Exercise 5A

1 $8\mathbf{i}-7\mathbf{j}=0.25\mathbf{v}-0.25(12\mathbf{i}+4\mathbf{j})$

Use impulse = $m\mathbf{v}-m\mathbf{u}$, then make $\mathbf{v}$ the subject of the formula.

$8\mathbf{i}-7\mathbf{j}=0.25\mathbf{v}-3\mathbf{i}-\mathbf{j}$

$\therefore 0.25\mathbf{v}=11\mathbf{i}-6\mathbf{j}$

$\mathbf{v}=44\mathbf{i}-24\mathbf{j}$

The new velocity is $(44\mathbf{i}-24\mathbf{j})$ m s^{-1}

2 $3\mathbf{i}+5\mathbf{j}=0.5\mathbf{v}-0.5(2\mathbf{i}-2\mathbf{j})$

Use impulse = $m\mathbf{v}-m\mathbf{u}$ (change in momentum).

$=0.5\mathbf{v}-\mathbf{i}+\mathbf{j}$

$\therefore 0.5\mathbf{v}=4\mathbf{i}+4\mathbf{j}$

$\mathbf{v}=8\mathbf{i}+8\mathbf{j}$

The new velocity is $(8\mathbf{i}+8\mathbf{j})$ m s^{-1}

3 $4\mathbf{i}+8\mathbf{j}=2\times(3\mathbf{i}+2\mathbf{j})-2\mathbf{u}$

Use impulse = change in momentum.

$=6\mathbf{i}+4\mathbf{j}-2\mathbf{u}$

$\therefore\quad 2\mathbf{u}=6\mathbf{i}+4\mathbf{j}-4\mathbf{i}-8\mathbf{j}$

$=2\mathbf{i}-4\mathbf{j}$

$\mathbf{u}=\mathbf{i}-2\mathbf{j}$

The original velocity was $(\mathbf{i}-2\mathbf{j})$ m s^{-1}

4 $3\mathbf{i}-6\mathbf{j}=1.5(5\mathbf{i}-8\mathbf{j})-1.5\mathbf{u}$

$\therefore\ 1.5\mathbf{u}=7.5\mathbf{i}-12\mathbf{j}-3\mathbf{i}+6\mathbf{j}$

$=4.5\mathbf{i}-6\mathbf{j}$

$\mathbf{u}=3\mathbf{i}-4\mathbf{j}$

The original velocity was $(3\mathbf{i}-4\mathbf{j})$ m s^{-1}

5 Impulse = force × time

Use impulse = force × time.

impulse $=(6\mathbf{i}-8\mathbf{j})\times 3$

$=18\mathbf{i}-24\mathbf{j}$

The impulse exerted is $(18\mathbf{i}-24\mathbf{j})$ N s

But impulse = change in momentum

Then use impulse = change in momentum = $m\mathbf{v}-m\mathbf{u}$.

$18\mathbf{i}-24\mathbf{j}=3(\mathbf{v}-(\mathbf{i}+\mathbf{j}))$

$18\mathbf{i}-24\mathbf{j}+3\mathbf{i}+3\mathbf{j}=3\mathbf{v}$

$\therefore\quad 3\mathbf{v}=21\mathbf{i}-21\mathbf{j}$

$\mathbf{v}=7\mathbf{i}-7\mathbf{j}$

When the force ceases to act the velocity is $(7\mathbf{i}-7\mathbf{j})$ m s^{-1}

6 Impulse $=$ force $\times$ time

$$= (2\mathbf{i} - \mathbf{j}) \times 5$$
$$= 10\mathbf{i} - 5\mathbf{j}$$

The impulse exerted is $(10\mathbf{i} - 5\mathbf{j})$ N s.

Use impulse = force × time.

But impulse $=$ change in momentum.

Then use impulse = change in momentum = $m\mathbf{v} - m\mathbf{u}$.

$$10\mathbf{i} - 5\mathbf{j} = 0.5(\mathbf{v} - (5\mathbf{i} + 12\mathbf{j}))$$
$$10\mathbf{i} - 5\mathbf{j} + 2.5\mathbf{i} + 6\mathbf{j} = 0.5\mathbf{v}$$
$$\therefore \quad 0.5\mathbf{v} = 12.5\mathbf{i} + \mathbf{j}$$
$$\mathbf{v} = 25\mathbf{i} + 2\mathbf{j}$$

When the force ceases to act the velocity is $(25\mathbf{i} + 2\mathbf{j})$ m s^{-1}

7 Impulse $=$ change in momentum

$$= 2(-\mathbf{i} - 3\mathbf{j}) - 2(5\mathbf{i} + 3\mathbf{j})$$
$$= -12\mathbf{i} - 12\mathbf{j}$$

Use impulse = $m\mathbf{v} - m\mathbf{u}$.

The impulse exerted by the wall on the particle is $(-12\mathbf{i} - 12\mathbf{j})$ N s

8 Impulse $=$ change in momentum

$$= 0.5 \times (-\mathbf{i} + 7\mathbf{j}) - 0.5 \times (11\mathbf{i} - 2\mathbf{j})$$
$$= -6\mathbf{i} + 4.5\mathbf{j}$$

The impulse exerted by the wall on the particle is $(-6\mathbf{i} + 4.5\mathbf{j})$ N s

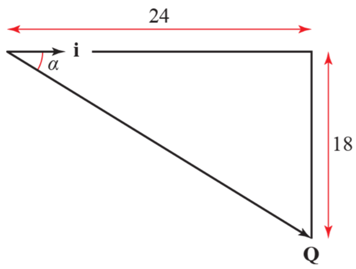
9 $\mathbf{Q} = m\mathbf{v} - m\mathbf{u}$

$$= 3(13\mathbf{i} - 6\mathbf{j}) - 3(5\mathbf{i})$$
$$= 24\mathbf{i} - 18\mathbf{j}$$

Use impulse = change in momentum.

$$\therefore |\mathbf{Q}| = \sqrt{(24)^2 + (-18)^2}$$
$$= 30$$

Find the magnitude of **Q** by using Pythagoras' theorem, and find the angle between **Q** and **i** by using trigonometry.



Let α be the acute angle between **i** and **Q**

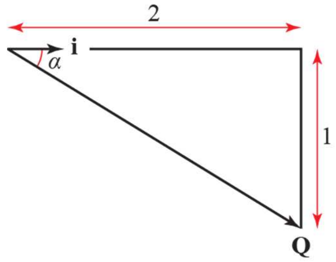
Then

$$\tan \alpha = \frac{18}{24}$$

$$\therefore \quad \alpha = 37° \text{ (nearest degree)}$$

10 Use impulse = change in momentum.

$$\mathbf{Q} = 0.5(3\mathbf{i} - 4\mathbf{j}) - 0.5(-\mathbf{i} - 2\mathbf{j})$$
$$= 2\mathbf{i} - \mathbf{j}$$
$$\therefore |\mathbf{Q}| = \sqrt{2^2 + (-1)^2}$$
$$= \sqrt{5} = 2.24 \text{ (3 s.f.)}$$



Let α be the acute angle between **Q** and **i**.

Then

$$\tan\alpha = \frac{1}{2}$$
$$\therefore \alpha = 27° \text{ (nearest degree)}$$

11 Impulse = change in momentum

$$= m\mathbf{v} - m\mathbf{u}$$
$$= 0.5 \times (-16\mathbf{i} + 8\mathbf{j}) - 0.5 \times (20\mathbf{i} - 4\mathbf{j})$$
$$= -8\mathbf{i} + 4\mathbf{j} - 10\mathbf{i} + 2\mathbf{j}$$
$$= -18\mathbf{i} + 6\mathbf{j}$$

$\therefore$ Magnitude of the impulse $= \sqrt{(-18)^2 + 6^2} = 6\sqrt{10}$

$= 19.0 \text{ N s (3 s.f.)}$

12 Use impulse = change in momentum

$$2\mathbf{i} + 6\mathbf{j} = 0.2\mathbf{v} - 0.2(-15\mathbf{i})$$
$$= 0.2\mathbf{v} + 3\mathbf{i}$$
$$\therefore 0.2\mathbf{v} = 2\mathbf{i} + 6\mathbf{j} - 3\mathbf{i}$$
$$= -\mathbf{i} + 6\mathbf{j}$$
$$\therefore \mathbf{v} = -5\mathbf{i} + 30\mathbf{j}$$

The velocity of the ball after the impact is $(-5\mathbf{i} + 30\mathbf{j})$ m s^{-1}

13 $\mathbf{v} = (t^2 - 3)\mathbf{i} + 4t\mathbf{j}$

When $t = 3$ let $\mathbf{v} = \mathbf{u}$

$\mathbf{u} = 6\mathbf{i} + 12\mathbf{j}$

Substitute $t = 3$ into the expression for velocity, to find the velocity before the impact.

Use impulse = change in momentum

Then $2\mathbf{i} + 2\mathbf{j} = 0.25\mathbf{v} - 0.25(6\mathbf{i} + 12\mathbf{j})$

$\therefore 0.25\mathbf{v} = 2\mathbf{i} + 2\mathbf{j} + 0.25(6\mathbf{i} + 12\mathbf{j})$

$= 3.5\mathbf{i} + 5\mathbf{j}$

$\therefore \quad \mathbf{v} = 14\mathbf{i} + 20\mathbf{j}$

The velocity of the particle after the impulse is (14**i** + 20**j**) m s^{-1}

14 Use impulse = change in momentum.

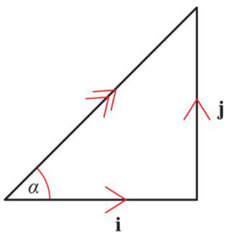
$2\mathbf{j} = 2\mathbf{v} - 2(\mathbf{i} + \mathbf{j})$

$\therefore 2\mathbf{v} = 2\mathbf{j} + 2(\mathbf{i} + \mathbf{j})$

$= 2\mathbf{i} + 4\mathbf{j}$

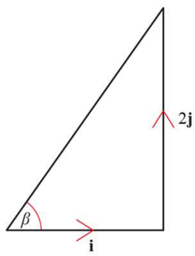
$\mathbf{v} = \mathbf{i} + 2\mathbf{j}$

Immediately after the impulse the velocity is (**i** + 2**j**) m s^{-1}



Before impact the velocity was **i** + **j** and so the direction of the ball was at an angle α with **i**, where $\tan\alpha = \frac{1}{1}$, i.e. $\alpha = 45°$

Find the angle between the direction of the velocity and the direction **i**, both before and after the impulse.



After impact the velocity is **i** + 2**j** and so the direction of the ball is at an angle β with **i**, where $\tan\beta = \frac{2}{1}$, i.e. $\beta = 63.4°$

Then calculate the angle of deflection.

$\therefore$ The ball is deflected through an angle of 63.4 – 45 ≈ 18° (nearest degree).

15 Let the new velocity be $x\mathbf{i}$

Let the new velocity be $x\mathbf{i}$ and use conservation of momentum. Equate $\mathbf{i}$ components to find x.

Using conservation of momentum:

$$(0.5\times 3\mathbf{i})+(0.25\times 12\mathbf{i})=0.75x\mathbf{i}$$
$$1.5\mathbf{i}+3\mathbf{i}=0.75x\mathbf{i}$$
$$\therefore\ 0.75x\mathbf{i}=4.5\mathbf{i}$$
$$x=\frac{4.5}{0.75}$$
$$=6$$

So the velocity of the combined particle is $6\mathbf{i}$ m s^{-1}

16 Let the new velocity $\mathbf{v}$ be $x\mathbf{i}+y\mathbf{j}$
Use conservation of momentum:

Use conservation of momentum to find $\mathbf{v}$, then use Pythagoras' theorem and trigonometry to find $|\mathbf{v}|$

$$5(\mathbf{i}-\mathbf{j})+2(-\mathbf{i}+\mathbf{j})=7(x\mathbf{i}+y\mathbf{j})$$
$$5\mathbf{i}-5\mathbf{j}-2\mathbf{i}+2\mathbf{j}=7x\mathbf{i}+7y\mathbf{j}$$
$$3\mathbf{i}-3\mathbf{j}=7x\mathbf{i}+7y\mathbf{j}$$

Equate coefficients of $\mathbf{i}$ and $\mathbf{j}$ to give

$$7x=3 \text{ and } 7y=-3$$
$$\therefore x=\frac{3}{7} \text{ and } y=-\frac{3}{7}$$
$$\therefore \text{velocity is } \frac{3}{7}\mathbf{i}-\frac{3}{7}\mathbf{j}$$

The magnitude of the velocity $\mathbf{v}$ is $\sqrt{\left(\frac{3}{7}\right)^2+\left(-\frac{3}{7}\right)^2}=\frac{3}{7}\sqrt{2}$

Challenge

Let the impulse be $\mathbf{I}$

$$\mathbf{I}=m\mathbf{v}-m\mathbf{u}$$
$$=m(c\mathbf{i}+d\mathbf{j})-m(a\mathbf{i}+b\mathbf{j})$$
$$=m(c-a)\mathbf{i}+m(d-b)\mathbf{j}$$

Now $\mathbf{I}$ makes an angle of 45° above $\mathbf{i}$, so

$$\tan 45=\frac{d-b}{c-a}=1$$

Hence $b+c=a+d$