OCR Maths M2

Topic Questions from Papers

Centre of Mass

Answers

1	(i)	use of h/4	B1			
		com vert above lowest pt of contact	B1		can be implied	
		r = 5 x tan24°	M1			
		r = 2.2	A1	4	2.226	
	(ii)	No & valid reason (eg 24°☹26.6°)	B1√	1	√Yes if their r ☜ 2.5	**5**

(Q1, June 2005)

2	(iii)	1.6 $\bar{y}$ =	M1		must be moments with vert dists	
		20x0.2 + 20x0.2 +40x0.5	A1		or 1.6 $\bar{y}$ =20x0.2x2+40x0.7(22.5)	
		$\bar{y}$ = 17.5 cm	A1	3		

(Q4, Jan 2006)

3	(i)	d = 2.25 h = 1.125 or 1.12 or 1.13 or 9/8	B1 B1	2	3/8x6 *OG* (be generous) horizontal distance	
	(ii)	$T_1 + T_2 = 12$ resolving vertically	M1		if not then next M1 ok	
		T_1 x 6cos30° = 12xh (their h)	M1 A1		or mom(*A*)T_2x6cos30°= 12(6cos30° — h)	
		mom(*O*) (their h ok for A1)				
		T_1 = 2.60 N or 3√3/2	A1		or T_2 = 9.40	
		T_2 = 9.40 N √(12 — T_1)	A1√	5	or T_1=2.60 or √(12—T_2)	**7**
		above √ depends on at least one of the M marks (T_s > 0)				

(Q3, June 2006)

4	(i)	$\bar{x}$ = 9	B1		ignore any working	
		c of m of Δ 4 cm above BD	B1		8 cm below C/see their diagram	
		(324 + 108) (m) $\bar{y}$ = 324(m)x9 + 108(m)x(18+4)	M1		432 $\bar{y}$ = 108x8 + 18^2 (12 + 9) from C	
		432 $\bar{y}$	A1		left hand side	
		324 x 9 (18^2 x 9)	A1		1st term on right hand side 2916	
		108 x (18 +4)	A1		2nd term on right hand side 2376	
		$\bar{y}$ = 12.25	A1	7	5292÷432 or 49/4	
	(ii)	tanθ = 5.75/9	M1		must be …/9	
		θ = 32.6° or 147.4°	A1√	2	√$\tan^{-1}$ ((18-their $\bar{y}$)/9) or 180°..	**9**

(Q5, June 2006)

5		com directly above lowest point	B1			
		tan α = 6/10	M1			
		α = 31.0	A1	3	or 0.540 rads	**3**

(Q1, Jan 2007)

6	(i)	com of Δ 4 cm right of *C*	B1			
		1.5 x 10 + 7 x 20 = $\bar{x}$ x 30	M1			
			A1			
		$\bar{x}$ = 5.17	A1		5 1/6 31/6	
		com of Δ 6 cm above *E*	B1		or 3 cm below *C*	
		4.5 x 10 + 6 x 20 = $\bar{y}$ x 30	M1			
			A1			
		$\bar{y}$ = 5.5	A1	8		
	(ii)	tanθ = 5.17/3.5	M1		right way up and (9– $\bar{y}$)	
		55.9° or 124°	A1√	2	√ their $\bar{x}$ /(9 – $\bar{y}$)	
	(iii)	d = 15sin45° (10.61)	B1		dist to line of action of T	
		Td = 30 x 5.17	M1		allow Tx15 i.e. T vertical	
		T = 14.6	A1	3		**13**

(Q6, Jan 2007)

7	com of hemisphere 0.3 from *O*	B1	**or** 0.5 from base
	com of cylinder *h*/2 from *O*	B1	
	0.6x45 = 40x0.5 + (0.8+*h*/2) x 5 **or**	M1	**or** 40x0.3 – 5xh/2 = 45 x 0.2
	45(h+0.2) = 5h/2 +40(h+0.3)	A1	**or** 5(0.2 + h/2) = 40x0.1
	27 = 20 + (0.8+*h*/2) x 5	M1	solving
	h = 1.2	A1 **6**	**AG**

(Q8, June 2007)

8 (i)	(2x4xsinΠ/2)/3xΠ/2 1.70	M1 A1 **2**	or 4r/3Π **AG**
(ii)(a)	$\bar{x}$ xd(8x20–Πx4^2/2)=10x8x20d–12xΠx4^2/2xd 10x8x20(d) (1600) (8x20–Πx4^2/2) (d) (134.9) (12xΠx4^2/2) (d) (301.6) $\bar{x}$ = 9.63 cm	M1 A1 A1 A1 A1 **5**	or 134.9 $\bar{x}$ = 64x4+38.9x12+32x18 (1298.8) 64x4 38.9x12 32x18 **AG**
(ii)(b)	$\bar{y}$ xd(8x20–Πx4^2/2)=4x8x20d–1.7xΠx4^2/2xd 4x8x20 (d) 1.7d x Π x 4^2 /2 (13.6Π) $\bar{y}$ = 4.43 cm	M1 A1 A1M1 A1 **4**	or 64x4=42.7+38.9 $\bar{y}$ $\bar{y}$ = 5.49 135 $\bar{y}$ =32x4+38.9x5.49+64x4

(Q8, Jan 2008)

9 (i)	3/8 x 3 (1.125) 0.53d = 5x0.02 + (10 + 3/8x3) x 0.5 d = 10.7	B1 M1 A1 A1 **4**	c.o.m. hemisphere 0.53e=3x5/8x0.5+8x0.02+13x .01 0.53f=3x3/8x0.5-5x0.02-10x0.01 **AG (**e = 2.316 f = 0.684**)**

(Q5, June 2008)

10 (i)	com of Δ 3 cm right of C (48+27) $\bar{x}$ = 48x4 + 27x11 $\bar{x}$ = 6.52 com of Δ 2 cm above AD (48+27) $\bar{y}$ = 48x3 + 27x2 $\bar{y}$ = 2.64	B1 M1 A1 A1 B1 M1 A1 A1 **8**	

(Q8, June 2008)

11 (ii)	đ = (2x40sinΠ/2)÷3Π/2	M1	must be radians
		A1	
	đ = 17.0	A1	16.98 160/3Π (8/15Π m)
	70 $\bar{y}$ = 100x60 + 217 x 10	M1	
		A1 ft	ft 200 + their đ or 2 + their đ (m)
	$\bar{y}$ = 117	A1 **6**	116.7

(Q3, Jan 2009)

12(i)	$\bar{d}$ = (2× 6 sinπ/4)/3π/4 $\bar{d}$ = 3.60	M1 A1 **2**	must be correct formula with rads AG
(ii)	$\bar{d}$ cos45° = "2.55" 5 $\bar{x}$ = 3 × 3 + 2 × "2.55" $\bar{x}$ = 2.82 5 $\bar{y}$ = 3 × 6 + 2 × (12 + "2.55") $\bar{y}$ = 9.42	B1 M1 A1 A1 M1 A1 A1 **7**	may be implied moments must not have areas 2kg/3kg misread (swap) gives (2.73,11.13) θ = 21.7° (MR – 2) (max 7 for (ii) + (iii)) SR -1 for $\bar{x}$, $\bar{y}$ swap
(iii)	tanθ = 2.82/8.58 θ = 18.2°	M1 A1 **2**	M0 for their $\bar{x}$ / $\bar{y}$ their $\bar{x}$ /(18 - $\bar{y}$) **11**

(Q5, June 2009)

13 (i)	ū = 0.2 (from vertex) or 0.8 or 0.1 0.5đ = 0.2×ū + 0.3×0.65 đ = 0.47	B1 M1 A1 A1 **[4]**	com of conical shell **AG**
(ii)	s = 0.5 Tsin80°× 0.5 = 0.47 × 0.5 × 9.8 T = 4.68 N	B1 M1 A1 A1 **[4]**	slant height, may be implied **8**

(Q3, Jan 2010)

14 (i)	$(6\sin \Pi/2) \div (\Pi/2)$ 3.82	M1 A1 **2**	Use of correct formula **AG**
(ii)	$8đ = 3(6-3.82) + 5x9.82$ or $8x = \pm\{3(-3.82) + 5x3.82\}$ $đ = 6.95$ or 6.96 or $x = +/-0.955$ $\tan\theta = 0.96/6$ $\theta = 9°$	M1 A1 A1 M1 A1 **5** **7**	Method to find centre of mass Attempt to find the required angle

(Q2, June 2010)

15	**(i)**		$3x_G = 2x0.3 + 1x0.6$ OR $3x_G = 2x0.3 + 0$ OR $3x_G = 4x0.3$ OR $3y_G = 1x0.3 + 1x0.6 + 0$ OR $3y_G = 4x0.3 - 1x0.3$ $x_G = 0.4$ (from AD) OR $x_G = 0.2$ (from BC) $y_G = 0.3$m from AB or CD $AG^2 = 0.4^2 + 0.3^2$ AG = 0.5 m	M1 A1 A1 M1 A1 **[5]**	Table of moments idea. M0 for reducing to 1D problem. Masses/weights may be included. Pythagoras with 2 appropriate distances. This may only be seen in (ii), allow M1A1 in this case.

(Q1, Jan 2011)

16	**(i)**		$x_H = 3x0.6/8$ $\pi(0.6^2x0.6)(0.6/2) - (0.6^3x2\pi/3)0.225$ $= \pi x0.6^3(1+2/3)x_G$ $x_G = 0.09$ m **AG**	B1 M1 A1 A1 A1 **[5]**	CoM hemisphere ($x_H = 0.225$), may be implied Use of table of moments idea SC Volume of sphere used, max B1M1A1, moment equation fully correct for A1 (3/5) Accept -0.09

(Q5, Jan 2011)

17 i	- $(8\cos30/3)(8^2\sin60/2)$ + $(4)(8^2)$ = $(8^2 + 8^2\sin60/2)(x_G)$ $x_G = 2.09$ cm	M1 A1 A1 A1 A1 [5]	Table of moments idea, may include g and/or density. -2.309 x 27.7
ii	$\tan\theta = (2.09/4)$ $\theta = 27.6°$	M1 A1ft [2]	ft cv(x_G)

(Q3, June 2011)

18	(i)		$h = r\tan\alpha$ $x(\frac{2}{3}\pi r^3 + \frac{1}{3}\pi r^2 h) = \frac{1}{3}\pi r^2 h \times \frac{h}{4} - \frac{2}{3}\pi r^3 \times \frac{3}{8}r$ $x = \frac{r(\tan^2\alpha - 3)}{8 + 4\tan\alpha}$	B1 M1 A1 A1 [4]	Seen anywhere and in any form. Table of values idea. **AG** www
	(ii)		$x < 0$ Solve $\tan^2\alpha - 3 < 0$ $\alpha < 60$	B1 M1 A1 [3]	May be implied. Condone = Condone $\leq$ throughout. SC Use of = or > throughout. Max B0 M1 A0

(Q2, Jan 2012)

19	(i)		⅓ a $(25 + 2.5a)x_G$ $= 25 \times 2.5 + 2.5a \times (5 + ⅓\ a)$ $x_G = \dfrac{a^2 + 15a + 75}{3(a + 10)}$ **AG**	B1 M1 A1 A1 A1 **[5]**	Centre of mass of triangle Table of values idea, using any fixed axis Relative to the axis they are using
	(ii)		$\dfrac{a^2 + 15a + 75}{3(a + 10)} = 5$ Solving for a $a = 8.66$ or $5\sqrt{3}$	*M1 dep*M1 A1 **[3]**	Substitute x_G as 5 $a \leq 8.66$
	(iii)		$(25 + 2.5a)y_G = 25 \times 2.5 + 2.5a \times (⅔ \times 5)$ $y_G = \dfrac{10a + 75}{3(a + 10)}$ or 2.89 $\tan\theta = x_G / y_G$ $= 5 / y_G$ $\theta = 60$	*M1 A1ft A1ft dep*M1 A1ft A1 **[6]**	Method to find centre of mass from AB (or CD) with or without a substituted. ft their a from (ii), from CD y_G=2.11 Using trig to find an appropriate angle, eg complement of θ. ft their a from (ii), but not an incorrect y_G $\theta \leq 60$ (anything that rounds to 60)

(Q7, June 2012)

20	**(i)**		$(2 \times 3\sin(\pi/2))/(3\pi/2)$ or equivalent 3×6^2 $-(\pi \times 3^2/2) \times (6 - 4/\pi)$ $= (6^2 - \pi \times 3^2/2)x_G$ $x_G = 1.88$ cm	B1 M1 A1 A1 A1 A1 **[6]**	Centre of mass of semicircle; $4/\pi$ Table of moments idea about any axis.	
	(ii)		$\tan\theta = 1.88/3$ $\theta = 32.1^o$	M1 A1ft **[2]**	Attempt at a relevant angle allow 180-θ & radians (0.561 or 0.560)	

(Q4, Jan 2013)

21		**(i)**	$4.4x_G = 4 \times ¼ \times 8$ $- 0.4 \times 1/3 \times 10$ $x_G = 1.52$ cm	M1 A1 A1 A1 **[4]**	Table of moments idea. Moments about other axes acceptable Allow $^{50}/_{33}$
		(ii)	$T_{shell} \times 18 = 4.4g \times (8 - 1.52)$ or $T_{cone} \times 18 = 4.4g \times (10 + 1.52)$ $T_{shell} + T_{cone} = 4.4g$ $T_{shell} = 15.5$ and $T_{cone} = 27.6$	M1 A1ft M1 A1 **[4]**	Or any other correct moment equation. ft on x_G from **(i)** May use a second moments equation For both

(Q4, June 2013)