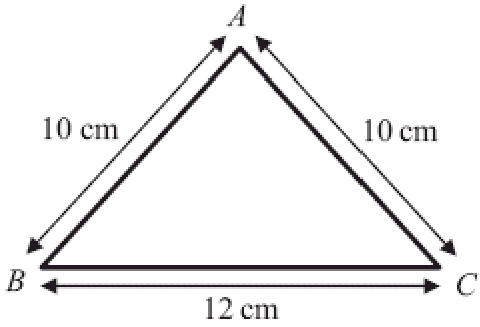
1.



A triangular frame is formed by cutting a uniform rod into 3 pieces which are then joined to form a triangle ABC, where $AB = AC = 10$ cm and $BC = 12$ cm, as shown in the diagram above.

(a) Find the distance of the centre of mass of the frame from BC.

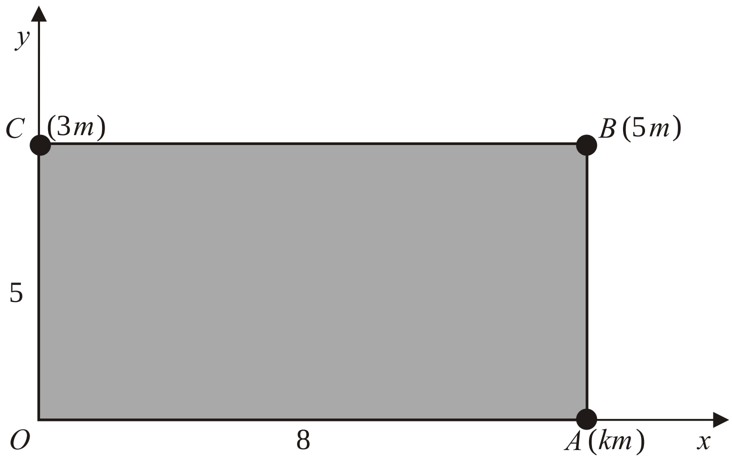
(5)

The frame has total mass M. A particle of mass M is attached to the frame at the mid-point of BC. The frame is then freely suspended from B and hangs in equilibrium.

(b) Find the size of the angle between BC and the vertical.

(4)
(Total 9 marks)

2.



The diagram above shows a rectangular lamina $OABC$. The coordinates of O, A, B and C are (0, 0), (8, 0), (8, 5) and (0, 5) respectively. Particles of mass km, $5m$ and $3m$ are attached to the lamina at A, B and C respectively.

The x-coordinate of the centre of mass of the three particles *without the lamina* is 6.4.

(a) Show that $k = 7$.

(4)

The lamina $OABC$ is uniform and has mass $12m$.

(b) Find the coordinates of the centre of mass of the combined system consisting of the three particles and the lamina.

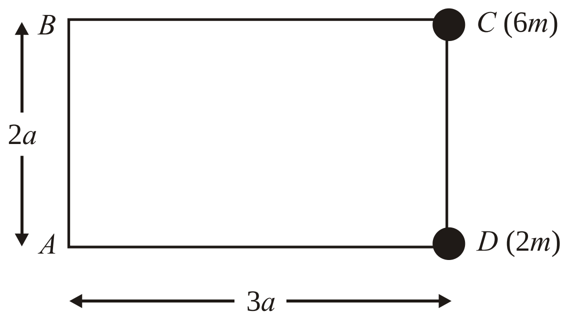
(6)

The combined system is freely suspended from O and hangs at rest.

(c) Find the angle between OC and the horizontal.

(3)
(Total 13 marks)

3.



The figure above shows four uniform rods joined to form a rigid rectangular framework $ABCD$, where $AB = CD = 2a$, and $BC = AD = 3a$. Each rod has mass m. Particles, of mass $6m$ and $2m$, are attached to the framework at points C and D respectively.

(a) Find the distance of the centre of mass of the loaded framework from

(i) AB,

(ii) AD.

(7)

The loaded framework is freely suspended from B and hangs in equilibrium.

(b) Find the angle which BC makes with the vertical.

(3)
(Total 10 marks)

4. A uniform ladder AB, of mass m and length $2a$, has one end A on rough horizontal ground. The coefficient of friction between the ladder and the ground is 0.6. The other end B of the ladder rests against a smooth vertical wall.

A builder of mass $10m$ stands at the top of the ladder. To prevent the ladder from slipping, the builder's friend pushes the bottom of the ladder horizontally towards the wall with a force of magnitude P. This force acts in a direction perpendicular to the wall. The ladder rests in equilibrium in a vertical plane perpendicular to the wall and makes an angle α with the horizontal, where $\tan \alpha = \frac{3}{2}$.

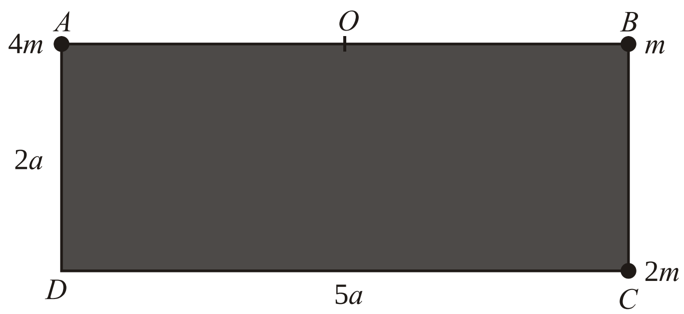
(a) Show that the reaction of the wall on the ladder has magnitude $7mg$.

(5)

(b) Find, in terms of m and g, the range of values of P for which the ladder remains in equilibrium.

(7)
(Total 12 marks)

5.



A loaded plate L is modelled as a uniform rectangular lamina $ABCD$ and three particles. The sides CD and AD of the lamina have lengths $5a$ and $2a$ respectively and the mass of the lamina is $3m$. The three particles have mass $4m$, m and $2m$ and are attached at the points A, B and C respectively, as shown in the diagram above.

(a) Show that the distance of the centre of mass of L from AD is $2.25a$.

(3)

(b) Find the distance of the centre of mass of L from AB.

(2)

The point O is the mid-point of AB. The loaded plate L is freely suspended from O and hangs at rest under gravity.

(c) Find, to the nearest degree, the size of the angle that AB makes with the horizontal.

(3)

A horizontal force of magnitude P is applied at C in the direction CD. The loaded plate L remains suspended from O and rests in equilibrium with AB horizontal and C vertically below B.

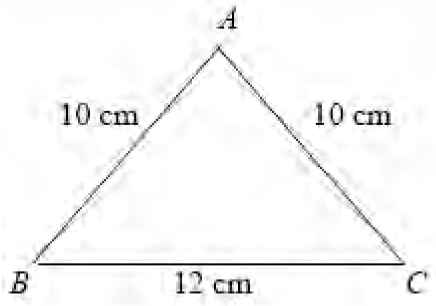
(d) Show that $P = \frac{5}{4}mg$.

(4)

(e) Find the magnitude of the force on L at O.

(4)
(Total 16 marks)

1. (a)



	AB	AC	BC	frame	
mass ratio	10	10	12	32	B1
dist. from BC	4	4	0	$\bar{x}$	B1

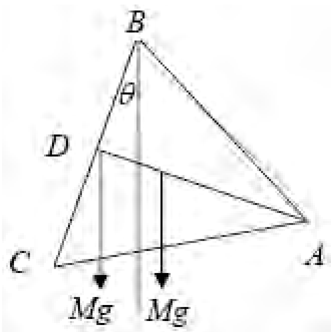
Moments about BC:

$$10\times 4+10\times 4+0=32\bar{x}$$ M1 A1

$$\bar{x}=\frac{80}{32}$$

$$\bar{x}=2\tfrac{1}{2}\quad (2.5)$$ A1 5

(b)



Moments about B:

$$Mg\times 6\sin\theta = Mg\times\left(\bar{x}\cos\theta - 6\sin\theta\right)$$ M1 A1 A1

$$12\sin\theta=\bar{x}\cos\theta$$

$$\tan\theta=\frac{\bar{x}}{12}$$

θ = 11.768..... = 11.8° A1 4

Alternative method :

C of M of loaded frame at distance $\frac{1}{2}\bar{x}$ from D along DA B1

$$\tan\theta = \frac{\frac{1}{2}\bar{x}}{6}$$ M1 A1

θ = 11.768..... = 11.8° A1

[9]

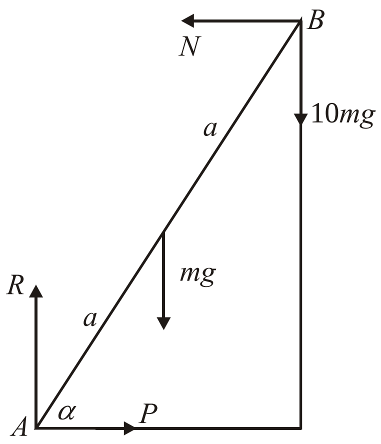
2.	(a)	$M(Oy)$	$(8 + k)m \times 6.4 = 5m \times 8 + km \times 8$		M1A1	
			$1.6k = 11.2 \Rightarrow k = 7$ *	cso	DM1A1	4
	(b)	$M(Oy)$	$27m\bar{x} = 12m \times 4 + 5m \times 8 + 7m \times 8$		M1A1	
			$\bar{x} = \frac{16}{3}$	5.3 or better	A1	
		$M(Ox)$	$27m\bar{y} = 12m \times 2.5 + 8m \times 5$		M1A1	
			$\bar{y} = \frac{70}{27}$	2.6 or better	A1	6
	(c)	$\tan\theta = \frac{\bar{y}}{\bar{x}} = \frac{35}{72}$			M1A1ft	
		$\theta \approx 26°$		awrt 25.9 °	A1	3

[13]

3.	(a)	Total mass = $12m$ (used)		M1	
		(i)	M(AB): $m.3a/2 + m.3a/2 + m.3a + 6m.3a + 2m.3a = 12m.x$	indep M1 A1	
			$\Rightarrow x = \frac{5}{2}a$	A1	
		(ii)	M(AD): $m.a + m.a + m.2a + 6m.2a = 12m.y$	indep M1 A1	
			$y = \frac{4}{3}a$	A1	7
	(b)	$\tan\alpha = \frac{2a - 4a/3}{5a/2}$		M1 A1 f.t	
		$\Rightarrow \alpha \approx 14.9°$		A1cao	3

[10]

4.



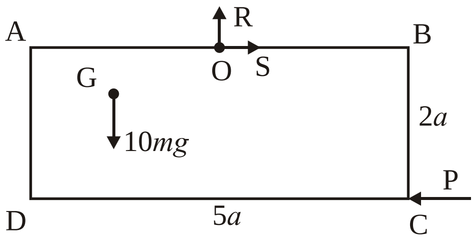
(a)	$\boldsymbol{M}(\text{a})\quad N \times 2a\sin\alpha = mg \times a\cos\alpha + 10mg \times 2a\cos\alpha$	M1 A2(1, 0)	
	$2N\tan\alpha = 21mg$		
	$N = 7mg$ (*)	cso M1 A1	5
(b)	$\uparrow R = 11mg$	B1	
	$F_r = 0.6 \times 11mg = 6.6mg$	B1	
	For min P $F_r \rightarrow P_{\text{min}} = 7mg - 6.6mg = 0.4mg$	M1 A1	
	For max P $F_r \leftarrow P_{\text{max}} = 7mg + 6.6mg = 13.6mg$	M1 A1	
	$0.4mg \le P \le 13.6mg$	cso A1	7

[12]

Note: In (a), if moments are taken about a point other than A, a complete set of equations for finding N is needed for the first M1. *If this* M1 *is gained, the* A2(1, 0) *is awarded for the moments equation as it first appears.*

5.	(a)	AD: $10m\bar{x} = 3m\dfrac{5a}{2} + 3m \times 5a$	M1 A1	
		$\bar{x} = 2.25a$ *	A1	3
	(b)	AB: $10m\bar{y} = 2m \times 2a + 3m \times a$	M1	
		$\bar{y} = 0.7a$	A1	2
	(c)	$\tan\theta = \dfrac{2.5a - \bar{x}}{\bar{y}}$	M1 A1 f.t.	
		$\theta = 20°$ (nearest degree)	A1	3

(d)



M(0), $10mg \times \frac{a}{4} = P \times 2a$ M1 A1 A1

(**OR**: $4mg \times \frac{5a}{2} - 3\,mg \times \frac{5a}{2} = P \times 2a$)

$P = \frac{5mg}{4}$ * (exact) A1 4

(e) $S = \frac{5mg}{4}$; $R = 10mg$ B1; B1

$F = \sqrt{S^2 + R^2} = \frac{5mg\sqrt{65}}{4}$ (10.1 mg) M1 A1 4

[16]

1. Some candidates struggled with this question. Despite the question being explained clearly with reference to rods it was not uncommon to see the triangle treated as a lamina. Another common error was to treat the rods as being of equal mass.

The geometry of the symmetrical triangular figure was appreciated by nearly all candidates with the height of the triangle correctly calculated as 8 cm, although it was disappointing to find several candidates not recognising the 3-4-5 triangle and engaging in more work than expected to find the height of the triangle.

For part (a) those candidates who answered the question as set and worked with three rods had little difficulty in producing a relevant moments equation and arriving at the correct result. However it was disappointing to find a significant number of candidates treating the triangle as a lamina, and they were happy simply to write down the answer as $\frac{8}{3}$ cm.

For part (b) most candidates could either write down or calculate the distance of the new centre of mass from *BC* and proceed to find the required angle. 3 out of 4 marks were available for those who had treated the shape as a lamina. A number of candidates ignored the extra particle added to the framework and answered their own question. Very few students used the method of taking moments about *B* to find the angle.

2. This question was often answered very well. Most candidates took moments about *Oy* or *Ox*, although alternatives were seen. Weaker candidates did not seem to be very confident in dealing with a lamina plus a set of particles.

(a) Full marks were usually seen. The alternative method of taking moments about axes through the centre of mass was seen and was usually implemented successfully.
A small number were too casual in claiming the printed result from a correct moment's equation – candidates need to remember that with a given answer a little more detail is required.

(b) This was usually correct. Many candidates calculated the *y* coordinate of the centre of mass of the three particles as $\frac{8}{3}$ and then used that to calculate the centre of mass of the system.
Any errors were usually in the total mass (e.g. taken to be $40m$ or equal to the total mass of only those which contributed to the moment). A few responses did not include the lamina and scored nothing. Some candidates treated *OABC* as a set of connected rods, rather than as a lamina.

(c) Candidates who answered part (b) incorrectly often picked up two marks here from the follow through. The majority of candidates recognised the correct triangle, although many of them then calculated the incorrect angle. A number of responses used 5 and / or 8, which scored nothing. A few candidates lost the final mark by using rounded values from part (b) and arrived at an incorrect value for the angle.

3. This question was well answered by many candidates and the method in part (a) was well known. Some, however, used "*m*" as mass per unit length for the framework, or counted the masses of the particles more than once in an attempt to consider each rod separately. Common sense often failed to prevail, with the mass of the whole system sometimes appearing as different values in the two equations. Most were able to attempt part (b), but many failed to use $(2a - y)$ in their ratio. The very few who decided to use sine instead of tangent were usually successful.

4. Part (a) was generally well done. The majority of candidates found the most efficient method, which is to take moments about *A*. The use of a calculator is not appropriate in questions like this and, for full marks, the candidate is expected to obtain the exact answer, as printed, using a simple trigonometric identity or equivalent argument. Part (b) was rarely answered fully. Most could find one limiting value of *P*, usually $0.4mg$, the value associated with friction acting towards the wall, but very few realised that, if the builder's friend pushed hard enough, the direction of friction reversed. The commonest way of obtaining a range was to say that the maximum value of *P* was associated with a frictional force of zero. However some candidates, having obtained their first value efficiently by resolving, thought they might get a second value by taking moments about one or more points and it was possible to waste much time this way.

5. Many candidates found the paper too long and relatively few attempts at this question went beyond part (c) and the majority of candidates scored less than half marks. The method for (a) and (b) was well known and there were many correct solutions. The principal error was failure to include the mass of the lamina, producing equations which were missing a term, and divisions by $7m$ instead of $10m$. Some candidates tried to include the lamina by using its area $10a^2$ instead of its given mass. Marks were often lost unnecessarily in part (c) by candidates, who knew exactly what to do, doing it carelessly.

The required angle was found probably less frequently than its complement and those who found the correct answer often failed to round it to the nearest degree. Huge numbers of candidates lost easy marks by not making it clear that they knew the basic method. Re-drawing the diagram to show the centre of mass below the point of suspension is a far less popular option than it used to be. It is therefore essential that candidates indicate that the line they draw through *G* represents the vertical.

It is difficult to know the extent to which (d) and (e) were omitted due to difficulties rather than lack of time but good attempts were rare. Of those who tried (d) a significant proportion were attempting to answer a different question about attaching an extra particle at *C*. Some of those who attempted moments took them about points other than *O* but failed to realise that the unknown force at *O* would contribute.

Far too many attempted to arrive at the printed answer of $1.25mg$ by number juggling, concocting any old equation which looked half-way convincing. The same was true to a lesser extent in part (a). It is sad to see candidates losing marks they possibly deserve by inventing extra terms to produce the right numerical answer and so turning what may have been a minor numerical slip into a wrong method.

Part (e) was rarely attempted and even more rarely correctly. A common misconception was that the force at C was vertical and this led to some worthless moments attempts about C. Some candidates correctly combined the known $10mg$ and $1.25mg$ forces without any reference to the point of suspension, and others simply added up everything which had been mentioned so far.