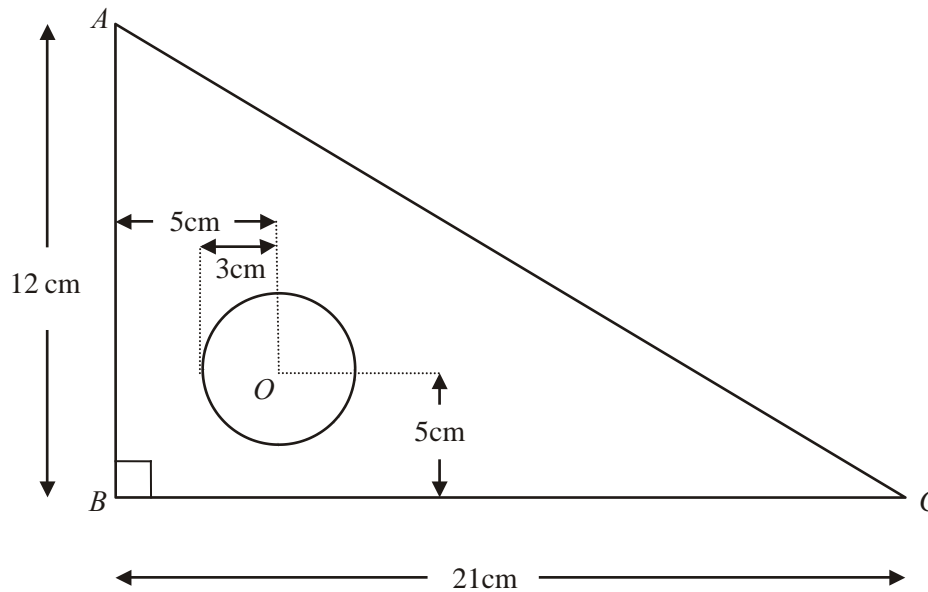


1.



A set square S is made by removing a circle of centre O and radius 3 cm from a triangular piece of wood. The piece of wood is modelled as a uniform triangular lamina ABC , with $\angle ABC = 90^\circ$, $AB = 12$ cm and $BC = 21$ cm. The point O is 5 cm from AB and 5 cm from BC , as shown in the diagram above.

- (a) Find the distance of the centre of mass of S from
- (i) AB ,
 - (ii) BC .

(9)

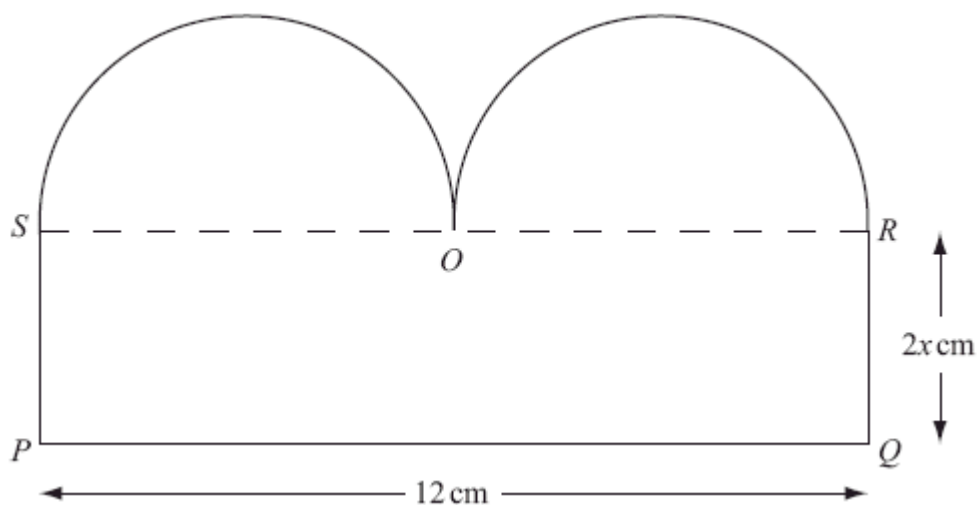
The set square is freely suspended from C and hangs in equilibrium.

- (b) Find, to the nearest degree, the angle between CB and the vertical.

(3)

(Total 12 marks)

2. [The centre of mass of a semi-circular lamina of radius r is $\frac{4r}{3\pi}$ from the centre]



A template T consists of a uniform plane lamina $PQROS$, as shown in the diagram above. The lamina is bounded by two semicircles, with diameters SO and OR , and by the sides SP , PQ and QR of the rectangle $PQRS$. The point O is the mid-point of SR , $PQ = 12$ cm and $QR = 2x$ cm.

- (a) Show that the centre of mass of T is a distance $\frac{4|2x^2 - 3|}{8x + 3\pi}$ cm from SR .

(7)

The template T is freely suspended from the point P and hangs in equilibrium.

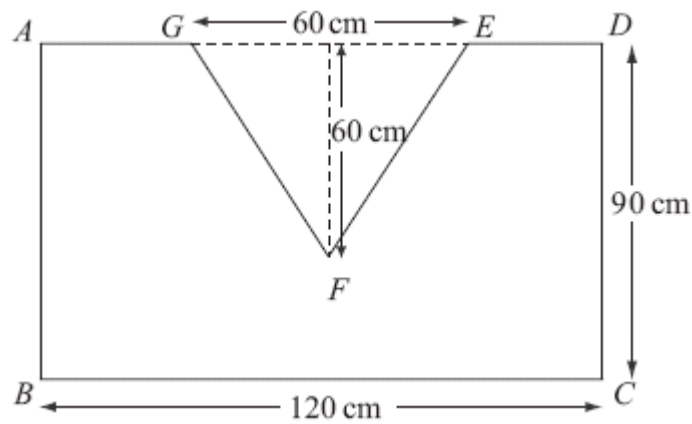
Given that $x = 2$ and that θ is the angle that PQ makes with the horizontal,

- (b) show that $\tan \theta = \frac{48 + 9\pi}{22 + 6\pi}$.

(4)

(Total 11 marks)

3.



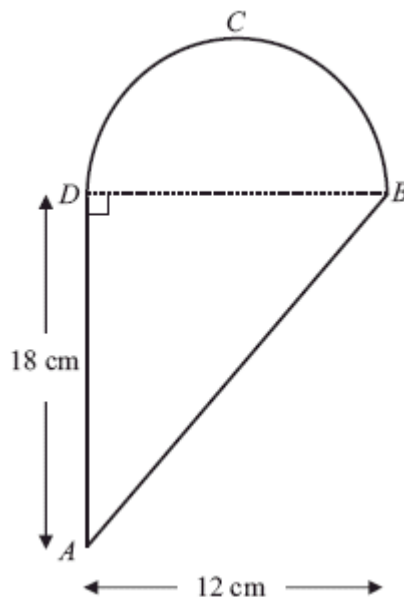
A shop sign $ABCDEFG$ is modelled as a uniform lamina, as illustrated in the diagram above. $ABCD$ is a rectangle with $BC = 120$ cm and $DC = 90$ cm. The shape EFG is an isosceles triangle with $EG = 60$ cm and height 60 cm. The mid-point of AD and the mid-point of EG coincide.

- (a) Find the distance of the centre of mass of the sign from the side AD . (5)

The sign is freely suspended from A and hangs at rest.

- (b) Find the size of the angle between AB and the vertical. (4)
(Total 9 marks)

4.



A uniform lamina $ABCD$ is made by joining a uniform triangular lamina ABD to a uniform semi-circular lamina DBC , of the same material, along the edge BD , as shown in the diagram above. Triangle ABD is right-angled at D and $AD = 18$ cm. The semi-circle has diameter BD and $BD = 12$ cm.

- (a) Show that, to 3 significant figures, the distance of the centre of mass of the lamina $ABCD$ from AD is 4.69 cm.

(4)

Given that the centre of mass of a uniform semicircular lamina, radius r , is at a distance $\frac{4r}{3\pi}$ from the centre of the bounding diameter,

- (b) find, in cm to 3 significant figures, the distance of the centre of mass of the lamina $ABCD$ from BD .

(4)

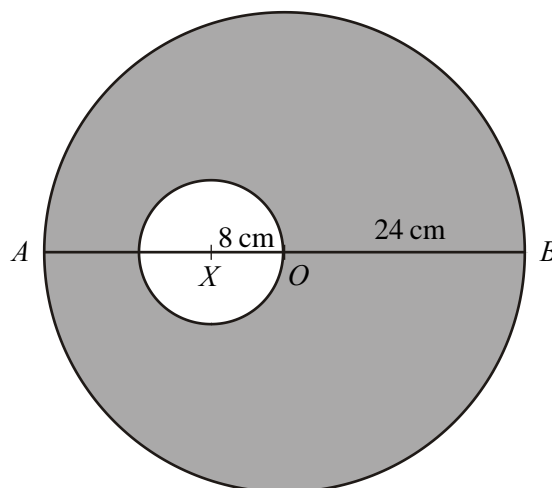
The lamina is freely suspended from B and hangs in equilibrium.

- (c) Find, to the nearest degree, the angle which BD makes with the vertical.

(4)

(Total 12 marks)

5.



The diagram above shows a template T made by removing a circular disc, of centre X and radius 8 cm, from a uniform circular lamina, of centre O and radius 24 cm. The point X lies on the diameter AOB of the lamina and $AX = 16$ cm. The centre of mass of T is at the point G .

- (a) Find AG .

(6)

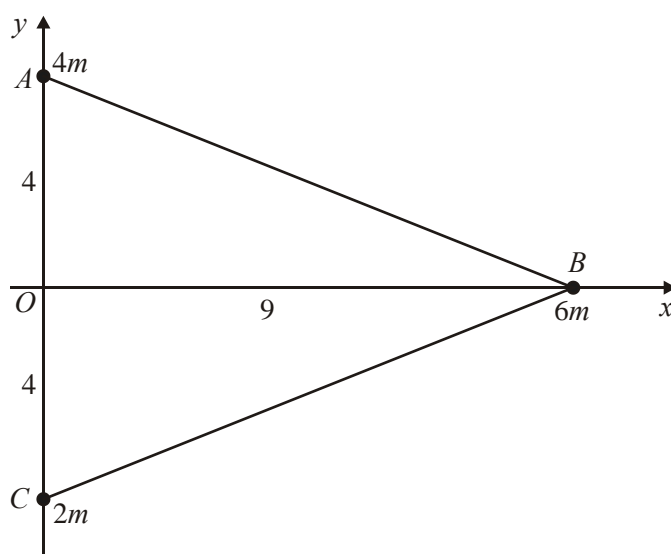
The template T is free to rotate about a smooth fixed horizontal axis, perpendicular to the plane of T , which passes through the mid-point of OB . A small stud of mass $\frac{1}{4}m$ is fixed at B , and T and the stud are in equilibrium with AB horizontal. Modelling the stud as a particle,

- (b) find the mass of T in terms of m .

(4)

(Total 10 marks)

6.



The figure above shows a triangular lamina ABC . The coordinates of A , B and C are $(0, 4)$, $(9, 0)$ and $(0, -4)$ respectively. Particles of mass $4m$, $6m$ and $2m$ are attached at A , B and C respectively.

- (a) Calculate the coordinates of the centre of mass of the three particles, *without the lamina*.

(4)

The lamina ABC is uniform and of mass km . The centre of mass of the combined system consisting of the three particles and the lamina has coordinates $(4, \lambda)$.

- (b) Show that $k = 6$.

(3)

- (c) Calculate the value of λ .

(2)

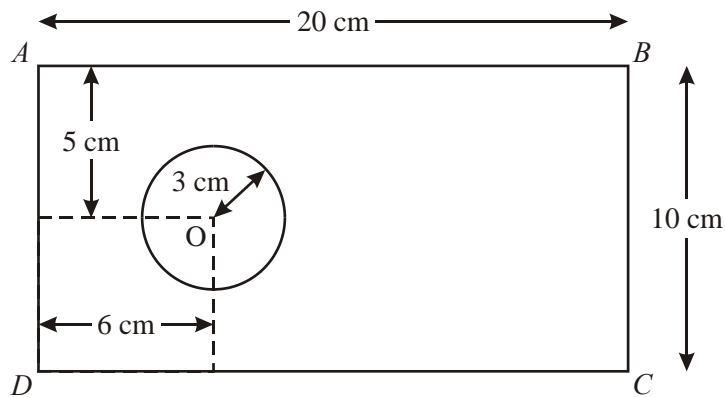
The combined system is freely suspended from O and hangs at rest.

- (d) Calculate, in degrees to one decimal place, the angle between AC and the vertical.

(3)

(Total 12 marks)

7.



This diagram shows a metal plate that is made by removing a circle of centre O and radius 3 cm from a uniform rectangular lamina $ABCD$, where $AB = 20$ cm and $BC = 10$ cm. The point O is 5 cm from both AB and CD and is 6 cm from AD .

(a) Calculate, to 3 significant figures, the distance of the centre of mass of the plate from AD .

(5)

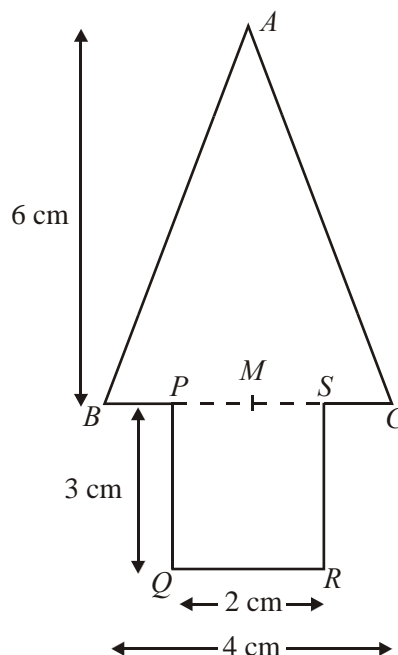
The plate is freely suspended from A and hangs in equilibrium.

(b) Calculate, to the nearest degree, the angle between AB and the vertical.

(3)

(Total 8 marks)

8.



The diagram above shows a decoration which is made by cutting the shape of a simple tree from a sheet of uniform card. The decoration consists of a triangle ABC and a rectangle $PQRS$. The points

P and S lie on BC and M is the mid-point of both BC and PS . The triangle ABC is isosceles with $AB = AC$, $BC = 4$ cm, $AM = 6$ cm, $PS = 2$ cm and $PQ = 3$ cm.

- (a) Find the distance of the centre of mass of the decoration from BC .

(5)

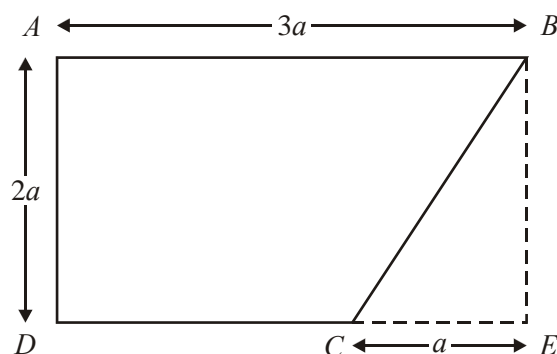
The decoration is suspended from Q and hangs freely.

- (b) Find, in degrees to one decimal place, the angle between PQ and the vertical.

(4)

(Total 9 marks)

9.



A uniform lamina $ABCD$ is made by taking a uniform sheet of metal in the form of a rectangle $ABED$, with $AB = 3a$ and $AD = 2a$, and removing the triangle BCE , where C lies on DE and $CE = a$, as shown in the diagram above.

- (a) Find the distance of the centre of mass of the lamina from AD .

(5)

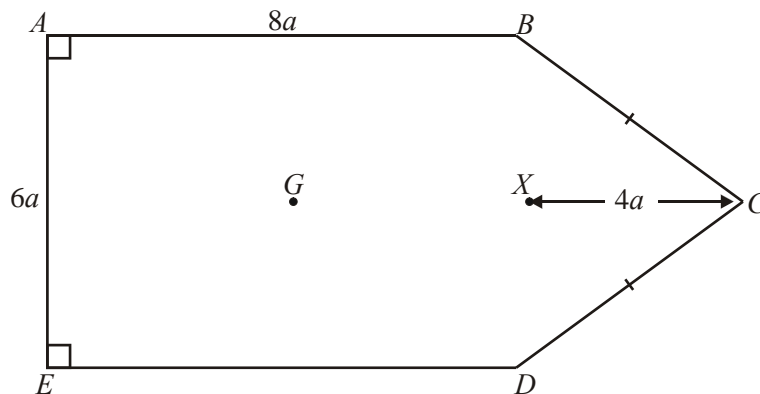
The lamina has mass M . A particle of mass m is attached to the lamina at B . When the loaded lamina is freely suspended from the mid-point of AB , it hangs in equilibrium with AB horizontal.

- (b) Find m in terms of M .

(4)

(Total 9 marks)

10.



The diagram above shows a uniform lamina $ABCDE$ such that $ABDE$ is a rectangle, $BC = CD$, $AB = 8a$ and $AE = 6a$. The point X is the mid-point of BD and $XC = 4a$. The centre of mass of the lamina is at G .

(a) Show that $GX = \frac{44}{15}a$.

(6)

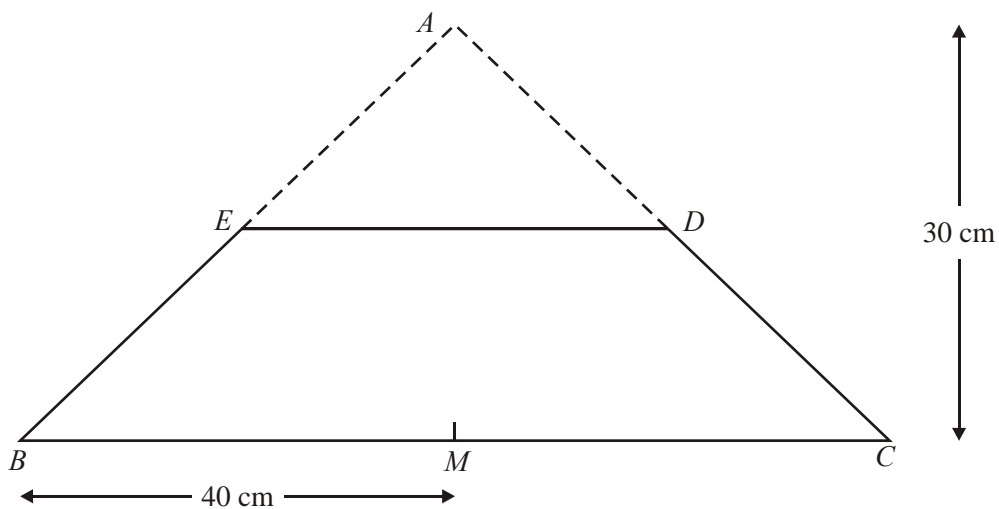
The mass of the lamina is M . A particle of mass λM is attached to the lamina at C . The lamina is suspended from B and hangs freely under gravity with AB horizontal.

(b) Find the value of λ .

(3)

(Total 9 marks)

11.



A uniform plane lamina is in the shape of an isosceles triangle ABC , where $AB = AC$. The mid-point of BC is M , $AM = 30$ cm and $BM = 40$ cm. The mid-points of AC and AB are D and E respectively. The triangular portion ADE is removed leaving a uniform plane lamina $BCDE$ as shown in the diagram above.

- (a) Show that the centre of mass of the lamina $BCDE$ is $6\frac{2}{3}$ cm from BC . (6)

The lamina $BCDE$ is freely suspended from D and hangs in equilibrium.

- (b) Find, in degrees to one decimal place, the angle which DE makes with the vertical. (3)
- (Total 9 marks)**

1. (a)
- | | Triangle | Circle | S | | |
|--|----------|------------------|------------------------|-----------|---------|
| Mass ratio | 126 | 9π (28.3) | $126 - 9\pi$ (97.7) | | B1 B1ft |
| \bar{x} | 7 | 5 | \bar{x} | | |
| \bar{y} | 4 | 5 | \bar{y} | 4, 7 seen | B1 |
| $126 \times 7 = 9\pi \times 5 + (126 - 9\pi) \times \bar{x}$ ft their table values | | | | | M1 A1ft |
| $\bar{x} \approx 7.58 \left(\frac{882 - 45\pi}{126 - 9\pi} \right)$ | | | | awrt 7.6 | A1 |
| $126 \times 4 = 9\pi \times 5 + (126 - 9\pi) \times \bar{y}$ ft their table values | | | | | M1 A1ft |
| $\bar{y} \approx 3.71 \left(\frac{504 - 45\pi}{126 - 9\pi} \right)$ | | | | awrt 3.7 | A1 9 |
- (b) $\tan \theta = \frac{\bar{y}}{21 - \bar{x}}$ ft their \bar{x}, \bar{y} M1 A1ft
- $\theta \approx 15^\circ$ A1 3

[12]

2. (a)
- | Rectangle | Semicircles | | Template, T | |
|--|---------------------------|---------------------------|---------------|-------|
| $24x$ | 4.5π | 4.5π | $24x + 9\pi$ | B2 |
| x | $\frac{4 \times 3}{3\pi}$ | $\frac{4 \times 3}{3\pi}$ | \bar{x} | B2 |
| $24x^2 - 4.5\pi \times \left(\frac{4 \times 3}{3\pi} \right) - 4.5\pi \times \left(\frac{4 \times 3}{3\pi} \right) = (24x + 9\pi) \bar{x}$ | | | | M1 A1 |
| distance = $ \bar{x} = \frac{4 2x^2 - 3 }{(8x + 3\pi)}$ ** | | | | A1 7 |
- (b) When $x = 2$, $|\bar{x}| = \frac{20}{16 + 3\pi}$ B1
- $\tan \theta = \frac{6}{4 - |\bar{x}|} = \frac{6}{4 - \frac{20}{16 + 3\pi}}$ M1 A1
- $= \frac{48 + 9\pi}{22 + 6\pi}$ A1 4

[11]

- 3 (a) Ratio of areas triangle:sign:rectangle = 1 : 5 : 6
(1800:9000:10800) B1
- Centre of mass of the triangle is 20cm down from AD (seen or implied) B1

$$\Rightarrow 6 \times 45 - 1 \times 20 = 5 \times \bar{y}$$

$$\bar{y} = 50 \text{ cm}$$

M1A1

A1 5

(b) Distance of centre of mass from AB is 60cm

B1

$$\text{Required angle is } \tan^{-1} \frac{60}{50}$$

(their values)




M1A1ft

$$= 50.2^\circ \text{ (0.876 rads)}$$

A1 4

[9]

4. (a)

| | | | | | |
|--------------------------------------|---|---|--|--|----|
| |  |  |  | | |
| MR | 108 | 18π | $108 + 18\pi$ | | B1 |
| x_1 (\rightarrow) from AD | 4 | 6 | \bar{x} | | B1 |
| y_i (\downarrow) from BD | 6 | $-\frac{8}{\pi}$ | \bar{y} | | |




$$AD(\rightarrow): 108(4) + 18\pi(6) = (108 + 18\pi) \bar{x}$$

M1

$$\bar{x} = \frac{432 + 108\pi}{108 + 18\pi} = 4.68731\dots = \underline{4.69} \text{ (cm) (3sf) AG}$$

A1 4

(b)

| | | | | | |
|-------------------------------------|---|---|--|--|-------|
| |  |  |  | | |
| y_i (\downarrow) from BD | 6 | $-\frac{8}{\pi}$ | \bar{y} | | B1 oe |

$$BD(\downarrow): 108(6) + 18\pi(-\frac{8}{\pi}) = (108 + 18\pi) \bar{y}$$

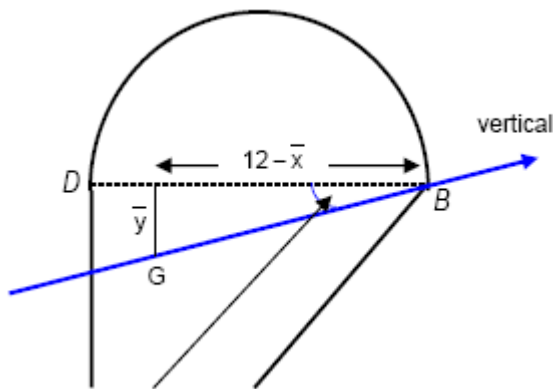
M1

A1ft

$$\bar{y} = \frac{504}{108 + 18\pi} = 3.06292\dots = \underline{3.06} \text{ (cm) (3sf)}$$

A1 4

(c)



$\theta =$ required angle

$$\tan \theta = \frac{\bar{y}}{12 - 4.68731..}$$

$$= \frac{3.06392..}{12 - 4.68731..}$$

$$\theta = 22.72641... = \underline{23} \text{ (nearest degree)}$$

dM1

A1

A1

4

[12]

5. (a)

| | | | | |
|-------------|---------|---------|----------|----------------------------|
| | Large | Small | Template | |
| Mass ratios | 24^2 | 8^2 , | 512 | anything in ratio 9: 1 : 8 |
| | (c.1810 | c.200 | c.1610) | |

B1,B1ft

$$M(A) 9 \times 24 = 16 \times 1 + 8\bar{x}$$

M1*A1

$$\bar{x} = 25 \text{ (cm) exact}$$

DM1*A1

6

(b) $M(\text{axis}) 1LM = 12 \times \frac{1}{4} m$

ft their \bar{x}

M1†A1ft

$$\left((36 - \bar{x})M = 12 \times \frac{1}{4} m \right)$$

$$M = \frac{3}{11} m \text{ (o.e.e.)}$$

DM1†A1

4

[10]

6. (a)

$$12m\bar{x} = 6m \times 9$$

M1

$$\bar{x} = 4\frac{1}{2}$$

A1

$$12m\bar{y} = 16m - 8m$$

M1

$$\bar{y} = \frac{2}{3}$$

A1

4

(b) $(12 + k)m \times 4 = 12m \times 4\frac{1}{2} + km \times 3$ ft their \bar{x} M1 A1ft
 $k = 6$ A1 3

(c) $18m \times \lambda = 12m \times \frac{2}{3}, \Rightarrow \lambda = \frac{4}{9}$ M1A1 2

(d) $\tan \theta = \frac{4}{4/9}, \Rightarrow \theta \approx 83.7^\circ$ ft their λ , cao M1 A1ft A1 3

[12]

7. (a)

| | | | | |
|-----------------|--------|-----------|--------------|----------|
| | circle | rectangle | plate | |
| Mass ratios | 9π | 200; | $200 - 9\pi$ | B1; B1ft |
| Centres of mass | 6 | 10 | \bar{x} | B1 |

 $9\pi \times 6 + (200 \times 9\pi)\bar{x} = 200 \times 10$ M1
 $\bar{x} \approx 10.7$ (cm) A1 5
 cao

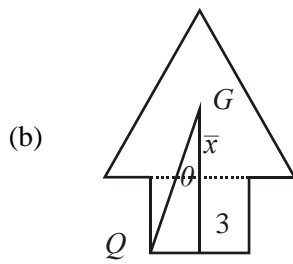
(b) $\tan \theta = \frac{5}{10.7}$ M1 A1ft
 ft their \bar{x}
 $\theta \approx 25^\circ$ A1 3
 cao

[8]

8. (a)

| | | | |
|------------|------------------|-------------|----|
| Rectangle | Triangle | Decoration | |
| Mass Ratio | 6 12 18 | Ratio 1:2:3 | B1 |
| CM from BG | $(-)\frac{1}{2}$ | 2 \bar{x} | B1 |

 $18 \times \bar{x} = -6 \times 1\frac{1}{2} + 12 \times 2$ M1 A1
 $\bar{x} = \frac{5}{6}$ accept exact equivalents A1 5



Identification and use of correct triangle

$$\tan \theta = \frac{1}{3 + \bar{x}} \text{ ft their } \bar{x}$$

$$\theta \text{ } 14.6^\circ \text{ cao}$$

M1

M1 A1ft

A1 4

[9]

9. (a)



Area: $6a^2$ a^2 $5a^2$ (Ratio)

CM from AD: $\frac{3a}{2}$ $\left(2a + \frac{2a}{3}\right)$ \bar{x}

B1 B1

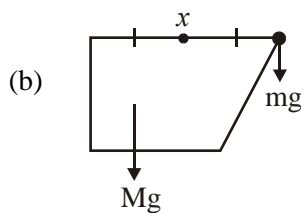
$$6 \times \frac{3a}{2} - 1 \times \frac{8a}{3} = 5 \bar{x}$$

M1

$$\bar{x} = \frac{19a}{15}$$

A1 5

AWRT 1.27a



$M(X)$,

$$Mg \left(\frac{3a}{2} - \frac{19a}{15} \right) = mg \times \frac{3a}{2}$$




M1 A1 ft A1

$$\Rightarrow m = \frac{7M}{45}, 0.156M, 0.16M$$

A1 4

[9]

10. (a)

| | | | | |
|----|---|---|---|-----------|
| |  |  |  | |
| MR | $48a^2$ | $12a^2$ | $60a^2$ | B1, B1 ft |
| CM | $4a$ | $(-)\frac{1}{3} \times 4a$ | \bar{x} | B1 |

$$48a^2 \times 4a - 12a^2 \times \frac{4}{3}a = 60\bar{x}$$

M1 A1

Solving to $\bar{x} = \frac{44}{15}a$ (*)

A1 6

(b) $\lambda M \times 4a = M \times \frac{44}{15}a$

M1 A1

$$\lambda = \frac{11}{15}$$

A1 3

[9]

11. (a)

| | | | |
|---|------------|------------|-------------|
| | <i>ABC</i> | <i>ADE</i> | <i>BCDE</i> |
| Relative mass | 4 | 1 | 3 |
| Distance of centre of mass from <i>BC</i> | 10 | 20 | \bar{y} |

(-1 each error or omission)

B3

$$(4 \times 10) - (1 \times 20) = 3\bar{y}$$

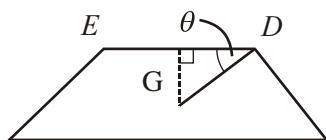
M1 A1

$$6\frac{2}{3} = \frac{20}{3} = \bar{y} \quad (\text{T})$$

A1 6

(b) $\tan \theta = \frac{15 - \bar{y}}{20}$

M1



$$= \frac{15 - \frac{20}{3}}{20} = \frac{5}{12}$$

A1

$$\theta = 22.6^\circ \text{ (1 d.p.)}$$

A1 3

[9]

1. Many candidates achieved full marks on this question, whilst others just missed out on the final mark because they did not notice the instruction to give their final answer correct to the nearest degree.
 - (a) Where there were difficulties these usually arose when a candidate tried to work with the geometry of the triangle, finding lengths of medians, etc in order to find the location of the centre of mass of the triangle – many seemed to be completely unaware of the simple result they could apply and invariably made algebraic errors in their work. Most candidates understood that the circle had been cut out of the triangle, but quite a few added the circle to the triangle in their working. A few candidates treated the triangle as if it were just three rods, and others confused the centre of mass of the triangle with the centre of mass of the set square in the course of their working.
 - (b) Most candidates correctly identified the required angle. A few did not use their answers from part (a) at all, they simply found the angle BCA .
2.
 - (a) The method was understood by most candidates and there was no problem in forming a moments equation for the centre of mass. Common errors included simplifying $\frac{4 \times 3}{3\pi}$ to 4π rather than $\frac{4}{\pi}$, using the area of a circle rather than a semicircle, and the use of 6 for the radius of the semicircle. From a correct table, accuracy marks were often lost in the moments equation because of a sign error. In general, those candidates who set out the masses and distances in a table tended to make fewer errors. Many candidates made it more difficult to obtain the given answer by taking their measurements from PQ and attempting to subtract their result from $2x$, although this was often successfully completed. One advantage of this approach was that they were less likely to make a sign error in their moments equation. Candidates very rarely justified the modulus sign at all, with most candidates simply writing the final answer after their last line of working. Students who had a negative coefficient for the x^2 term in the numerator were more likely to deal with this.
 - (b) The fact that the answer was given did guide some candidates to the correct result, indicating that they clearly appreciated the ‘show that’ nature of the question. Most candidates substituted $x = 2$ correctly into the given result and went on to find the tangent of an angle. Many candidates did identify the correct triangle although some went to great lengths to find the distance of the centre of mass from SP as an expression in x , not realising that it could be found by symmetry. Furthermore, they often did not then realise that their expression cancelled to 6. Many of those who made progress with this part found the angle to the vertical, with quite a few unconvincingly converting to the given result or simply leaving it as the reciprocal.

3. This question was generally well answered by most candidates. It was pleasing to see many correct evaluations of the centre of mass of the triangle and many completely correct solutions to part (a). A few candidates attempted breaking up the lamina into rectangles and triangles rather than subtracting the moment of the triangle from the moment of the rectangle and so made the question much more difficult. Of those who calculated the areas of the rectangle and triangle correctly some failed to subtract these and added them instead. Most seemed to be happy with the use of large numbers for areas and only a few reduced these to a ratio. A very small number of candidates tried to replace the lamina by a framework of rods.

In part (b) a significant number of candidates failed to recognise that the lamina was symmetrical and wasted time in finding the distance to the centre of mass from AB using the same method, rather than using symmetry to write it down.

Many candidates lost the last two marks by finding the angle between AD and the vertical instead of the angle required.

4. The majority of candidates applied the correct mechanical principles to solve this problem. Most were able to find the relative masses and the centres of mass of the semi-circle and the triangle and obtain a correct moments equation. Many candidates did not show sufficient working to demonstrate that their equation led to the given result in part (a).

In part (b) the most common error was to fail to realise that the two centres of mass were on opposite sides of the line BD and they hence had a sign error in their expression. Those who decided to take moments about a line through A , perpendicular to AD avoided this problem.

Candidates were generally able to use the given result to find the centre of mass of the semicircle, although it was quite common to see it written incorrectly as 8π .

A clear diagram tended to lead candidates to identify the correct angle in part (c) and the correct method for finding it.

5. Many candidates reached for calculators in this question – it was unusual to see the simplified form (9 : 1 : 8) for the ratio of areas. Despite this, many solved the problem successfully. The most common error was in failing to subtract the area of the disc removed in the moments equation or in failing to subtract the moment of the disc removed. Candidates did not always choose to take moments about A , but many correct solutions were seen. A minority of candidates were either searching the formula booklet for inspiration or confused by more advanced work that they have studied, and attempted to use the formula for centre of mass of a sector of a circle. Candidates generally scored either full marks or no marks for (b). Some tried to bring in the areas from (a) and ended up with dimensionally incorrect equations that earned no marks. Several chose to take moments about a different axis and then usually neglected the reaction at the pivot.

6. In part (a) many saw intuitively that $\bar{x} = 4.5$ because of the distribution of the masses. Some made life more difficult by taking moments about different axes – especially to find the y -coordinate. Many completed the second part easily, but weaker candidates tried to find the area of the triangle and somehow got the required answer by devious means thereafter. A worrying number thought that 6×9 was 36. A small minority failed to complete this part through not knowing the position of the centre of mass of a triangular lamina. Part (c) was usually completely correct or else they had absolutely no idea. In the final part there were the usual problems of identifying the angle correctly with some attempting to find sine or cosine, using Pythagoras or the cosine rule.
7. This question was also well done. A few added the masses of the circle and rectangle, rather than subtract them, and a few errors of sign in the moments equation were seen. Some candidates ignored the obvious symmetry of the diagram and took moments to find the distance of the centre of mass from AB and this, besides being unnecessary work, caused further difficulties if the distance found was incorrect. When an accuracy is specified in a question, the candidate is expected to give their answer to that accuracy to gain full marks.
8. This was another question in which full marks were common. In part (a), some candidates, when taking moments, ignored the fact that the centre of mass of the rectangle was below BC and the centre of mass of the triangle was above BC . Many candidates produce solutions in which it is not at all clear what coordinate system they are using and what axis they are taking moments about. In part (b) most could identify a suitable triangle and trigonometric manipulation was usually completed correctly and to the degree of accuracy requested. However, the length of one side to the triangle was often given as 2 cm when it was 1 cm.
9. (a) The method here was generally well known but there were often mistakes in the mass ratios and the distance for the triangle.
(b) This proved to be a lot more demanding with some candidates not knowing where to start. Most who took moments about AD missed out the third term.
10. Part (a) was done very well although some lost unnecessary marks by the partial omission of a from their solution or from the use of approximate decimals on the way to the required exact answer. One error commonly seen was to find the distance of the centre of mass of the triangle from AE as $10\frac{2}{3}a$. Part (b) proved more difficult. Many were unclear how they should be using the given masses, attempting, often unsuccessfully, to write an equation involving both masses and areas. The simplest correct solution is to take moments about the X , obtaining the equation $\lambda M \times 4a = M \times \frac{44}{15}a$, which leads quickly to the answer. This part proved effective in distinguishing candidates who gained high grades.
11. No Report available for this question.