## Exercise 5B

**1** Use Newton's law of restitution $e = \dfrac{\text{speed of separation}}{\text{speed of approach}}$

**a** $e = \dfrac{4-0}{6-0} = \dfrac{2}{3}$

**b** $e = \dfrac{3-2}{4-2} = \dfrac{1}{2}$

**c** $e = \dfrac{2-(-3)}{9-(-6)} = \dfrac{5}{15} = \dfrac{1}{3}$

**2** **a** Using conservation of linear momentum for the system $(\rightarrow)$:

$0.25 \times 6 + 0.5 \times 0 = 0.25v_1 + 0.5v_2$

Multiply this equation by 4:

$6 = v_1 + 2v_2$ **(1)**

Using Newton's law of restitution:

$\dfrac{1}{2} = \dfrac{v_2 - v_1}{6-0}$

$\Rightarrow 3 = v_2 - v_1$ **(2)**

Add equations **(1)** and **(2)**:

$9 = 3v_2$

$\Rightarrow v_2 = 3$

Substituting this value into equation **(1)** gives:

$6 = v_1 + 2 \times 3$

$\Rightarrow v_1 = 0$

After the collision, $A$ is at rest and $B$ moves at $3\,\text{m s}^{-1}$.

**2** **b** Using conservation of linear momentum for the system $(\rightarrow)$:

$2\times4+3\times2=2v_1+3v_2$

$\Rightarrow 14=2v_1+3v_2$ **(1)**

Using Newton's law of restitution:

$0.25=\dfrac{v_2-v_1}{4-2}$

$\Rightarrow 0.5=v_2-v_1$ **(2)**

Multiply equation **(2)** by 2 and add to equation **(1)**:

$15=5v_2$

$\Rightarrow v_2=3$

Substituting this value into equation **(1)** gives:

$14=2v_1+3\times3$

$\Rightarrow v_1=\dfrac{5}{2}=2.5$

After the collision, $A$ and $B$ move with speeds of $2.5\,\text{m s}^{-1}$ and $3\,\text{m s}^{-1}$ respectively.

**c** Using conservation of linear momentum for the system $(\rightarrow)$:

$3\times8+1\times(-6)=3v_1+1v_2$

$\Rightarrow 18=3v_1+v_2$ **(1)**

Note that in deriving equation **(1)** the speed of particle $B$ appears in the equation as –6 because it is directed to the left in the diagram.

Using Newton's law of restitution:

$\dfrac{1}{7}=\dfrac{v_2-v_1}{8-(-6)}$

$\Rightarrow 2=v_2-v_1$ **(2)**

Subtracting equation **(2)** from equation **(1)** gives:

$16=4v_1$

$\Rightarrow v_1=4$

Substituting this value into equation **(1)** gives:

$18=3\times4+v_2$

$\Rightarrow v_2=6$

This answer may be checked by using equation **(2)**.

After the collision, $A$ and $B$ move with speeds of $4\text{ m s}^{-1}$ and $6\text{ m s}^{-1}$ respectively.

**2 d** Using conservation of linear momentum for the system $(\rightarrow)$:

$$0.4\times 6+0.4\times(-6)=0.4v_1+0.4v_2$$

$$\Rightarrow 0=v_1+v_2 \qquad \textbf{(1)}$$

Using Newton's law of restitution:

$$\frac{2}{3}=\frac{v_2-v_1}{6-(-6)}=\frac{v_2-v_1}{12}$$

$$\Rightarrow v_2-v_1=8 \qquad \textbf{(2)}$$

Adding equations **(1)** and **(2)** gives:

$$2v_2=8$$

$$\Rightarrow v_2=4$$

Substituting this value into equation **(1)** gives:

$$v_1=-4$$

After the collision, the speeds of $A$ and $B$ are 4 m s$^{-1}$, and both particles change direction.

**e** Noting that the particle moving in the opposite direction (i.e. to the left) has a negative velocity in the equation, using conservation of linear momentum for the system $(\rightarrow)$:

$$5\times 3+4\times(-12)=5v_1+4v_2$$

$$\Rightarrow -33=5v_1+4v_2 \qquad \textbf{(1)}$$

Using Newton's law of restitution:

$$\frac{1}{5}=\frac{v_2-v_1}{3-(-12)}=\frac{v_2-v_1}{15}$$

$$\Rightarrow 3=v_2-v_1 \qquad \textbf{(2)}$$

Multiply equation **(2)** by 5 and add to equation **(1)** to obtain:

$$-18=9v_2$$

$$\Rightarrow v_2=-2$$

Substituting this value into equation **(1)** gives:

$$-33=5v_1-8$$

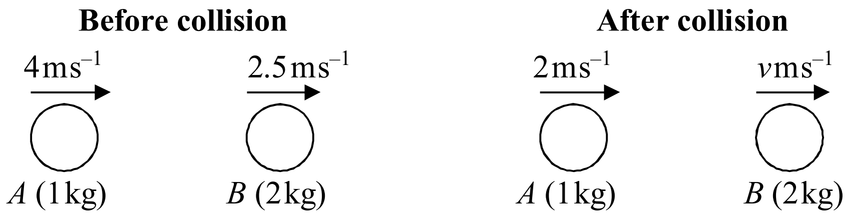
$$-25=5v_1$$

$$\Rightarrow v_1=-5$$

This answer may be checked by using equation **(2)**.

After the collision, the speeds of $A$ and $B$ are 5 m s$^{-1}$ and 2 m s$^{-1}$ respectively, and both particles move to the left, i.e. particle $A$ changes direction in the collision.

**3 a** Draw a clearly labelled diagram




Using conservation of linear momentum for the system $(\rightarrow)$:

$1 \times 4 + 2 \times 2.5 = 1 \times 2 + 2v$

$9 = 2 + 2v$

$2v = 7$

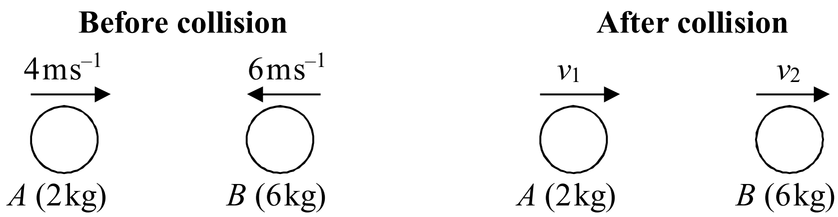
$\Rightarrow v = 3.5$

Speed of $B$ after the collision is $3.5\,\text{m s}^{-1}$.

**b** Using Newton's law of restitution:

$$e = \frac{v-2}{4-2.5} = \frac{3.5-2}{4-2.5} = \frac{1.5}{1.5} = 1$$

**4**




Using conservation of linear momentum for the system $(\rightarrow)$:

$2\times 4+6\times(-6)=2v_1+6v_2$

$\Rightarrow\ -14=v_1+3v_2$ **(1)**

Using Newton's law of restitution:

$\frac{1}{5}=\frac{v_2-v_1}{4-(-6)}=\frac{v_2-v_1}{10}$

$\Rightarrow\ 2=v_2-v_1$ **(2)**

Adding equations **(1)** and **(2)** gives:

$-12=4v_2\Rightarrow v_2=-3$

Substituting this value into equation **(2)** gives:

$2=-3-v_1\Rightarrow v_1=-5$

After the collision, the speeds of $A$ and $B$ are $5\,\text{m s}^{-1}$ and $3\,\text{m s}^{-1}$ respectively, and both particles move in the direction sphere $B$ was moving before the impact.

The impulse of sphere $B$ on sphere $A$ = change in momentum of sphere $A$

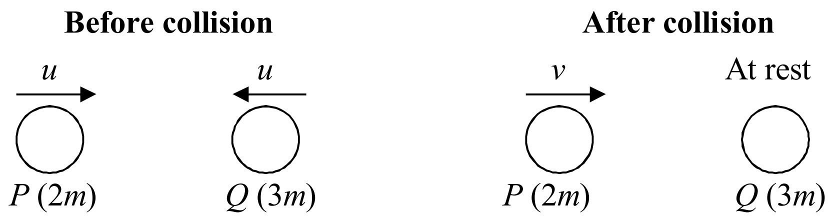
$=2\times(-5)-2\times 4=-18\,\text{N s}$

The impulse of sphere $A$ on sphere $B$ = change in momentum of sphere $B$

$=6\times(-3)-6\times(-6)=18\,\text{N s}$

Spheres $A$ and $B$ experience equal and opposite impulses of magnitude $18\,\text{N s}$.

**5**




Using conservation of linear momentum for the system $(\rightarrow)$:

$2mu - 3mu = 2mv + 3m \times 0$

$-mu = 2mv$

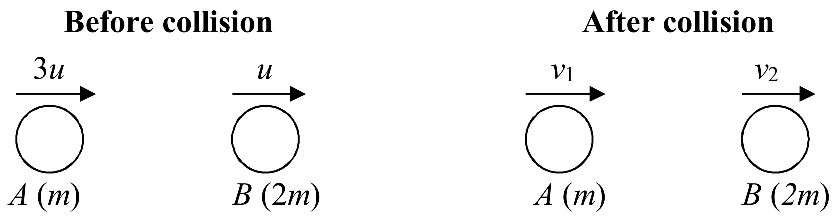
$\Rightarrow v = -\frac{u}{2}$

After the collision, particle $P$ changes direction and has a speed of $0.5u\,\text{m s}^{-1}$

Using Newton's law of restitution:

$e = \frac{0 - v}{u - (-u)} = \frac{\frac{u}{2}}{2u} = \frac{1}{4}$

**6**




Using conservation of linear momentum for the system $(\rightarrow)$:

$m \times 3u + 2m \times u = mv_1 + 2mv_2$

$\Rightarrow v_1 + 2v_2 = 5u$ (cancelling out the common factor $m$) **(1)**

Using Newton's law of restitution:

$e = \frac{v_2 - v_1}{3u - u} = \frac{v_2 - v_1}{2u}$

$\Rightarrow v_2 - v_1 = 2ue$ **(2)**

Adding equations **(1)** and **(2)** gives:

$3v_2 = u(5 + 2e) \Rightarrow v_2 = \frac{u}{3}(5 + 2e)$

Substituting into equation **(1)** gives:

$\frac{u}{3}(5 + 2e) - v_1 = 2ue$

$3v_1 = 5u + 2ue - 6ue = u(5 - 4e)$

$\Rightarrow v_1 = \frac{u}{3}(5 - 4e)$

**7**


Using conservation of linear momentum for the system $(\rightarrow)$:

$m \times 2u + m \times (-3u) = m(-v_1) + mv_2$

$\Rightarrow \; v_2 - v_1 = -u$ (cancelling out the common factor $m$) **(1)**

Using Newton's law of restitution:

$e = \dfrac{v_2 - (-v_1)}{2u + 3u} = \dfrac{v_2 + v_1}{5u}$

$\Rightarrow v_2 + v_1 = 5ue$ **(2)**

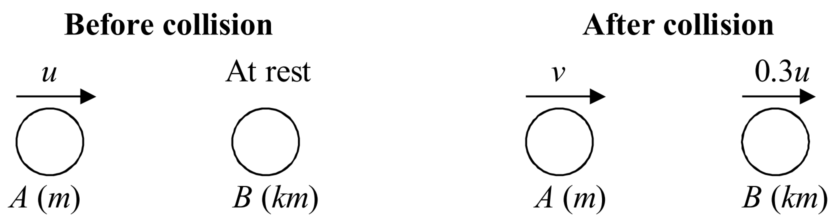
Adding equations **(1)** and **(2)** gives:

$2v_2 = -u + 5eu \; \Rightarrow \; v_2 = \dfrac{u}{2}(5e - 1)$

As $v_2 > 0, \; \dfrac{u}{2}(5e - 1) > 0$

And as $u > 1 \Rightarrow (5e - 1) > 0 \Rightarrow 5e > 1 \Rightarrow e > \dfrac{1}{5}$

**8 a**




Using conservation of linear momentum for the system $(\rightarrow)$:

$mu = mv + km \times 0.3u$

$\Rightarrow \; v = u(1 - 0.3k)$ (cancelling out the common factor $m$)

**8 b** Using Newton's law of restitution:

$$\frac{0.3u - v}{u - 0} = e$$

So using the result from part **a**

$$0.3u - u(1 - 0.3k) = eu$$

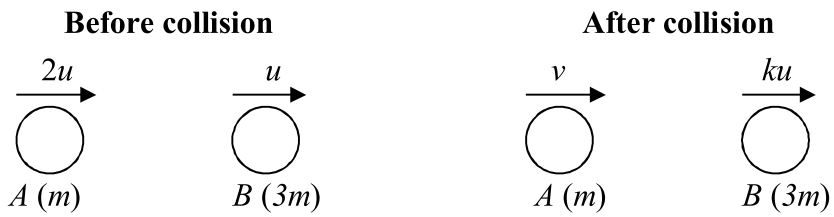
$$\Rightarrow e = 0.3k - 0.7$$

As $0 \leqslant e \leqslant 1$, therefore $0 \leqslant 0.3k - 0.7 \leqslant 1$

$$\Rightarrow 0.7 \leqslant 0.3k \leqslant 1.7$$

$$\Rightarrow \frac{7}{3} \leqslant k \leqslant \frac{17}{3}$$

**9 a**




Using conservation of linear momentum for the system $(\rightarrow)$:

$$2mu + 3mu = vm + 3kmu$$

$$\Rightarrow \; v = u(5 - 3k) \qquad \text{(cancelling out the common factor m)}$$

**b** Using Newton's law of restitution:

$$\frac{ku - v}{2u - u} = e$$

So using the result from part **a**

$$ku - u(5 - 3k) = eu$$

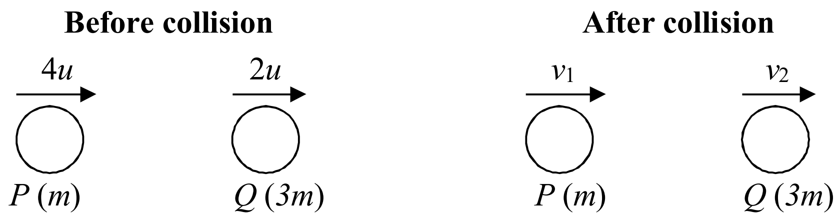
$$\Rightarrow e = 4k - 5$$

As $0 \leqslant e \leqslant 1$, therefore $0 \leqslant 4k - 5 \leqslant 1$

$$\Rightarrow 5 \leqslant 4k \leqslant 6$$

$$\Rightarrow \frac{5}{4} \leqslant k \leqslant \frac{3}{2}$$

**10 a**




Using conservation of linear momentum for the system $(\rightarrow)$:

$m \times 4u + 3m \times 2u = mv_1 + 3mv_2$

$\Rightarrow\ 3v_2 + v_1 = 10u$ (cancelling out the common factor $m$) **(1)**

Using Newton's law of restitution:

$e = \dfrac{v_2 - v_1}{4u - 2u} = \dfrac{v_2 - v_1}{2u}$

$\Rightarrow v_2 - v_1 = 2ue$ **(2)**

Adding equations **(1)** and **(2)** gives:

$4v_2 = 10u + 2ue$

$\Rightarrow\ v_2 = \dfrac{u}{4}(10 + 2e) = \dfrac{u}{2}(5 + e)$

**b** Substituting into equation **(1)** gives:

$\dfrac{3u}{2}(5 + e) + v_1 = 10u$

$2v_1 = 20u - 15u - 3ue$

$\Rightarrow v_1 = \dfrac{u}{2}(5 - 3e)$

**c** The direction of motion of $P$ is unchanged provided that $\dfrac{u}{2}(5 - 3e) > 0$, i.e. $e < \dfrac{5}{3}$

This must be the case as $0 \leqslant e \leqslant 1$

**d** Change of momentum of $Q$ $= 3m(v_2 - 2u)$

$= 3m\left(\dfrac{5u}{2} + \dfrac{eu}{2} - 2u\right)$

$= \dfrac{3mu}{2}(1 + e)$

As impulse of $P$ on $Q$ = change in momentum of $Q$, this gives:

$2mu = \dfrac{3mu}{2}(1 + e)$

$1 + e = \dfrac{4}{3}$

$\Rightarrow e = \dfrac{1}{3}$

**Challenge**


Using conservation of linear momentum for the system $(\rightarrow)$:

$3m\times 2+m\times(-u)=3mv+2mv$

$\Rightarrow\ 5v=6-u$ (cancelling out the common factor $m$) **(1)**

Using Newton's law of restitution:

$\frac{1}{4}=\frac{2v-v}{2+u}$

$\Rightarrow\ 4v=2+u$ **(2)**

Eliminating $v$ from equations **(1)** and **(2)** gives:

$\frac{6-u}{5}=\frac{2+u}{4}$

So $24-4u=10+5u$

$14=9u$

$\Rightarrow u=\frac{14}{9}$