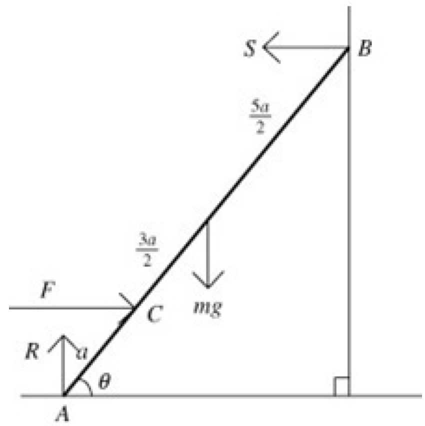
Chapter Review

1 Let S be the reaction of the wall on the ladder at B.
Let R be the reaction of the ground on the ladder at A.
(Both surfaces are smooth, so no friction.)



$R(\rightarrow)$: $F = S$

Taking moments about A:
$mg \times \frac{5}{2}a \times \cos\theta + F \times a \times \sin\theta = S \times 5a \times \sin\theta$

$$\frac{5mg}{2} + F\tan\theta = 5S\tan\theta \qquad \text{(dividing by } a\cos\theta\text{)}$$

$$\frac{5mg}{2} + F\tan\theta = 5F\tan\theta \qquad \text{(Since } F = S\text{)}$$

$$\frac{5mg}{2} = 4F\tan\theta$$

$$= 4 \times \tfrac{9}{5}F \qquad \text{(Since } \tan\theta = \tfrac{9}{5}\text{)}$$

$$= 7.2F$$

$$F = \frac{5mg}{2 \times 7.2}$$

$$= \frac{25mg}{72} \text{ as required.}$$

2 Let N be the reaction of the wall on the ladder at B.
Let R be the reaction of the ground on the ladder at A,
Let F the friction between the ladder and the ground at A.
$\tan\alpha = \frac{3}{4} \Rightarrow \sin\alpha = \frac{3}{5}$ and $\cos\alpha = \frac{4}{5}$

$R(\uparrow)$: $R = mg + 2mg = 3mg$

Taking moments about B:
$mg \times a\sin\alpha + 2mg \times \frac{4}{3}a\sin\alpha + F \times 2a\cos\alpha = R \times 2a\sin\alpha$

$$mga \times \frac{3}{5} + \frac{8mga}{3} \times \frac{3}{5} + F \times 2a \times \frac{4}{5} = 6mga \times \frac{3}{5}$$

$$F \times \frac{8a}{5} = \frac{18mga}{5} - \frac{8mga}{5} - \frac{3mga}{5}$$

$$F \times \frac{8a}{5} = \frac{7mga}{5}$$

$$F = \frac{7mga}{5} \times \frac{5}{8a}$$

$$= \frac{7mg}{8}$$

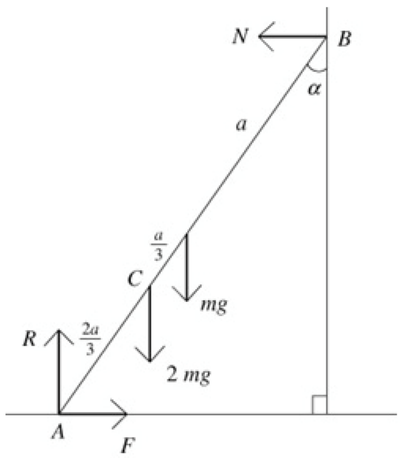
The ladder and the child are in equilibrium, so

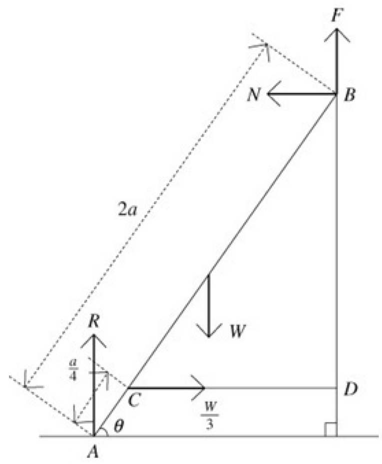
$$F \le \mu R$$

$$\frac{7mg}{8} \le \mu \times 3mg$$

$$\mu \ge \frac{7}{24}$$

The least possible value for μ is $\frac{7}{24}$





3 Let R be the reaction of the ground on the ladder at A.
Let N be the reaction of the wall on the ladder at B.
Let F be the friction between the wall and the ladder at B.

a Since you do not know the magnitude of F, you cannot resolve vertically to find R.
Therefore, take moments about B (since this eliminates F):

$$\frac{W}{3}\times\frac{7a}{4}\sin\theta+W\times a\cos\theta=R\times 2a\cos\theta$$

$$\frac{7W}{12}\times\tan\theta+W=2R \qquad \text{(dividing through by } a\cos\theta\text{)}$$

$$\frac{7W}{12}\times\frac{4}{3}+W=2R \qquad \text{(since } \tan\theta=\tfrac{4}{3}\text{)}$$

$$\frac{16W}{9}=2R$$

$$R=\frac{8W}{9}$$

b $R(\rightarrow)$: $\mathrm{N}=\frac{W}{3}$

$R(\uparrow)$:

$$R+F=W$$

$$F=W-R$$

$$=W-\tfrac{8}{9}W$$

$$=\frac{W}{9}$$

For the ladder to remain in equilibrium,

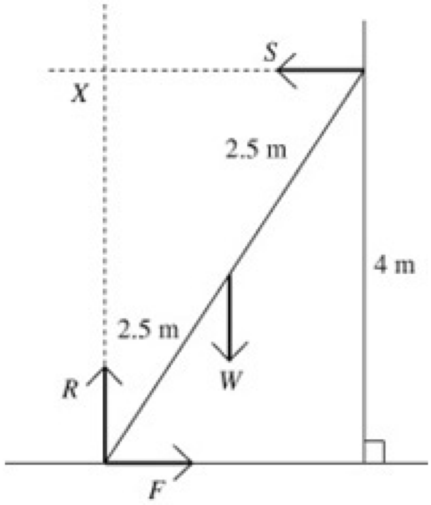
$$F\le\mu N$$

$$\frac{W}{9}\le\mu\frac{W}{3}$$

$$\mu\ge\tfrac{1}{3}$$

c The ladder had negligible thickness / the ladder does not bend.

4 a Let S be the reaction of the wall on the ladder.
Let R be the reaction of the ground on the ladder.
Let F the friction between the ladder and the ground.
Let X be the point where the lines of action of R and S intersect, as shown in the diagram.



By Pythagoras's Theorem, distance from base of ladder to wall is 3 m.

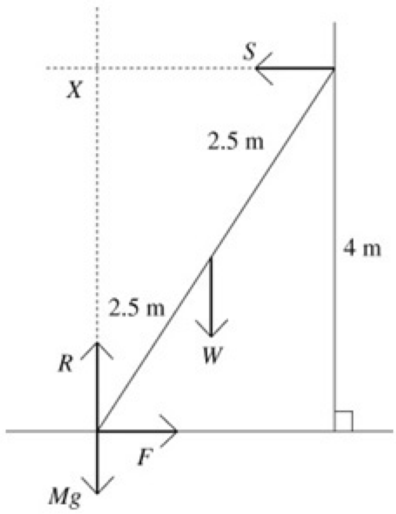
$R(\rightarrow)$: $F = S$
$R(\uparrow)$: $R = W$

Taking moments about X:
$1.5W = 4F$

Suppose the ladder can rest in equilibrium in this position. Then
$F \leq \mu R$
$\frac{1.5W}{4} \leq 0.3 \times W$
$\frac{3W}{8} \leq \frac{3W}{10}$
$30 \leq 24$

which is false, therefore the assumption that $F \leq \mu R$ must be false – the ladder cannot be resting in equilibrium.

b With the brick in place, take moments about X:
$1.5W = 4F$ so
$F = \frac{1.5W}{4} = \frac{3W}{8}$
which is independent of M, the mass of the brick.



c $R(\uparrow)$ $R = W + Mg$
$R(\rightarrow)$ $F = S$

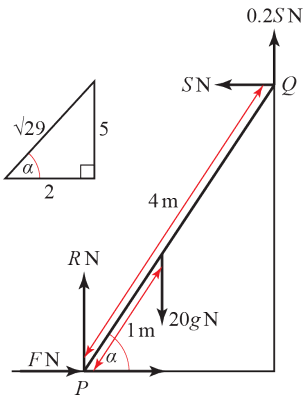
$F \leq \mu R \Rightarrow \frac{3W}{8} \leq 0.3(W + Mg) = \frac{3(W + Mg)}{10}$
$\Rightarrow 10W \leq 8W + 8Mg$
$8Mg \geq 2W$, $M \geq \frac{W}{4g}$

So the smallest value for M is $\frac{W}{4g}$

5 Let S be the reaction of the wall on the ladder at Q
Let R be the reaction of the ground on the ladder at P

$\tan\alpha = \frac{5}{2} \Rightarrow \sin\alpha = \frac{5}{\sqrt{29}}$ and $\cos\alpha = \frac{2}{\sqrt{29}}$

Since the ladder is in limiting equilibrium,
frictional force at the wall = $\mu S = 0.2S$.



Taking moments about P:
$20g \times 1\cos\alpha = S \times 4\sin\alpha + 0.2S \times 4 \times \cos\alpha$

$$\frac{20\times 2}{\sqrt{29}}g = \left(\frac{4\times 5}{\sqrt{29}} + \frac{0.8\times 2}{\sqrt{29}}\right)S$$

$$40g = 21.6S$$

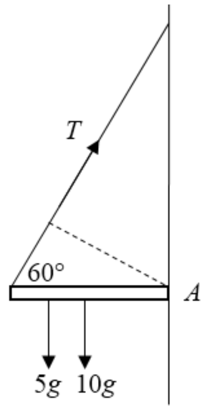
$$S = \frac{392}{21.6} = 18.148...$$

R(→): $F = S$

The force F required to hold the ladder still is 18 N (2 s.f.).

Challenge

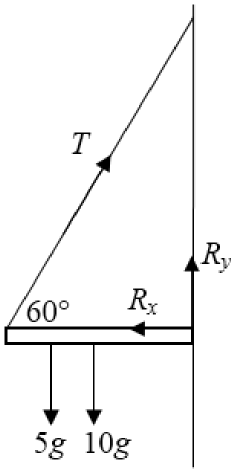
1



a Taking moments about A
$3T\sin 60 = 5g \times 2 + 10g \times 1.5$

$$T = \frac{50}{3\sqrt{3}}g \text{ N}$$

1 b



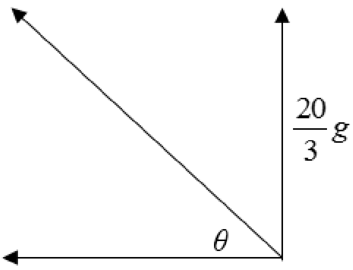
Res(→) $R_x = T\cos 60$

$= \frac{50}{6\sqrt{3}}g$

Res(↑) $T\sin 60 + R_y = 15g$

$R_y = 15g - \frac{25}{3}g$

$= \frac{20}{3}g$



$R = \sqrt{R_x^2 + R_y^2}$

$= \sqrt{\left(\frac{50}{6\sqrt{3}}g\right)^2 + \left(\frac{20}{3}g\right)^2}$

$= 80.570...$

$= 80.6$ N (3 s.f.)

$\tan\theta = \frac{\frac{20}{3}g}{\frac{50}{6\sqrt{3}}g}$

$= \frac{120\sqrt{3}}{150}$

$\theta = 54.182...$

$= 54.2°$ (3 s.f.)

Therefore at 54.2° to the horizontal.