projectile lands at a point P, lower down the inclined plane, as shown in the diagram.



(a) Find, in terms of u, g, θ and α, the greatest perpendicular distance of the projectile from the plane. *(4 marks)*

(b) (i) Find, in terms of u, g, θ and α, the time of flight from O to P. *(2 marks)*

(ii) By using the identity $\cos A \cos B + \sin A \sin B = \cos(A - B)$, show that the distance OP is given by $\dfrac{2u^2 \sin\theta \cos(\theta - \alpha)}{g\cos^2\alpha}$. *(6 marks)*

(iii) Hence, by using the identity $2\sin A \cos B = \sin(A + B) + \sin(A - B)$ or otherwise, show that, as θ varies, the maximum possible distance OP is $\dfrac{u^2}{g(1 - \sin\alpha)}$. *(5 marks)*

END OF QUESTIONS

General Certificate of Education
June 2009
Advanced Level Examination

MATHEMATICS **MM03**
Unit Mechanics 3

Wednesday 17 June 2009 9.00 am to 10.30 am

For this paper you must have:
- a 12-page answer book
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Time allowed: 1 hour 30 minutes

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Information
- The maximum mark for this paper is 75.
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Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 A ball of mass m is travelling vertically downwards with speed u when it hits a horizontal floor. The ball bounces vertically upwards to a height h.

It is thought that h depends on m, u, the acceleration due to gravity g, and a dimensionless constant k, such that

$$h = km^{\alpha}u^{\beta}g^{\gamma}$$

where α, β and γ are constants.

By using dimensional analysis, find the values of α, β and γ. *(5 marks)*

2 A particle is projected from a point O on a horizontal plane and has initial velocity components of $2\,\text{m}\,\text{s}^{-1}$ and $10\,\text{m}\,\text{s}^{-1}$ parallel to and perpendicular to the plane respectively. At time t seconds after projection, the horizontal and upward vertical distances of the particle from the point O are x metres and y metres respectively.

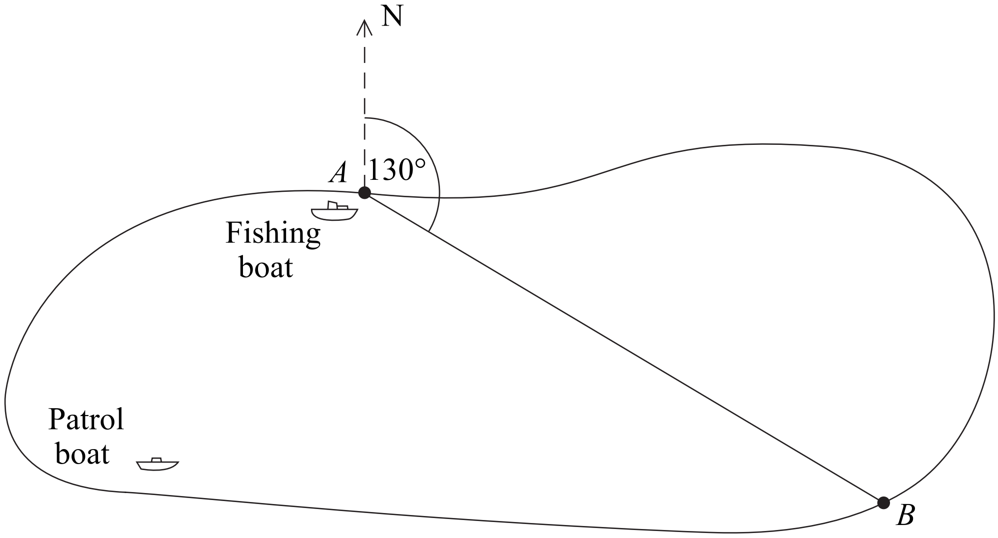
(a) Show that x and y satisfy the equation

$$y = -\frac{g}{8}x^2 + 5x$$ *(4 marks)*

(b) By using the equation in part (a), find the horizontal distance travelled by the particle whilst it is more than 1 metre above the plane. *(4 marks)*

(c) Hence find the time for which the particle is more than 1 metre above the plane. *(2 marks)*

3 A fishing boat is travelling between two ports, A and B, on the shore of a lake. The bearing of B from A is 130°. The fishing boat leaves A and travels directly towards B with speed $2\,\mathrm{m\,s^{-1}}$. A patrol boat on the lake is travelling with speed $4\,\mathrm{m\,s^{-1}}$ on a bearing of 040°.



(a) Find the velocity of the fishing boat relative to the patrol boat, giving your answer as a speed together with a bearing. *(5 marks)*

(b) When the patrol boat is 1500 m due west of the fishing boat, it changes direction in order to intercept the fishing boat in the shortest possible time.

(i) Find the bearing on which the patrol boat should travel in order to intercept the fishing boat. *(4 marks)*

(ii) Given that the patrol boat intercepts the fishing boat before it reaches B, find the time, in seconds, that it takes the patrol boat to intercept the fishing boat after changing direction. *(4 marks)*

(iii) State a modelling assumption necessary for answering this question, other than the boats being particles. *(1 mark)*

4 A particle of mass 0.5 kg is initially at rest. The particle then moves in a straight line under the action of a single force. This force acts in a constant direction and has magnitude $(t^3 + t)$ N, where t is the time, in seconds, for which the force has been acting.

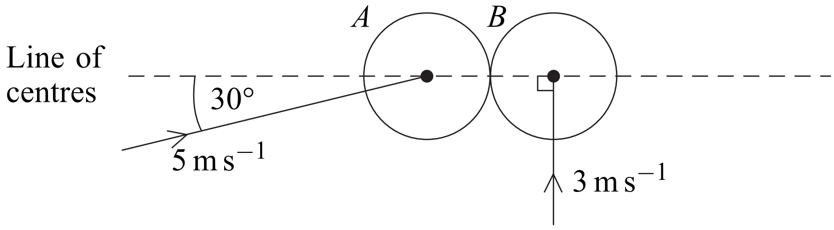
(a) Find the magnitude of the impulse exerted by the force on the particle between the times $t = 0$ and $t = 4$. *(3 marks)*

(b) Hence find the speed of the particle when $t = 4$. *(2 marks)*

(c) Find the time taken for the particle to reach a speed of $12\,\mathrm{m\,s^{-1}}$. *(5 marks)*

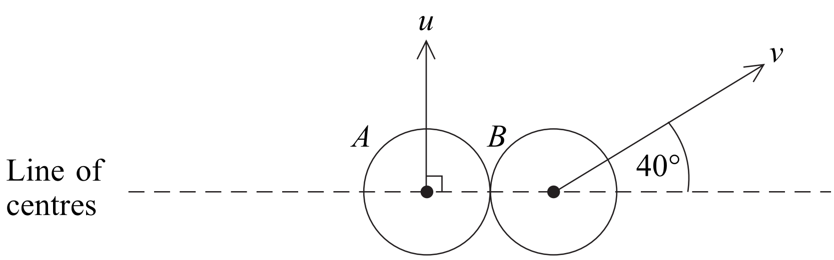
5 Two smooth spheres, A and B, of equal radii and different masses are moving on a smooth horizontal surface when they collide.

Just before the collision, A is moving with speed $5\,\text{m s}^{-1}$ at an angle of 30° to the line of centres of the spheres, and B is moving with speed $3\,\text{m s}^{-1}$ perpendicular to the line of centres, as shown in the diagram below.



Before collision

Immediately after the collision, A and B move with speeds u and v in directions which make angles of 90° and 40° respectively with the line of centres, as shown in the diagram below.



After collision

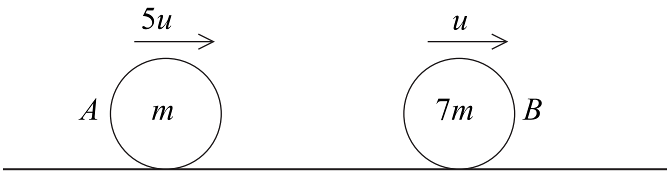
(a) Show that $v = 4.67\,\text{m s}^{-1}$, correct to three significant figures. *(3 marks)*

(b) Find the coefficient of restitution between the spheres. *(3 marks)*

(c) Given that the mass of A is 0.5 kg, show that the magnitude of the impulse exerted on A during the collision is 2.17 N s, correct to three significant figures. *(3 marks)*

(d) Find the mass of B. *(3 marks)*

6 A smooth sphere A of mass m is moving with speed $5u$ in a straight line on a smooth horizontal table. The sphere A collides directly with a smooth sphere B of mass $7m$, having the same radius as A and moving with speed u in the same direction as A. The coefficient of restitution between A and B is e.



Before collision

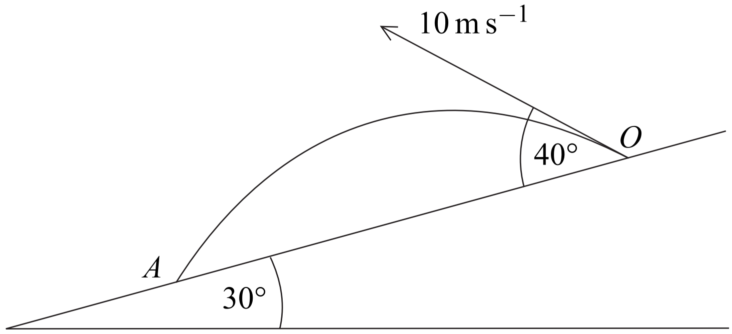
(a) Show that the speed of B after the collision is $\frac{u}{2}(e+3)$. *(5 marks)*

(b) Given that the direction of motion of A is reversed by the collision, show that $e > \frac{3}{7}$. *(4 marks)*

(c) Subsequently, B hits a wall fixed at right angles to the direction of motion of A and B. The coefficient of restitution between B and the wall is $\frac{1}{2}$. Given that after B rebounds from the wall both spheres move in the same direction and collide again, show also that $e < \frac{9}{13}$. *(4 marks)*

Turn over for the next question

7 A particle is projected from a point O on a smooth plane which is inclined at 30° to the horizontal. The particle is projected down the plane with velocity $10\,\text{m}\,\text{s}^{-1}$ at an angle of 40° above the plane and first strikes it at a point A. The motion of the particle is in a vertical plane containing a line of greatest slope of the inclined plane.



(a) Show that the time taken by the particle to travel from O to A is

$$\frac{20\sin 40°}{g\cos 30°}$$

(3 marks)

(b) Find the components of the velocity of the particle parallel to and perpendicular to the slope as it hits the slope at A. *(4 marks)*

(c) The coefficient of restitution between the slope and the particle is 0.5 . Find the speed of the particle as it rebounds from the slope. *(4 marks)*

END OF QUESTIONS

Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature										

For Examiner's Use	
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General Certificate of Education
Advanced Level Examination
June 2010

Mathematics MM03

Unit Mechanics 3

Tuesday 22 June 2010 1.30 pm to 3.00 pm

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Take $g = 9.8 \text{ m s}^{-2}$, unless stated otherwise.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions in the spaces provided.

1 A tank containing a liquid has a small hole in the bottom through which the liquid escapes. The speed, $u \text{ m s}^{-1}$, at which the liquid escapes is given by

$$u = CV\rho g$$

where $V \text{ m}^3$ is the volume of the liquid in the tank, $\rho \text{ kg m}^{-3}$ is the density of the liquid, g is the acceleration due to gravity and C is a constant.

By using dimensional analysis, find the dimensions of C. *(5 marks)*

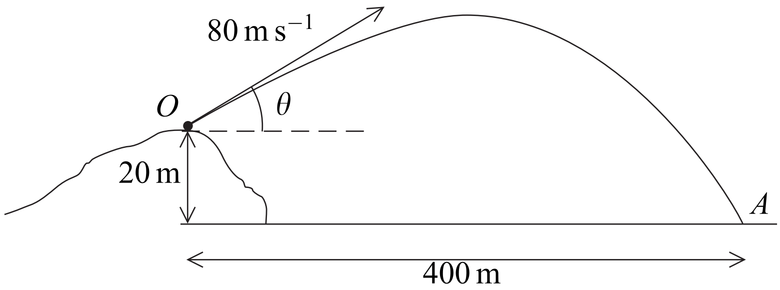
QUESTION PART REFERENCE

2 A projectile is fired from a point O on top of a hill with initial velocity $80\,\text{m s}^{-1}$ at an angle θ above the horizontal and moves in a vertical plane. The horizontal and upward vertical distances of the projectile from O are x metres and y metres respectively.

(a) (i) Show that, during the flight, the equation of the trajectory of the projectile is given by

$$y = x\tan\theta - \frac{gx^2}{12\,800}(1 + \tan^2\theta)$$ *(5 marks)*

(ii) The projectile hits a target A, which is 20 m vertically below O and 400 m horizontally from O.



Show that

$$49\tan^2\theta - 160\tan\theta + 41 = 0$$ *(2 marks)*

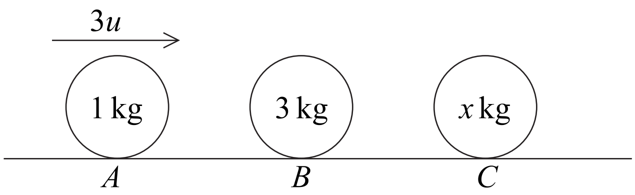
(b) (i) Find the two possible values of θ. Give your answers to the nearest 0.1°. *(3 marks)*

(ii) Hence find the shortest possible time of the flight of the projectile from O to A. *(2 marks)*

(c) State a necessary modelling assumption for answering part **(a)(i)**. *(1 mark)*

QUESTION PART REFERENCE

3 Three smooth spheres, A, B and C, of equal radii have masses 1 kg, 3 kg and x kg respectively. The spheres lie at rest in a straight line on a smooth horizontal surface with B between A and C. The sphere A is projected with speed $3u$ directly towards B and collides with it.



The coefficient of restitution between each pair of spheres is $\frac{1}{3}$.

(a) Show that A is brought to rest by the impact and find the speed of B immediately after the collision in terms of u. *(6 marks)*

(b) Subsequently, B collides with C.

Show that the speed of C immediately after the collision is $\dfrac{4u}{3+x}$.

Find the speed of B immediately after the collision in terms of u and x. *(6 marks)*

(c) Show that B will collide with A again if $x > 9$. *(2 marks)*

(d) Given that $x = 5$, find the magnitude of the impulse exerted on C by B in terms of u. *(2 marks)*

QUESTION PART REFERENCE

4 The unit vectors **i**, **j** and **k** are directed east, north and vertically upwards respectively.

At time $t = 0$, the position vectors of two small aeroplanes, A and B, relative to a fixed origin O are $(-60\mathbf{i} + 30\mathbf{k})$ km and $(-40\mathbf{i} + 10\mathbf{j} - 10\mathbf{k})$ km respectively.

The aeroplane A is flying with constant velocity $(250\mathbf{i} + 50\mathbf{j} - 100\mathbf{k})\,\text{km h}^{-1}$ and the aeroplane B is flying with constant velocity $(200\mathbf{i} + 25\mathbf{j} + 50\mathbf{k})\,\text{km h}^{-1}$.

(a) Write down the position vectors of A and B at time t hours. *(3 marks)*

(b) Show that the position vector of A relative to B at time t hours is $((-20 + 50t)\mathbf{i} + (-10 + 25t)\mathbf{j} + (40 - 150t)\mathbf{k})$ km. *(2 marks)*

(c) Show that A and B do not collide. *(4 marks)*

(d) Find the value of t when A and B are closest together. *(6 marks)*

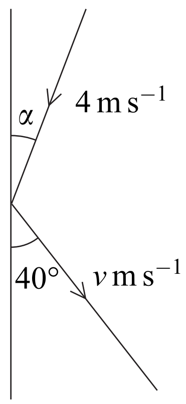
QUESTION PART REFERENCE

5 A smooth sphere is moving on a smooth horizontal surface when it strikes a smooth vertical wall and rebounds.

Immediately before the impact, the sphere is moving with speed $4\,\text{m}\,\text{s}^{-1}$ and the angle between the sphere's direction of motion and the wall is α.

Immediately after the impact, the sphere is moving with speed $v\,\text{m}\,\text{s}^{-1}$ and the angle between the sphere's direction of motion and the wall is 40°.

The coefficient of restitution between the sphere and the wall is $\frac{2}{3}$.



(a) Show that $\tan\alpha = \frac{3}{2}\tan 40°$. *(3 marks)*

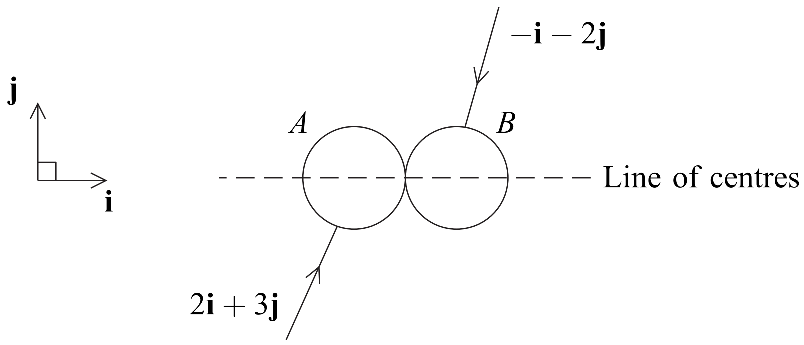
(b) Find the value of v. *(3 marks)*

QUESTION PART REFERENCE

6 Two smooth spheres, A and B, have equal radii and masses 1 kg and 2 kg respectively.

The sphere A is moving with velocity $(2\mathbf{i} + 3\mathbf{j})\,\mathrm{m\,s^{-1}}$ and the sphere B is moving with velocity $(-\mathbf{i} - 2\mathbf{j})\,\mathrm{m\,s^{-1}}$ on the same smooth horizontal surface.

The spheres collide when their line of centres is parallel to the unit vector $\mathbf{i}$, as shown in the diagram.



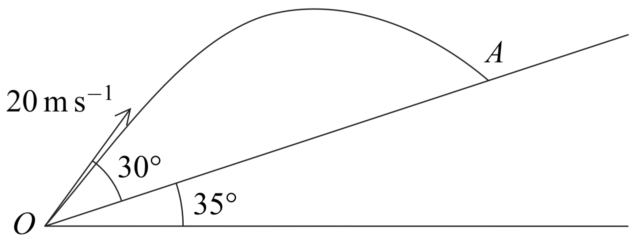
(a) Briefly state why the components of the velocities of A and B parallel to the unit vector $\mathbf{j}$ are not changed by the collision. *(1 mark)*

(b) The coefficient of restitution between the spheres is 0.5 .

Find the velocities of A and B immediately after the collision. *(6 marks)*

QUESTION PART REFERENCE

7 A ball is projected from a point O on a smooth plane which is inclined at an angle of 35° above the horizontal. The ball is projected with velocity $20\,\text{m}\,\text{s}^{-1}$ at an angle of 30° above the plane, as shown in the diagram. The motion of the ball is in a vertical plane containing a line of greatest slope of the inclined plane. The ball strikes the inclined plane at the point A.



(a) Find the components of the velocity of the ball, parallel and perpendicular to the plane, as it strikes the inclined plane at A. *(7 marks)*

(b) On striking the plane at A, the ball rebounds. The coefficient of restitution between the plane and the ball is $\frac{4}{5}$.

Show that the ball next strikes the plane at a point lower down than A. *(6 marks)*

QUESTION PART REFERENCE

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General Certificate of Education
Advanced Level Examination
June 2011

Mathematics MM03

Unit Mechanics 3

Wednesday 22 June 2011 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
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- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
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- Take $g = 9.8\ \text{m s}^{-2}$, unless stated otherwise.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Do not write outside the box

Answer **all** questions in the spaces provided.

1 A ball of mass 0.2 kg is hit directly by a bat. Just before the impact, the ball is travelling horizontally with speed $18\,\text{m s}^{-1}$. Just after the impact, the ball is travelling horizontally with speed $32\,\text{m s}^{-1}$ in the opposite direction.

(a) Find the magnitude of the impulse exerted on the ball. *(2 marks)*

(b) At time t seconds after the ball first comes into contact with the bat, the force exerted by the bat on the ball is $k(0.9t - 10t^2)$ newtons, where k is a constant and $0 \leqslant t \leqslant 0.09$. The bat stays in contact with the ball for 0.09 seconds.

Find the value of k. *(4 marks)*

QUESTION PART REFERENCE

2 The time, t, for a single vibration of a piece of taut string is believed to depend on

the length of the taut string, l,
the tension in the string, F,
the mass per unit length of the string, q, and
a dimensionless constant, k,

such that

$$t = kl^{\alpha}F^{\beta}q^{\gamma}$$

where α, β and γ are constants.

By using dimensional analysis, find the values of α, β and γ. *(5 marks)*

QUESTION PART REFERENCE

3 **(In this question, use $g = 10\,\text{m s}^{-2}$.)**

A golf ball is hit from a point O on a horizontal golf course with a velocity of $40\,\text{m s}^{-1}$ at an angle of elevation θ. The golf ball travels in a vertical plane through O. During its flight, the horizontal and upward vertical distances of the golf ball from O are x and y metres respectively.

(a) Show that the equation of the trajectory of the golf ball during its flight is given by

$$x^2 \tan^2 \theta - 320x \tan \theta + (x^2 + 320y) = 0$$

(6 marks)

(b) (i) The golf ball hits the top of a tree, which has a vertical height of 8 m and is at a horizontal distance of 150 m from O.

Find the two possible values of θ. *(5 marks)*

(ii) Which value of θ gives the shortest possible time for the golf ball to travel from O to the top of the tree? Give a reason for your choice of θ. *(2 marks)*

QUESTION PART REFERENCE

4 The unit vectors **i**, **j** and **k** are directed due east, due north and vertically upwards respectively.

A helicopter, A, is travelling in the direction of the vector $-2\mathbf{i} + 3\mathbf{j} + 6\mathbf{k}$ with constant speed $140\,\text{km}\,\text{h}^{-1}$. Another helicopter, B, is travelling in the direction of the vector $2\mathbf{i} - \mathbf{j} + 2\mathbf{k}$ with constant speed $60\,\text{km}\,\text{h}^{-1}$.

(a) Find the velocity of A relative to B. *(5 marks)*

(b) Initially, the position vectors of A and B are $(4\mathbf{i} - 2\mathbf{j} + 3\mathbf{k})$ km and $(-3\mathbf{i} + 6\mathbf{j} + 3\mathbf{k})$ km respectively, relative to a fixed origin.

Write down the position vector of A relative to B, t hours after they leave their initial positions. *(2 marks)*

(c) Find the distance between A and B when they are closest together. *(8 marks)*

QUESTION PART REFERENCE

5 A ball is dropped from a height of 2.5 m above a horizontal floor. The ball bounces repeatedly on the floor.

(a) Find the speed of the ball when it first hits the floor. *(2 marks)*

(b) The coefficient of restitution between the ball and the floor is e.

(i) Show that the time taken between the first contact of the ball with the floor and the second contact of the ball with the floor is $\frac{10e}{7}$ seconds. *(3 marks)*

(ii) Find, in terms of e, the time taken between the second contact and the third contact of the ball with the floor. *(1 mark)*

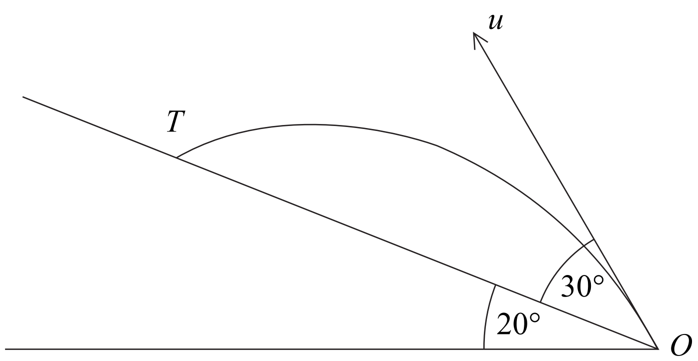
(c) Find, in terms of e, the total vertical distance travelled by the ball from when it is dropped until its third contact with the floor. *(5 marks)*

(d) State a modelling assumption for answering this question, other than the ball being a particle. *(1 mark)*

QUESTION PART REFERENCE

6 A projectile is fired from a point O on a plane which is inclined at an angle of $20°$ to the horizontal. The projectile is fired up the plane with velocity $u\,\text{m s}^{-1}$ at an angle of $30°$ to the inclined plane. The projectile travels in a vertical plane containing a line of greatest slope of the inclined plane.

The projectile hits a target T on the inclined plane.



(a) Given that $OT = 200\,\text{m}$, determine the value of u. *(7 marks)*

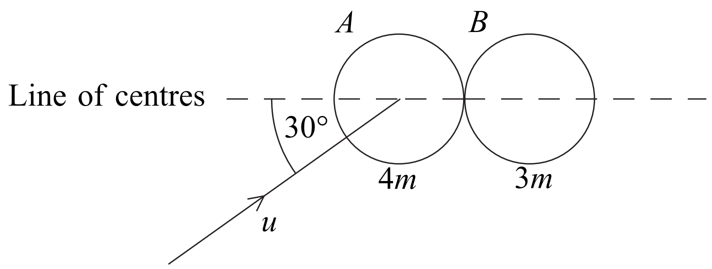
(b) Find the greatest perpendicular distance of the projectile from the inclined plane. *(4 marks)*

QUESTION PART REFERENCE

7 Two smooth spheres, A and B, have equal radii and masses $4m$ and $3m$ respectively. The sphere A is moving on a smooth horizontal surface and collides with the sphere B, which is stationary on the same surface.

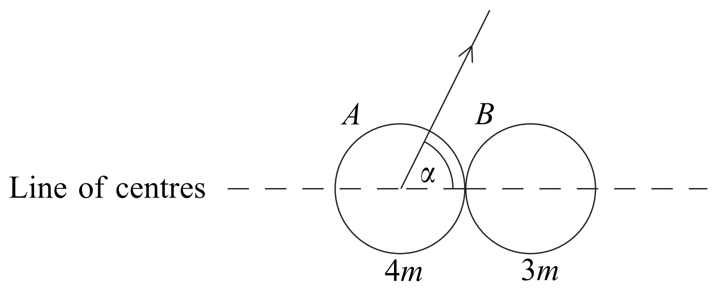
Just before the collision, A is moving with speed u at an angle of 30° to the line of centres, as shown in the diagram below.

Before collision



Immediately after the collision, the direction of motion of A makes an angle α with the line of centres, as shown in the diagram below.

After collision



The coefficient of restitution between the spheres is $\frac{5}{9}$.

(a) Find the value of α. *(10 marks)*

(b) Find, in terms of m and u, the magnitude of the impulse exerted on B during the collision. *(3 marks)*

QUESTION PART REFERENCE

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General Certificate of Education
Advanced Level Examination
June 2012

Mathematics MM03

Unit Mechanics 3

Friday 22 June 2012 1.30 pm to 3.00 pm

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
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- Take $g = 9.8\ \text{m s}^{-2}$, unless stated otherwise.

Information
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- The maximum mark for this paper is 75.

Advice
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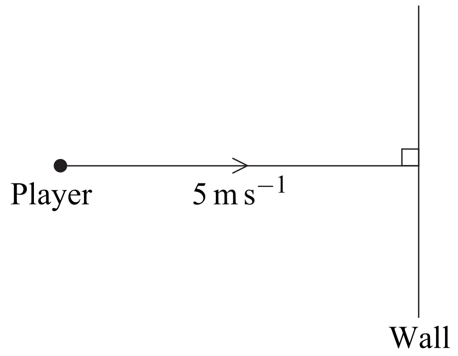
P47914/Jun12/MM03 6/6/6/

Answer **all** questions.

Answer each question in the space provided for that question.

1 An ice-hockey player has mass 60 kg. He slides in a straight line at a constant speed of $5\,\mathrm{m\,s^{-1}}$ on the horizontal smooth surface of an ice rink towards the vertical perimeter wall of the rink, as shown in the diagram.



The player collides directly with the wall, and remains in contact with the wall for 0.5 seconds.

At time t seconds after coming into contact with the wall, the force exerted by the wall on the player is $4 \times 10^4\, t^2\,(1 - 2t)$ newtons, where $0 \leqslant t \leqslant 0.5$.

(a) Find the magnitude of the impulse exerted by the wall on the player. *(4 marks)*

(b) The player rebounds from the wall. Find the player's speed immediately after the collision. *(3 marks)*

QUESTION PART REFERENCE

Answer space for question 1

2 A pile driver of mass m_1 falls from a height h onto a pile of mass m_2, driving the pile a distance s into the ground. The pile driver remains in contact with the pile after the impact. A resistance force R opposes the motion of the pile into the ground.

Elizabeth finds an expression for R as

$$R = \frac{g}{s}\left[s(m_1 + m_2) + \frac{h(m_1)^2}{m_1 + m_2}\right]$$

where g is the acceleration due to gravity.

Determine whether the expression is dimensionally consistent. *(4 marks)*

QUESTION PART REFERENCE

Answer space for question 2

3 **(In this question, take $g = 10\,\mathrm{m\,s^{-2}}$.)**

A projectile is fired from a point O with speed u at an angle of elevation α above the horizontal so as to pass through a point P. The projectile travels in a vertical plane through O and P. The point P is at a horizontal distance $2k$ from O and at a vertical distance k above O.

(a) Show that α satisfies the equation

$$20k\tan^2\alpha - 2u^2\tan\alpha + u^2 + 20k = 0$$

(7 marks)

(b) Deduce that

$$u^4 - 20ku^2 - 400k^2 \geqslant 0$$

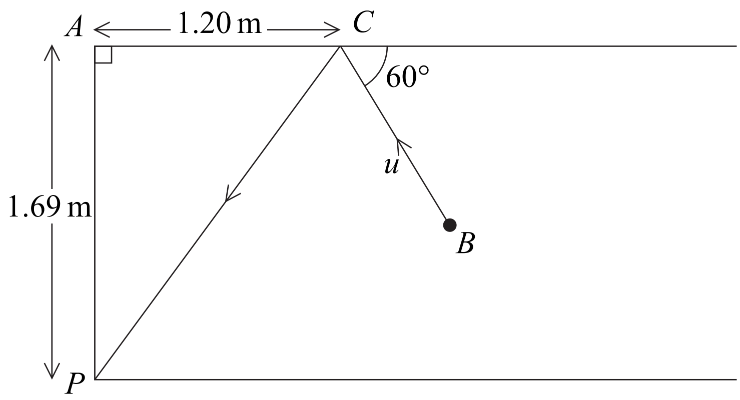
(3 marks)

QUESTION PART REFERENCE

Answer space for question 3

4 The diagram shows part of a horizontal snooker table of width 1.69 m.

A player strikes the ball B directly, and it moves in a straight line. The ball hits the cushion of the table at C before rebounding and moving to the pocket at P at the corner of the table, as shown in the diagram. The point C is 1.20 m from the corner A of the table. The ball has mass 0.15 kg and, immediately before the collision with the cushion, it has velocity u in a direction inclined at 60° to the cushion. The **table** and the **cushion** are modelled as smooth.



(a) Find the coefficient of restitution between the ball and the cushion. *(5 marks)*

(b) Show that the magnitude of the impulse on the cushion at C is approximately $0.236u$. *(4 marks)*

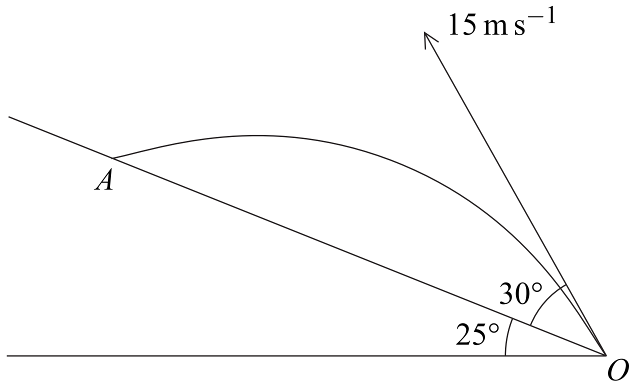
(c) Find, in terms of u, the time taken between the ball hitting the cushion at C and entering the pocket at P. *(3 marks)*

(d) Explain how you have used the assumption that the cushion is smooth in your answers. *(1 mark)*

QUESTION PART REFERENCE

Answer space for question 4

5 A particle is projected from a point O on a smooth plane, which is inclined at 25° to the horizontal. The particle is projected up the plane with velocity 15 m s^{-1} at an angle 30° above the plane. The particle strikes the plane for the first time at a point A. The motion of the particle is in a vertical plane containing a line of greatest slope of the inclined plane.



(a) Find the time taken by the particle to travel from O to A. *(4 marks)*

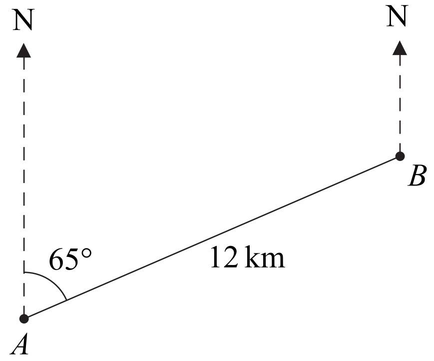
(b) The coefficient of restitution between the particle and the inclined plane is $\frac{2}{3}$.

Find the speed of the particle as it rebounds from the inclined plane at A. *(8 marks)*

QUESTION PART REFERENCE

Answer space for question 5

6 At noon, two ships, A and B, are a distance of 12 km apart, with B on a bearing of 065° from A. The ship B travels due north at a constant speed of $10\,\text{km h}^{-1}$. The ship A travels at a constant speed of $18\,\text{km h}^{-1}$.



(a) Find the direction in which A should travel in order to intercept B. Give your answer as a bearing. *(4 marks)*

(b) In fact, the ship A actually travels on a bearing of 065°.

(i) Find the distance between the ships when they are closest together. *(7 marks)*

(ii) Find the time when the ships are closest together. *(3 marks)*

QUESTION PART REFERENCE

Answer space for question 6

7 Two smooth spheres, A and B, have equal radii and masses $2m$ kg and m kg respectively. The spheres are moving on a smooth horizontal plane. The sphere A has velocity $(3\mathbf{i} + \mathbf{j})\ \mathrm{m\,s^{-1}}$ when it collides with the sphere B, which has velocity $(2\mathbf{i} - 5\mathbf{j})\ \mathrm{m\,s^{-1}}$. Immediately after the collision, the velocity of the sphere B is $(2\mathbf{i} + \mathbf{j})\ \mathrm{m\,s^{-1}}$.

(a) Find the velocity of A immediately after the collision. *(3 marks)*

(b) Show that the impulse exerted on B in the collision is $(6m\mathbf{j})$ Ns. *(3 marks)*

(c) Find the coefficient of restitution between the two spheres. *(4 marks)*

(d) After the collision, each sphere moves in a straight line with constant speed. Given that the radius of each sphere is 0.05 m, find the time taken, from the collision, until the centres of the spheres are 1.10 m apart. *(5 marks)*

QUESTION PART REFERENCE

Answer space for question 7

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General Certificate of Education
Advanced Level Examination
June 2013

Mathematics MM03

Unit Mechanics 3

Tuesday 18 June 2013 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
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- Take $g = 9.8\,\text{m}\,\text{s}^{-2}$, unless stated otherwise.

Information
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- The maximum mark for this paper is 75.

Advice
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- You do not necessarily need to use all the space provided.

P59194/Jun13/MM03 6/6/

Answer **all** questions.

Answer each question in the space provided for that question.

1 A stone, of mass 2 kg, is moving in a straight line on a smooth horizontal sheet of ice under the action of a single force which acts in the direction of motion. At time t seconds, the force has magnitude $(3t + 1)$ newtons, $0 \leqslant t \leqslant 3$.

When $t = 0$, the stone has velocity $1\,\text{m}\,\text{s}^{-1}$.
When $t = T$, the stone has velocity $5\,\text{m}\,\text{s}^{-1}$.

Find the value of T. *(6 marks)*

QUESTION PART REFERENCE

Answer space for question 1

2 A car has mass m and travels up a slope which is inclined at an angle θ to the horizontal. The car reaches a maximum speed v at a height h above its initial position. A constant resistance force R opposes the motion of the car, which has a maximum engine power output P.

Neda finds a formula for P as

$$P = mgv\sin\theta + Rv + \frac{1}{2}mv^3\frac{\sin\theta}{h}$$

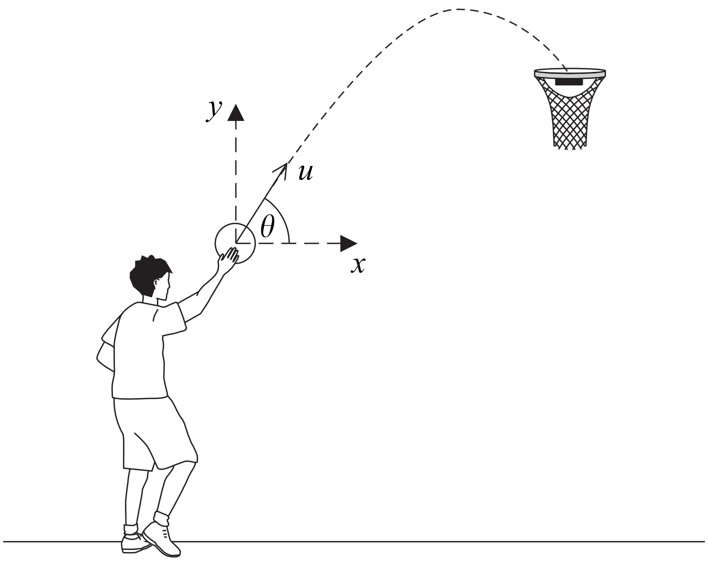
where g is the acceleration due to gravity.

Given that the engine power output may be measured in newton metres per second, determine whether the formula is dimensionally consistent. *(6 marks)*

QUESTION PART REFERENCE

Answer space for question 2

3 A player projects a basketball with speed $u\,\text{m}\,\text{s}^{-1}$ at an angle θ above the horizontal. The basketball travels in a vertical plane through the point of projection and goes into the basket. During the motion, the horizontal and upward vertical displacements of the basketball from the point of projection are x metres and y metres respectively.



(a) Find an expression for y in terms of x, u, g and $\tan\theta$. *(6 marks)*

(b) The player projects the basketball with speed $8\,\text{m}\,\text{s}^{-1}$ from a point 0.5 metres vertically below and 5 metres horizontally from the basket.

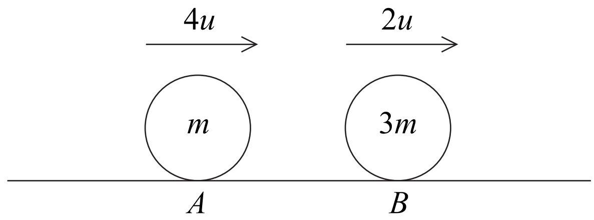
(i) Show that the two possible values of θ are approximately 63.1° and 32.6°, correct to three significant figures. *(5 marks)*

(ii) Given that the player projects the basketball at 63.1° to the horizontal, find the direction of the motion of the basketball as it enters the basket. Give your answer to the nearest degree. *(4 marks)*

(c) State a modelling assumption needed for answering parts **(a)** and **(b)** of this question. *(1 mark)*

QUESTION PART REFERENCE	**Answer space for question 3**

4 A smooth sphere A, of mass m, is moving with speed $4u$ in a straight line on a smooth horizontal table. A smooth sphere B, of mass $3m$, has the same radius as A and is moving on the table with speed $2u$ in the same direction as A.



The sphere A collides directly with sphere B. The coefficient of restitution between A and B is e.

(a) Find, in terms of u and e, the speeds of A and B immediately after the collision. *(6 marks)*

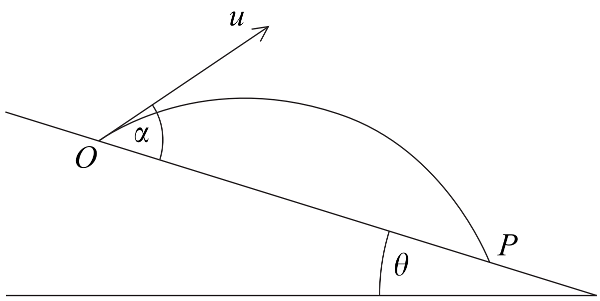
(b) Show that the speed of B after the collision cannot be greater than $3u$. *(2 marks)*

(c) Given that $e = \frac{2}{3}$, find, in terms of m and u, the magnitude of the impulse exerted on B in the collision. *(3 marks)*

QUESTION PART REFERENCE

Answer space for question 4

5 A particle is projected from a point O on a plane which is inclined at an angle θ to the horizontal. The particle is projected down the plane with velocity u at an angle α above the plane. The particle first strikes the plane at a point P, as shown in the diagram. The motion of the particle is in a vertical plane containing a line of greatest slope of the inclined plane.



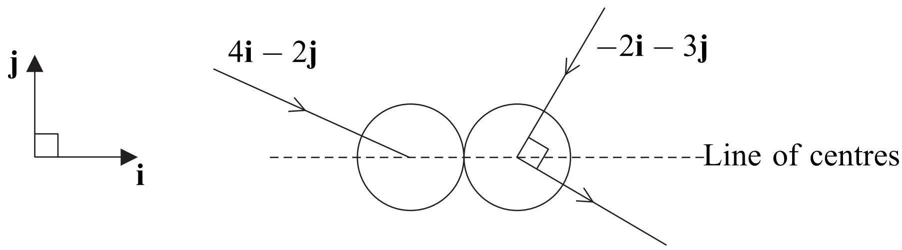
(a) Given that the time of flight from O to P is T, find an expression for u in terms of θ, α, T and g. *(4 marks)*

(b) Using the identity $\cos(X - Y) = \cos X \cos Y + \sin X \sin Y$, show that the distance OP is given by $\dfrac{2u^2 \sin\alpha \cos(\alpha - \theta)}{g \cos^2 \theta}$. *(6 marks)*

QUESTION PART REFERENCE

Answer space for question 5

6 Two smooth spheres, A and B, have equal radii and masses 4 kg and 2 kg respectively. The sphere A is moving with velocity $(4\mathbf{i} - 2\mathbf{j})\ \text{m s}^{-1}$ and the sphere B is moving with velocity $(-2\mathbf{i} - 3\mathbf{j})\ \text{m s}^{-1}$ on the same smooth horizontal surface. The spheres collide when their line of centres is parallel to unit vector $\mathbf{i}$. The direction of motion of B is changed through 90° by the collision, as shown in the diagram.



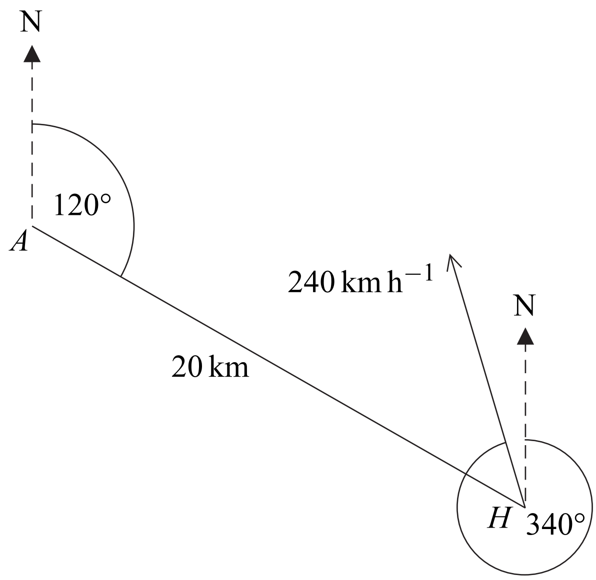
(a) Show that the velocity of B immediately after the collision is $\left(\frac{9}{2}\mathbf{i} - 3\mathbf{j}\right)\ \text{m s}^{-1}$. *(4 marks)*

(b) Find the coefficient of restitution between the spheres. *(5 marks)*

(c) Find the impulse exerted on B during the collision. State the units of your answer. *(3 marks)*

QUESTION PART REFERENCE	**Answer space for question 6**

7 From an aircraft A, a helicopter H is observed 20 km away on a bearing of 120°. The helicopter H is travelling horizontally with a constant speed 240 km h^{-1} on a bearing of 340°. The aircraft A is travelling with constant speed $v_A\text{ km h}^{-1}$ in a straight line and at the same altitude as H.



(a) Given that $v_A = 200$:

(i) find a bearing, to one decimal place, on which A could travel in order to intercept H; *(5 marks)*

(ii) find the time, in minutes, that it would take A to intercept H on this bearing. *(4 marks)*

(b) Given that $v_A = 150$, find the bearing on which A should travel in order to approach H as closely as possible. Give your answer to one decimal place. *(5 marks)*

QUESTION PART REFERENCE

Answer space for question 7

Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature										

For Examiner's Use	
Examiner's Initials	
Question	Mark
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TOTAL	

General Certificate of Education
Advanced Level Examination
June 2014

Mathematics — MM03

Unit Mechanics 3

Friday 6 June 2014 1.30 pm to 3.00 pm

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Take $g = 9.8\,\text{m}\,\text{s}^{-2}$, unless stated otherwise.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

Answer **all** questions.

Answer each question in the space provided for that question.

1 A tennis ball is projected from a point O with a velocity of $(4\sqrt{3}\mathbf{i} + 4\mathbf{j})\,\mathrm{m\,s^{-1}}$, where $\mathbf{i}$ and $\mathbf{j}$ are horizontal and vertical unit vectors respectively. The ball travels in a vertical plane through O which is $30\,\mathrm{cm}$ above the horizontal surface of a tennis court. During its flight, the horizontal and upward vertical distances of the ball from O are x metres and y metres respectively.

Model the ball as a particle.

(a) Show that, during the flight, the equation of the trajectory of the ball is given by

$$y = \frac{x}{\sqrt{3}} - \frac{49x^2}{480}$$

[4 marks]

(b) The ball hits a vertical net at a point A. The net is at a horizontal distance of $4\,\mathrm{m}$ from O.

Determine the height of the point A, above the surface of the tennis court. Give your answer to the nearest centimetre.

[2 marks]

(c) State a modelling assumption, other than the ball being a particle, that you need to make to answer this question.

[1 mark]

QUESTION PART REFERENCE

Answer space for question 1

2 A rod, of length x m and moment of inertia I kg m^2, is free to rotate in a vertical plane about a fixed smooth horizontal axis through one end.

When the rod is hanging at rest, its lower end receives an impulse of magnitude J Ns, which is just sufficient for the rod to complete full revolutions.

It is thought that there is a relationship between J, x, I, the acceleration due to gravity g m s^{-2} and a dimensionless constant k, such that

$$J = kx^{\alpha}I^{\beta}g^{\gamma}$$

where α, β and γ are constants.

Find the values of α, β and γ for which this relationship is dimensionally consistent.

[6 marks]

QUESTION PART REFERENCE

Answer space for question 2

3 A particle of mass $0.5\,\text{kg}$ is moving in a straight line on a smooth horizontal surface.

The particle is then acted on by a horizontal force for 3 seconds. This force acts in the direction of motion of the particle and at time t seconds has magnitude $(3t + 1)\ \text{N}$.

When $t = 0$, the velocity of the particle is $4\,\text{m}\,\text{s}^{-1}$.

(a) Find the magnitude of the impulse of the force on the particle between the times $t = 0$ and $t = 3$.

[3 marks]

(b) Hence find the velocity of the particle when $t = 3$.

[2 marks]

(c) Find the value of t when the velocity of the particle is $20\,\text{m}\,\text{s}^{-1}$.

[4 marks]

QUESTION PART REFERENCE

Answer space for question 3

4 Two boats, A and B, are moving on straight courses with constant speeds. At noon, A and B have position vectors $(\mathbf{i} + 2\mathbf{j})\,\text{km}$ and $(-\mathbf{i} + \mathbf{j})\,\text{km}$ respectively relative to a lighthouse. Thirty minutes later, the position vectors of A and B are $(-\mathbf{i} + 3\mathbf{j})\,\text{km}$ and $(2\mathbf{i} - \mathbf{j})\,\text{km}$ respectively relative to the lighthouse.

(a) Find the velocity of A relative to B in the form $(m\mathbf{i} + n\mathbf{j})\,\text{km h}^{-1}$, where m and n are integers.

[4 marks]

(b) The position vector of A relative to B at time t hours after noon is $\mathbf{r}\,\text{km}$.

Show that

$$\mathbf{r} = (2 - 10t)\mathbf{i} + (1 + 6t)\mathbf{j}$$

[3 marks]

(c) Determine the value of t when A and B are closest together.

[5 marks]

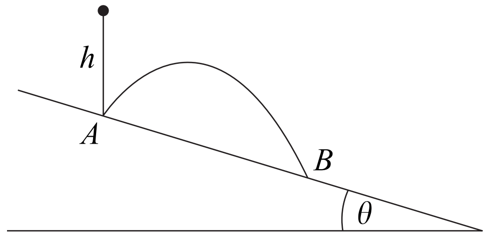
(d) Find the shortest distance between A and B.

[2 marks]

QUESTION PART REFERENCE

Answer space for question 4

5 A small smooth ball is dropped from a height of h above a point A on a fixed smooth plane inclined at an angle θ to the horizontal. The ball falls vertically and collides with the plane at the point A. The ball rebounds and strikes the plane again at a point B, as shown in the diagram. The points A and B lie on a line of greatest slope of the inclined plane.



(a) Explain whether or not the component of the velocity of the ball parallel to the plane is changed by the collision.

[2 marks]

(b) The coefficient of restitution between the ball and the plane is e.

Find, in terms of h, θ, e and g, the components of the velocity of the ball parallel to and perpendicular to the plane immediately after the collision.

[3 marks]

(c) Show that the distance AB is given by

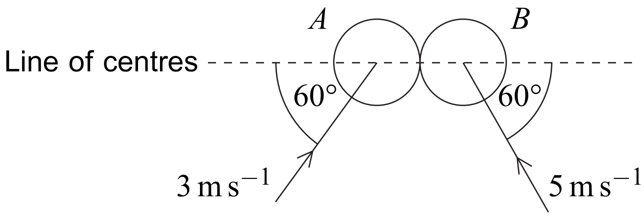
$$4he(e+1)\sin\theta$$

[7 marks]

QUESTION PART REFERENCE	**Answer space for question 5**

6 Two smooth spheres, A and B, have equal radii and masses $2\,\mathrm{kg}$ and $4\,\mathrm{kg}$ respectively.

The spheres are moving on a smooth horizontal surface and collide. As they collide, A has velocity $3\,\mathrm{m\,s^{-1}}$ at an angle of $60°$ to the line of centres of the spheres, and B has velocity $5\,\mathrm{m\,s^{-1}}$ at an angle of $60°$ to the line of centres, as shown in the diagram.



Just after the collision, B moves in a direction perpendicular to the line of centres.

(a) Find the speed of A immediately after the collision.

[6 marks]

(b) Find the acute angle, correct to the nearest degree, between the velocity of A and the line of centres immediately after the collision.

[2 marks]

(c) Find the coefficient of restitution between the spheres.

[2 marks]

(d) Find the magnitude of the impulse exerted on B during the collision.

[2 marks]

QUESTION PART REFERENCE

Answer space for question 6

7 Two small smooth spheres, A and B, are the same size and have masses $2m$ and m respectively. Initially, the spheres are at rest on a smooth horizontal surface. The sphere A receives an impulse of magnitude J and moves with speed $2u$ directly towards B.

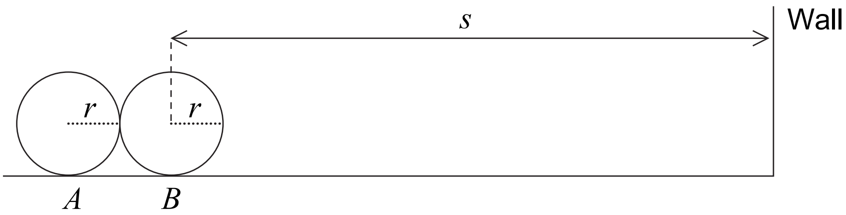
(a) Find J in terms of m and u.

[2 marks]

(b) The sphere A collides directly with B. The coefficient of restitution between A and B is $\frac{2}{3}$. Find, in terms of u, the speeds of A and B immediately after the collision.

[5 marks]

(c) At the instant of collision, the centre of B is at a distance s from a fixed smooth vertical wall which is at right angles to the direction of motion of A and B, as shown in the diagram.

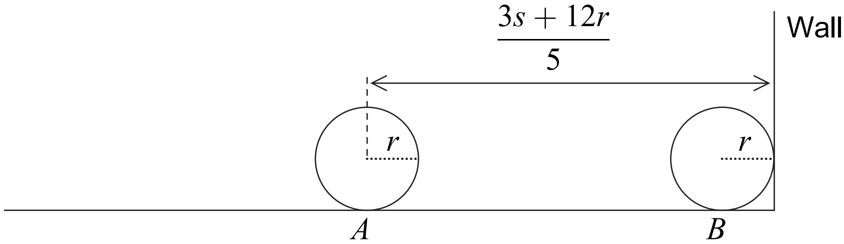


Subsequently, B collides with the wall. The radius of each sphere is r.

Show that the distance of the centre of A from the wall at the instant that B hits the wall is $\dfrac{3s + 12r}{5}$.

[4 marks]

(d) The diagram below shows the positions of A and B when B hits the wall.



The sphere B collides with A again after rebounding from the wall. The coefficient of restitution between B and the wall is $\frac{2}{5}$.

Find the distance of the **centre of** $\boldsymbol{B}$ from the wall at the instant when A and B collide again.

[4 marks]

Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature										

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TOTAL	

General Certificate of Education
Advanced Level Examination
June 2015

Mathematics MM03

Unit Mechanics 3

Wednesday 3 June 2015 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
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- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
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- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Take $g = 9.8\ \text{m s}^{-2}$, unless stated otherwise.

Information
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- The maximum mark for this paper is 75.

Advice
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- You do not necessarily need to use all the space provided.

Answer **all** questions.

Answer each question in the space provided for that question.

1 A formula for calculating the lift force acting on the wings of an aircraft moving through the air is of the form

$$F = k\, v^{\alpha} A^{\beta} \rho^{\gamma}$$

where F is the lift force in newtons,
k is a dimensionless constant,
v is the air velocity in $\mathrm{m\,s^{-1}}$,
A is the surface area of the aircraft's wings in $\mathrm{m^2}$, and
ρ is the density of the air in $\mathrm{kg\,m^{-3}}$.

By using dimensional analysis, find the values of the constants α, β and γ.

[6 marks]

QUESTION PART REFERENCE

Answer space for question 1

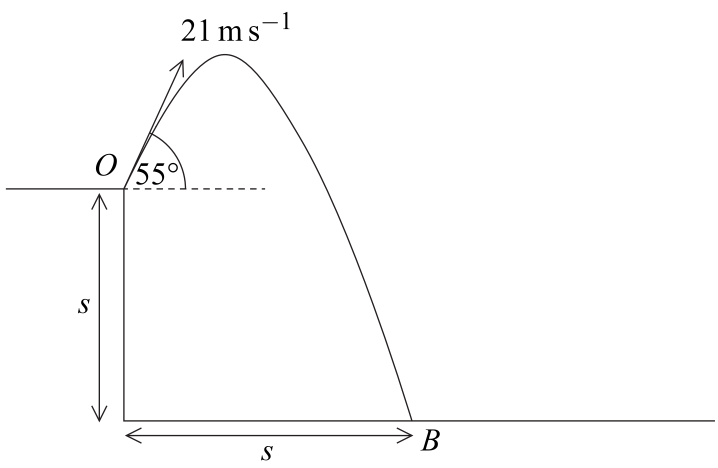
2 A projectile is launched from a point O on top of a cliff with initial velocity $u\,\mathrm{m\,s^{-1}}$ at an angle of elevation α and moves in a vertical plane. During the motion, the position vector of the projectile relative to the point O is $(x\mathbf{i} + y\mathbf{j})$ metres where $\mathbf{i}$ and $\mathbf{j}$ are horizontal and vertical unit vectors respectively.

(a) Show that, during the motion, the equation of the trajectory of the projectile is given by

$$y = x\tan\alpha - \frac{4.9x^2}{u^2\cos^2\alpha}$$

[5 marks]

(b) When $u = 21$ and $\alpha = 55°$, the projectile hits a small buoy B. The buoy is at a distance s metres vertically below O and at a distance s metres horizontally from O, as shown in the diagram.



(i) Find the value of s.

[3 marks]

(ii) Find the acute angle between the velocity of the projectile and the horizontal just before the projectile hits B, giving your answer to the nearest degree.

[5 marks]

QUESTION PART REFERENCE

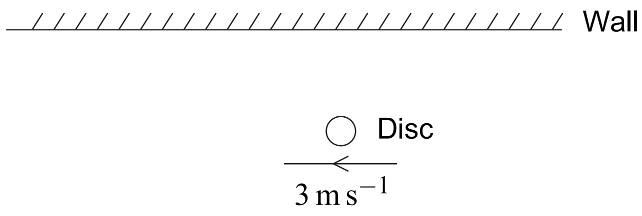
Answer space for question 2

3 A disc of mass $0.5\,\mathrm{kg}$ is moving with speed $3\,\mathrm{m\,s^{-1}}$ on a smooth horizontal surface when it receives a horizontal impulse in a direction **perpendicular to its direction of motion**. Immediately after the impulse, the disc has speed $5\,\mathrm{m\,s^{-1}}$.

(a) Find the magnitude of the impulse received by the disc.

[3 marks]

(b) Before the impulse, the disc is moving parallel to a smooth vertical wall, as shown in the diagram.



After the impulse, the disc hits the wall and rebounds with speed $3\sqrt{2}\,\mathrm{m\,s^{-1}}$.

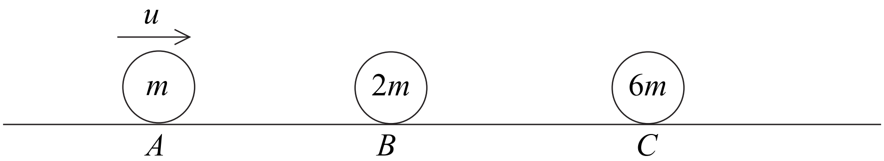
Find the coefficient of restitution between the disc and the wall.

[4 marks]

QUESTION PART REFERENCE

Answer space for question 3

4 Three uniform smooth spheres, A, B and C, have equal radii and masses m, $2m$ and $6m$ respectively. The spheres lie at rest in a straight line on a smooth horizontal surface with B between A and C. The sphere A is projected with speed u directly towards B and collides with it.



The coefficient of restitution between A and B is $\frac{2}{3}$.

(a) (i) Show that the speed of B immediately after the collision is $\frac{5}{9}u$.

(ii) Find, in terms of u, the speed of A immediately after the collision.

[6 marks]

(b) Subsequently, B collides with C. The coefficient of restitution between B and C is e.

Show that B will collide with A again if $e > k$, where k is a constant to be determined.

[8 marks]

(c) Explain why it is not necessary to model the spheres as particles in this question.

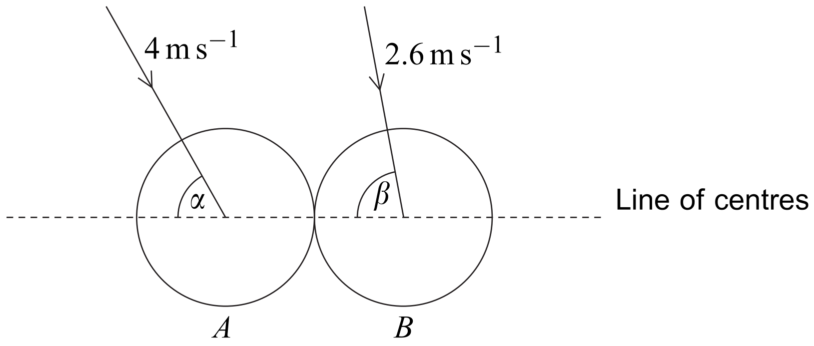
[2 marks]

QUESTION PART REFERENCE

Answer space for question 4

Do not write outside the box

5 Two smooth spheres, A and B, have equal radii and masses $2\,\text{kg}$ and $1\,\text{kg}$ respectively. The spheres move on a smooth horizontal surface and collide. As they collide, A has velocity $4\,\text{m s}^{-1}$ in a direction inclined at an angle α to the line of centres of the spheres, and B has velocity $2.6\,\text{m s}^{-1}$ in a direction inclined at an angle β to the line of centres, as shown in the diagram.



The coefficient of restitution between A and B is $\frac{4}{7}$.

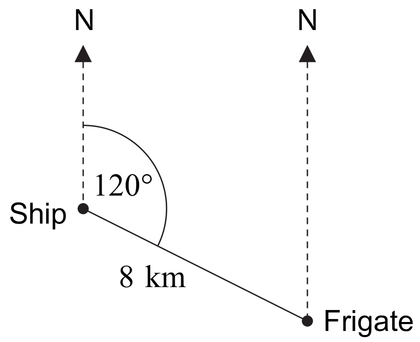
Given that $\sin\alpha = \frac{4}{5}$ and $\sin\beta = \frac{12}{13}$, find the speeds of A and B immediately after the collision.

[11 marks]

QUESTION PART REFERENCE

Answer space for question 5

6 A ship and a navy frigate are a distance of $8\,\text{km}$ apart, with the frigate on a bearing of $120°$ from the ship, as shown in the diagram.



The ship travels due east at a constant speed of $50\,\text{km h}^{-1}$. The frigate travels at a constant speed of $35\,\text{km h}^{-1}$.

(a) (i) Find the bearings, to the nearest degree, of the two possible directions in which the frigate can travel to intercept the ship.

[5 marks]

(ii) Hence find the **shorter** of the two possible times for the frigate to intercept the ship.

[5 marks]

(b) The captain of the frigate would like the frigate to travel at less than $35\,\text{km h}^{-1}$.

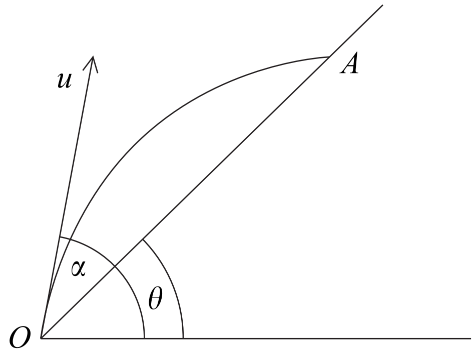
Find the minimum speed at which the frigate can travel to intercept the ship.

[3 marks]

QUESTION PART REFERENCE

Answer space for question 6

7 A particle is projected from a point O on a plane which is inclined at an angle θ to the horizontal. The particle is projected up the plane with velocity u at an angle α **above the horizontal**. The particle strikes the plane for the first time at a point A. The motion of the particle is in a vertical plane which contains the line OA.



(a) Find, in terms of u, θ, α and g, the time taken by the particle to travel from O to A.

[4 marks]

(b) The particle is moving horizontally when it strikes the plane at A.

By using the identity $\sin(P-Q) = \sin P \cos Q - \cos P \sin Q$, or otherwise, show that

$$\tan\alpha = k\tan\theta$$

where k is a constant to be determined.

[5 marks]

QUESTION PART REFERENCE

Answer space for question 7