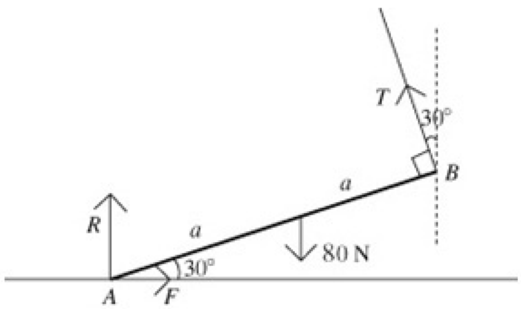
Exercise 6A

1 Suppose that the rod has length $2a$.



Taking moments about A:

$$2aT = 80 \times a\cos 30°$$
$$2T = 80 \times \frac{\sqrt{3}}{2}$$
$$T = 20\sqrt{3}$$
$$= 34.6 \text{ N}$$

$R(\rightarrow)$, $F = T\sin 30° = 10\sqrt{3} = 17.3 \text{ N}$

$R(\uparrow)$, $T\cos 30° + R = 80$

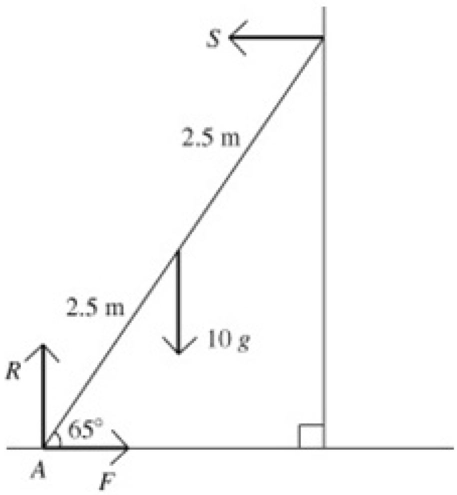
$$R = 80 - 20\sqrt{3} \times \frac{\sqrt{3}}{2}$$
$$= 50 \text{ N}$$

In order for the rod to remain in equilibrium, we must have $F \le \mu R$:

$$10\sqrt{3} \leqslant \mu \times 50$$
$$\mu \geqslant \frac{10\sqrt{3}}{50}$$
$$\mu \geqslant \frac{\sqrt{3}}{5}$$

$\therefore$ minimum $\mu = 0.35$ (2 s.f.)

2 Let A be the end of the ladder on the ground.
Let F be the frictional force at A.



a Taking moments about A:

$$10g \times 2.5\cos 65° = S \times 5\sin 65°$$
$$S = \frac{25g\cos 65°}{5\sin 65°}$$
$$= \frac{5g}{\tan 65°}$$
$$= 22.8 \text{ N}$$

b $R(\rightarrow)$, $F = S = 22.8 \text{ N}$

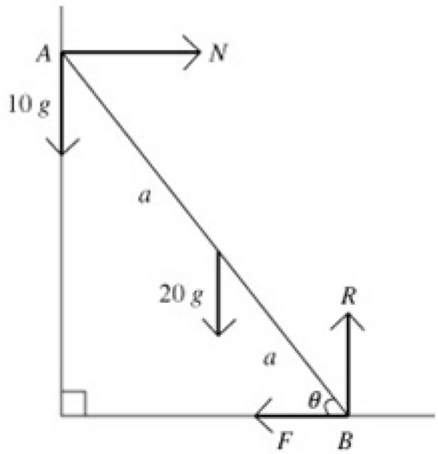
$R(\uparrow)$, $R = 10g = 98 \text{ N}$

c To ensure ladder remains in equilibrium, we must have

$$F \leqslant \mu R$$
$$22.8 \leqslant \mu \times 98$$
$$\mu \geqslant 0.233 \text{ (3 s.f.)}$$

2 d The weight is shown as acting through the midpoint of the ladder because of the assumption that the ladder is uniform.

3 Let the ladder have length $2a$, and be inclined at θ to the horizontal.



a $R(\uparrow),\quad R = 30g$

Taking moments about A:

$20g \times a\cos\theta + F \times 2a\sin\theta = R \times 2a\cos\theta$

$20g\cos\theta + 2F\sin\theta = 60g\cos\theta \quad (\text{using } R = 30g)$

$2F\sin\theta = 40g\cos\theta$

$$F = \frac{20g}{\tan\theta}$$

The ladder is on the point of slipping, so $F = \mu R$

$$\frac{20g}{\tan\theta} = \frac{3}{4} \times 30g$$

$$\therefore \tan\theta = \frac{2}{3} \times \frac{4}{3} = \frac{8}{9}$$

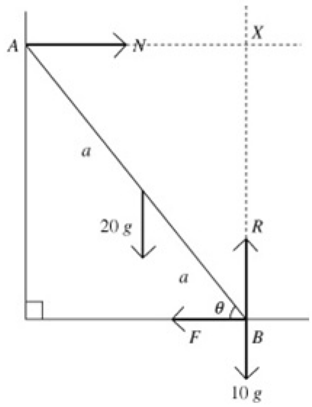
$$\therefore \theta = 41.6°$$

N is the normal reaction at A,
R is the normal reaction at B,
F is the frictional force at B.

b $R(\uparrow),\quad R = 30g$

$R(\rightarrow),\quad N - F = 0$

$N = F$



Taking moments about B:

$20g \times a\cos\theta = N \times 2a\sin\theta$

$20g \times a\cos\theta = F \times 2a\sin\theta$

$$F = \frac{10g\cos\theta}{\sin\theta}$$

$$F = \frac{10g}{\tan\theta}$$

The ladder is on the point of slipping, so $F = \mu R$

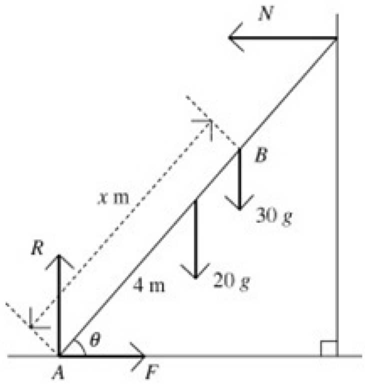
$$\frac{10g}{\tan\theta} = \frac{3}{4} \times 30g$$

$$\tan\theta = \frac{4}{9}$$

$$\theta = 24.0°$$

c The assumption that the wall is smooth means there is no friction between the ladder and the wall.

4 a Suppose that the boy reaches the point B, a distance x from A, whilst the end of the ladder is still in contact with the ground.



$R(\rightarrow)$, $F = N$

$R(\uparrow)$, $R = 50g$

Taking moments about A:

$$20g \times 4\cos\theta + 30g \times x\cos\theta = N \times 8\sin\theta$$

$$80g + 30gx = 8N\tan\theta$$

$$N = \frac{80g + 30gx}{8\tan\theta}$$

$$N = \frac{80g + 30gx}{16} \quad \text{(since } \tan\theta = 2\text{)}$$

$$F = \frac{80g + 30gx}{16} \quad \text{(since } F = N\text{)}$$

$$\mu R = \frac{80g + 30gx}{16} \quad \text{(in limiting equilibrium)}$$

$$0.3 \times 50g = \frac{80g + 30gx}{16}$$

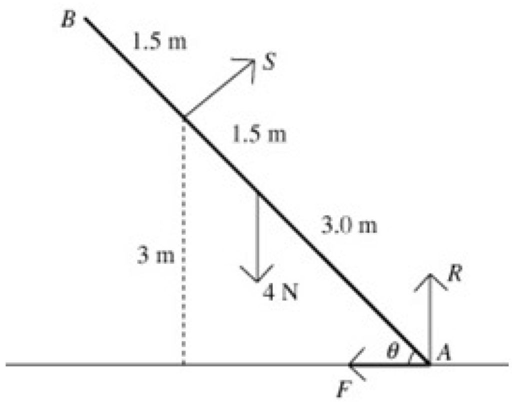
$$240 = 80 + 30x$$

$$x = 5\frac{1}{3} \text{ m}$$

b i The ladder may not be uniform.

ii There would be friction between the ladder and the wall.

5 Let:
S be the normal reaction of the rail on the pole at C,
R be the normal reaction of the ground on the pole at A,
F be the friction between the pole and the ground at A.
θ be the angle between the pole and the ground.



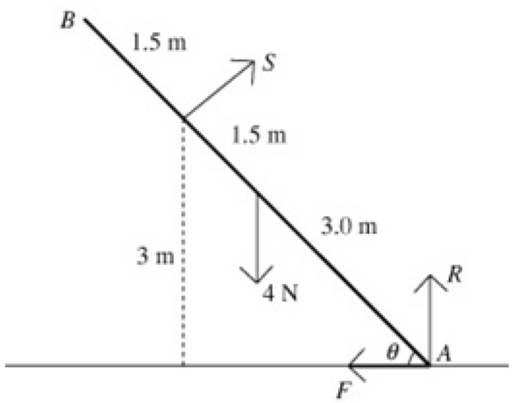
From the diagram,

$$\sin\theta = \frac{3}{4.5} = \frac{2}{3} \text{ and hence } \cos\theta = \frac{\sqrt{9-4}}{3} = \frac{\sqrt{5}}{3}$$

a Taking moments about A:

$$4.5S = 4\times 3\cos\theta$$
$$= \frac{12\sqrt{5}}{3}$$
$$= 4\sqrt{5}$$
$$S = \frac{8\sqrt{5}}{9}\text{ N}$$

b $R(\rightarrow)$



$$F = S\sin\theta$$
$$= \frac{8\sqrt{5}}{9}\times\frac{2}{3}$$
$$= \frac{16\sqrt{5}}{27}$$

$R(\uparrow)$

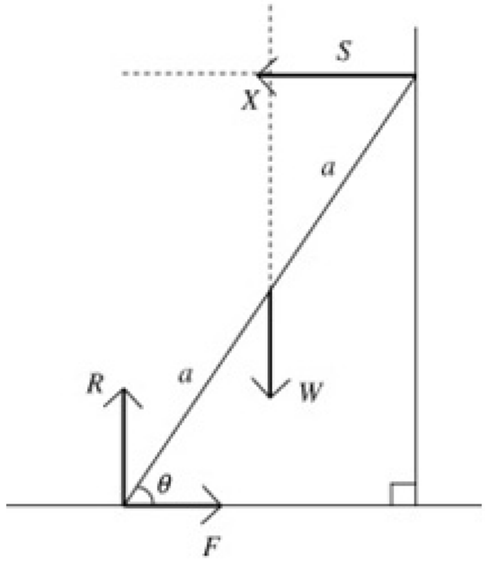
$$R + S\cos\theta = 4$$
$$R = 4 - \frac{8\sqrt{5}}{9}\times\frac{\sqrt{5}}{3}$$
$$= 4 - \frac{40}{27}$$
$$= \frac{68}{27}$$

Pole is in limiting equilibrium, so $F = \mu R$

$$\frac{16\sqrt{5}}{27} = \mu\times\frac{68}{27}$$
$$\therefore \mu = \frac{16\sqrt{5}}{68}$$
$$= \frac{4\sqrt{5}}{17}$$
$$= 0.526 \text{ (3 s.f.)}$$

c The assumption that the rail is smooth means there is no friction between the rail and the pole.

6 Suppose that the ladder has length $2a$ and weight W.
Let:
S be the normal reaction of the wall on the ladder,
R be the normal reaction of the floor on the ladder,
F be the friction between the floor and the ladder.
X be the point where the lines of action of W and S meet.



Taking moments about X:

$$2a\sin\theta \times F = R \times a\cos\theta$$

$$2F\sin\theta = R\cos\theta \qquad (\mathbf{1})$$

The ladder is in limiting equilibrium, so $F = \mu R$

Substituting $F = \mu R$ in (**1**):

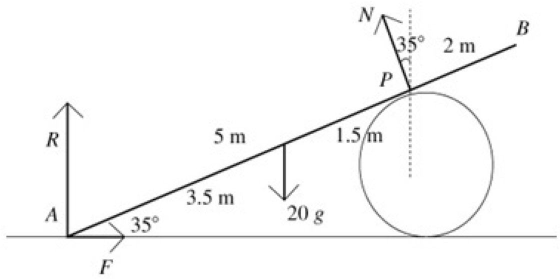
$$2\mu R\sin\theta = R\cos\theta$$

$$2\mu\sin\theta = \cos\theta$$

$$\frac{2\mu\sin\theta}{\cos\theta} = 1$$

$$2\mu\tan\theta = 1$$

7 Let:
N be the normal reaction of the drum on the ladder at P,
R be the normal reaction of the ground on the ladder at A,
F be the friction between the ground and the ladder at A.



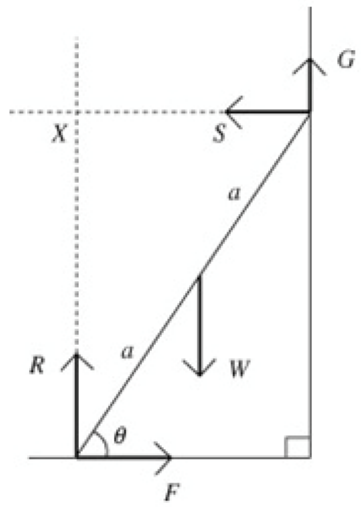
Taking moments about A:
$$20g \times 3.5\cos 35° = 5N$$
$$N = \frac{20g \times 3.5\cos 35°}{5}$$
$$= 14g\cos 35°$$

$R(\uparrow)$
$$N\cos 35° + R = 20g$$
$$R = 20g - 14g\cos 35° \times \cos 35°$$
$$= 103.9\ldots \text{N}$$

$R(\rightarrow)$
$$F = N\sin 35°$$
$$= 14g\cos 35° \times \sin 35°$$
$$= 64.46\ldots \text{N}$$

$F \le \mu R$ to maintain equilibrium:
$$14g\cos 35°\sin 35° \le \mu(20g - 14g\cos^2 35°)$$
$$\mu \ge \frac{14\cos 35°\sin 35°}{20 - 14\cos^2 35°}$$
$$\mu \ge 0.620 \text{ (3 s.f.)}$$
Least possible μ is 0.620 (3 s.f.)

8 Let:
R be the reaction of the ground on the ladder
F be the friction between the ground and the ladder
S be the reaction of the wall on the ladder
G be the friction between the wall and the ladder.
X be the point where the lines of action R and S meet.



Suppose that the ladder has length $2a$ and weight W.

As the ladder rests in limiting equilibrium, $F = \mu_1 R$ and $G = \mu_2 S$.

Taking moments about X:
$W \times a\cos\theta = F \times 2a\sin\theta + G \times 2a\cos\theta$
$W = 2F\tan\theta + 2G$ **(1)**

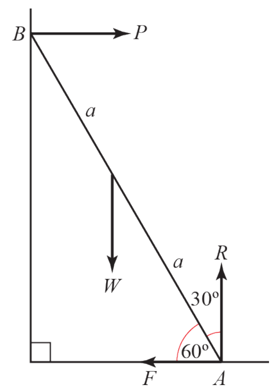
$R(\rightarrow)$, $F = S$
$R(\uparrow)$, $W = R + G$

Substituting for W and F in equation (**1**):

$R + G = 2\mu_1 R\tan\theta + 2G$
$R - G = 2\mu_1 R\tan\theta$
$R - \mu_1\mu_2 R = 2\mu_1 R\tan\theta$ (Since $G = \mu_2 S = \mu_2 F = \mu_2\mu_1 R$

Hence $\dfrac{1-\mu_1\mu_2}{2\mu_1} = \tan\theta$

9 Let:
A and B be the ends of the ladder.
P be the normal reaction of the wall on the ladder at B,
R the normal reaction of the ground on the ladder at A
F be the friction at between the ladder and the ground at A



Let the length of the ladder be $2a$.

a Taking moments about A:
$W \times a\cos 60° = P \times 2a\cos 30°$

$$P = \frac{Wa\cos 60°}{2a\cos 30°}$$

$$P = \frac{W \times \frac{1}{2}}{2 \times \frac{\sqrt{3}}{2}}$$

$$P = \frac{W}{2\sqrt{3}} \qquad \textbf{(1)}$$

b $R(\uparrow)$, $\quad R = W \qquad$ **(2)**

$R(\rightarrow)$, $\quad F = P \qquad$ **(3)**

Now $F \le \mu R$ since the ladder is in equilibrium (if not, ladder would slide)

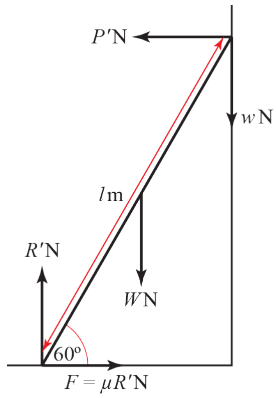
Hence, $\quad P \leqslant \mu R \quad$ (by (**3**))

$$\frac{W}{2\sqrt{3}} \leqslant \mu R \quad \text{(by (}\mathbf{1}\text{))}$$

$$\frac{W}{2\sqrt{3}} \leqslant \mu W \quad \text{(by (}\mathbf{2}\text{))}$$

$$\mu \geqslant \frac{\sqrt{3}}{6}$$

9 c Let:
R' be the normal reaction of the ground on the ladder at A
P' be the normal reaction of the wall on the ladder at B,
l be the length of the ladder



Since the ladder is in limiting equilibrium, $F' = \mu R'$

$R(\uparrow)$, $R' = W + w$

R($\rightarrow$), $\mu R' = P'$

Taking moments about B:

$$\frac{Wl\cos 60^\circ}{2} + \left(F' \times l\sin 60^\circ\right) = \left(R' \times l\cos 60^\circ\right)$$

$$\frac{W}{4} + \left(\mu R' \times \frac{\sqrt{3}}{2}\right) = \frac{R'}{2}$$

$$\frac{W}{4} + \left(\frac{\sqrt{3}}{5}(W+w) \times \frac{\sqrt{3}}{2}\right) = \frac{W+w}{2} \qquad \left(\text{since } R' = W + w \text{ and } \mu = \frac{\sqrt{3}}{5}\right)$$

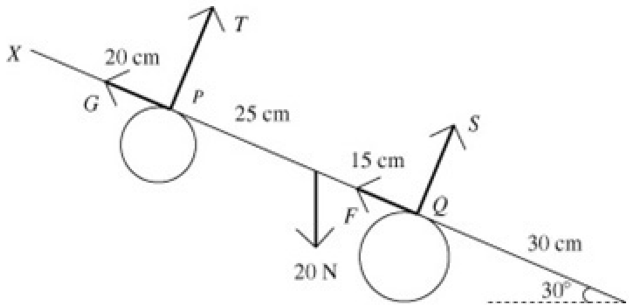
$$W + \frac{6}{5}(W+w) = 2(W+w)$$

$$5W + 6W + 6w = 10W + 10w$$

$$W = 4w$$

$$\Rightarrow w = \frac{W}{4}$$

10 Let:
T be the normal force of the peg on the rod at P,
G be the frictional force at P,
S be the normal force of the peg on the rod at Q,
F be the frictional force at Q.



a Taking moments about P:
$S \times 40 = 20 \times 25 \times \cos 30°$

$$S = \frac{20 \times 25 \times \frac{\sqrt{3}}{2}}{40}$$

$$S = \frac{25\sqrt{3}}{4}\text{ N}$$

Taking moments about Q:
$T \times 40 = 20 \times 15 \times \cos 30°$

$$T = \frac{20 \times 15 \times \frac{\sqrt{3}}{2}}{40}$$

$$T = \frac{15\sqrt{3}}{4}\text{ N}$$

b $R(\searrow)$
$G + F = 20\cos 60° = 10$ **(1)**

Since the rod is about to slip, friction is limiting and hence $G = \mu T$, $F = \mu S$.
From part **a**,

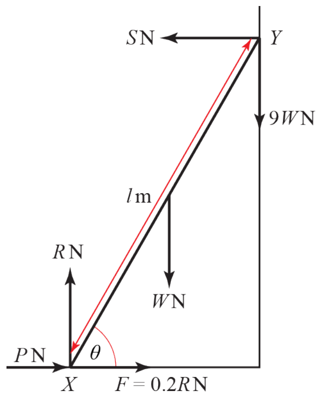
$$G + F = \mu T + \mu S = \mu \times \frac{40\sqrt{3}}{4} = 10\sqrt{3}\mu \quad \textbf{(2)}$$

$$\textbf{(1)} = \textbf{(2)} \Rightarrow \mu = \frac{1}{\sqrt{3}}$$

11 a Let:
S be the normal reaction of the wall on the ladder at Y,
R be the normal reaction of the ground on the ladder at X
F be the friction at between the ladder and the ground at X

$\tan\theta = \sqrt{3}$ so $\sin\theta = \frac{\sqrt{3}}{2}$ and $\cos\theta = \frac{1}{2}$

Ladder is in equilibrium.
Taking moments about X:



$$\frac{Wl\cos\theta}{2} + 9Wl\cos\theta = Sl\sin\theta$$

$$\frac{W}{4} + \frac{9W}{2} = \frac{\sqrt{3}S}{2}$$

$$\sqrt{3}S = \frac{W}{2} + 9W$$

$$\sqrt{3}S = \frac{19W}{2}$$

$$S = \frac{19W}{2\sqrt{3}}$$

b R(↑): $R = W + 9W = 10W$
For the ladder to be in limiting equilibrium,
$F = \mu R$

$F = \frac{1}{5} \times 10W$

$F = 2W$

R(→):
If $P + F > S$, ladder will slide towards and up the wall
If $P < S - F$, ladder will slide away from and down the wall
Therefore $S - F \leq P \leq S + F$

Substituting values for S & F from part **a** and above:

$$\frac{19W}{2\sqrt{3}} - 2W \leq P \leq \frac{19W}{2\sqrt{3}} + 2W$$

$$\left(\frac{19}{2\sqrt{3}} - 2\right)W \leq P \leq \left(\frac{19}{2\sqrt{3}} + 2\right)W$$

c Modelling the ladder as uniform allows us to assume the weight acts through the midpoint.

d i The reaction of the wall on the ladder will decrease. To understand why, consider how we took moments about X in part **a**

$$\frac{Wl\cos\theta}{2} + 9Wl\cos\theta = Sl\sin\theta$$

The first term in this equation is the turning moment of the weight of the ladder, which acts at a distance $\frac{l}{2}$ from X. If the centre of mass of the ladder is more towards X, say $\frac{l}{a}$ where $a > 2$, then this first term would decrease and hence S would also decrease.

11 d ii Ladder remains in equilibrium when $S - F \leq P \leq S + F$
If S were to decrease, then this range of values for P would also decrease