Exercise 5C

1 Using Newton's law of restitution $e = \dfrac{\text{speed of rebound}}{\text{speed of approach}}$

a $e = \dfrac{4}{10} = \dfrac{2}{5} = 0.4$

b $e = \dfrac{3}{6} = \dfrac{1}{2} = 0.5$

2 Using Newton's law of restitution $e = \dfrac{\text{speed of rebound}}{\text{speed of approach}}$ and making speed of rebound the subject of the formula, so speed of rebound $(v) = e \times$ speed of approach

a $v = e \times \text{speed of approach} = \dfrac{1}{2} \times 7 = \dfrac{7}{2} = 3.5$

Speed of sphere after collision is $3.5\,\text{m}\,\text{s}^{-1}$.

b $v = e \times \text{speed of approach} = \dfrac{1}{4} \times 12 = 3$

Speed of sphere after collision is $3\,\text{m}\,\text{s}^{-1}$.

3 Using Newton's law of restitution $e = \dfrac{\text{speed of rebound}}{\text{speed of approach}}$ and making speed of approach the subject of the formula, so speed of approach $(u) = \dfrac{\text{speed of rebound}}{e}$

a $u = \dfrac{\text{speed of rebound}}{e} = 2 \times 4 = 8$

Speed of sphere before collision is $8\,\text{m}\,\text{s}^{-1}$.

b $u = \dfrac{\text{speed of rebound}}{e} = \dfrac{4 \times 6}{3} = 8$

Speed of sphere before collision is $8\,\text{m}\,\text{s}^{-1}$.

4 Using Newton's law of restitution

$e = \dfrac{\text{speed of rebound}}{\text{speed of approach}} = \dfrac{7.5}{10} = 0.75$

5 The particle falls under gravity. Find the speed of the particle when it hits the plane by using the constant acceleration formula $v^2 = u^2 + 2as$, where $s = 2.5$, $a = g = 9.8$ and $u = 0$. This gives:

$v^2 = 2 \times g \times 2.5 = 5g = 5 \times 9.8 = 49$

$\Rightarrow\ v = \sqrt{49} = 7$

So the particle strikes the plane with a speed of $7\,\text{m s}^{-1}$.

After it rebounds the particle moves under gravity to a height of 1.5 m. Use the constant acceleration formula again to find its initial (rebound) speed, where in this case $s = 1.5$, $a = -g = -9.8$ and $v = 0$. This gives:

$0 = u^2 - 2g \times 1.5$

$u^2 = 3g = 3 \times 9.8 = 29.4$

$\Rightarrow u = \sqrt{29.4} = 5.422$

So the particle rebounds from the plane with a speed of $5.422\,\text{m s}^{-1}$.

Using Newton's law of restitution

$$e = \frac{\text{speed of rebound}}{\text{speed of approach}} = \frac{5.422}{7} = 0.77 \text{ (2 s.f.)}$$

6 a The particle falls under gravity. Find the speed of the particle when it hits the plane by using the constant acceleration formula $v^2 = u^2 + 2as$, where $s = 3$, $a = g$ and $u = 0$. This gives:

$v^2 = 2 \times g \times 3 = 6g$

$\Rightarrow\ v = \sqrt{6g}$

It hits the ground and rebounds. Use Newton's law of restitution to find the speed of rebound:

$$e = 0.25 = \frac{\text{speed of rebound}}{\text{speed of approach}} = \frac{\text{speed of rebound}}{\sqrt{6g}}$$

$\Rightarrow \text{speed of rebound} = 0.25\sqrt{6g}$

It rebounds and moves under gravity. Find the height the particle rebounds to by using the constant acceleration formula $v^2 = u^2 + 2as$, where $s = h$, $a = -g$, $u = 0.25\sqrt{6g}$ and $v = 0$. This gives:

$0 = \left(0.25\sqrt{6g}\right)^2 - 2gh$

$2gh = 0.625 \times 6g$

$$\Rightarrow h = \frac{0.375g}{2g} = 0.1875$$

So the particle rebounds to a height of 18.75 cm.

b If $e > 0.25$ the collision between the sphere and the plane would be more elastic, so the particle would rebound to a greater height.

7 The sphere falls under gravity. Find the speed of the particle when it hits the plane by using the constant acceleration formula $v = u + at$, where $t = 2, a = g = 9.8$ and $u = 0$. This gives:
$v = u + at \Rightarrow v = 2g$

The sphere then bounces and its speed of rebound is $2ge$, where e is the coefficient of restitution. It then moves under gravity for 2 seconds. Find e, by considering the motion after the first impact using the constant acceleration formula $s = ut + \frac{1}{2}at^2$, where $s = 0, t = 2, a = -g = -9.8$ and $u = 2ge$.

This gives:

$$0 = 2ge \times 2 - \frac{1}{2}g \times 4 = 4ge - 2g$$

So $4ge = 2g$

$$\Rightarrow e = \frac{2g}{4g} = \frac{1}{2}$$

The coefficient of restitution is $\frac{1}{2}$

8 The sphere falls under gravity. Find the speed of the particle when it hits the plane by using the constant acceleration formula $v = u + at$, where $t = 3, a = g = 9.8$ and $u = 0$. This gives:
$v = u + at = 3g$

The sphere then bounces and its speed of rebound is $3ge$, where e is the coefficient of restitution, i.e. speed of rebound = $3 \times 0.49g = 1.47g$. It then moves under gravity for t seconds before bouncing a second time. Find t, by considering the motion after the first impact using the constant acceleration formula $s = ut + \frac{1}{2}at^2$, where $s = 0, a = -g = -9.8$ and $u = 1.47g$. This gives:

$0 = 1.47gt - \frac{1}{2}gt^2$ (note $t = 0$ is the time at which the sphere bounces up from the plane)

$$\Rightarrow t = \frac{2 \times 1.47g}{g} = 2.94 \text{ s}$$

9 After it rebounds the particle moves under gravity to a height of $0.5h$ m. Find the speed of the particle when it rebounds from the plane by using the constant acceleration formula $v^2 = u^2 + 2as$, where $s = 0.5h, a = -g$ and $v = 0$. This gives:

$$0 = u^2 - 2g \times 0.5h$$

$$u^2 = gh$$

$$\Rightarrow u = \sqrt{gh}$$

Find the height of the particle 1 second after it rebounds from the plane by using the constant acceleration formula $s = ut + \frac{1}{2}at^2$, where $t = 1, a = -g = -9.8$ and $u = \sqrt{gh}$. This gives:

$$s = \left(\sqrt{gh} - \frac{g}{2}\right) \text{ m}$$

Challenge

The particle falls under gravity. Find the speed of the particle when it hits the plane by using the constant acceleration formula $v^2 = u^2 + 2as$, where $s = h, a = g$ and $u = 0$. This gives:

$v^2 = 2gh$

$\Rightarrow v = \sqrt{2gh}$

Use Newton's law of restitution to find the speed of rebound:

$e = \frac{\text{speed of rebound}}{\text{speed of approach}} = \frac{\text{speed of rebound}}{\sqrt{2gh}}$

$\Rightarrow \text{speed of rebound} = e\sqrt{2gh}$

Newton's law of restitution gives speed of separation from floor as $e\sqrt{2gh}$

It rebounds and moves under gravity. Find the height the particle rebounds to by using the constant acceleration formula $v^2 = u^2 + 2as$, where $a = -g, u = e\sqrt{2gh}$ and $v = 0$. This gives:

$0 = \left(e\sqrt{2gh}\right)^2 - 2gs$

$\Rightarrow s = \frac{e^2 2gh}{2g} = he^2$