Exercise 4C

1 **a** P.E. lost $= mgh = 0.4 \times 9.8 \times 7$

$$= 27.44$$

The P.E. lost is 27.4 J (3 s.f.)

b K.E. gained $= \frac{1}{2}mv^2 - \frac{1}{2}mu^2$

$$= \frac{1}{2} \times 0.4 \times v^2 - 0$$

P.E. lost = K.E. gained

$$27.44 = \frac{1}{2} \times 0.4 \times v^2$$

$$v^2 = \frac{27.44}{0.2}$$

$$v = 11.71\ldots$$

The final speed of the particle is 11.7 m s^{-1} (3 s.f.)

2 **a** K.E. gained $= \frac{1}{2}mv^2 - \frac{1}{2}mu^2$

$$= \frac{1}{2} \times 0.5 \times 12^2 - 0$$

$$= 36$$

The K.E. gained by the stone is 36 J

b P.E. lost = K.E. gained

= 36 J

The P.E. lost by the stone is 36 J

c P.E. lost $= mgh$

$$36 = 0.5 \times 9.8 \times h$$

$$h = \frac{36}{0.5 \times 9.8}$$

$$h = 7.346\ldots$$

The height of the tower is 7.35 m (3 s.f.)

3

a Increase in K.E. $= \frac{1}{2}mv^2 - \frac{1}{2}mu^2$

$$= \frac{1}{2} \times 6 \times 5^2 - \frac{1}{2} \times 6 \times 2.5^2$$

$$= 56.25$$

The increase in K.E. of the box is 56.3 J (3 s.f.)

b The work done by the force is 56.3 J

3 **c** $F = ma$

$10 = 6a$

$a = \frac{5}{3}$

Substituting into:

$v^2 = u^2 + 2as$

With $u = 2.5\ \text{m s}^{-1}$, $v = 5\ \text{m s}^{-1}$ and $a = \frac{5}{3}\ \text{m s}^{-2}$ gives:

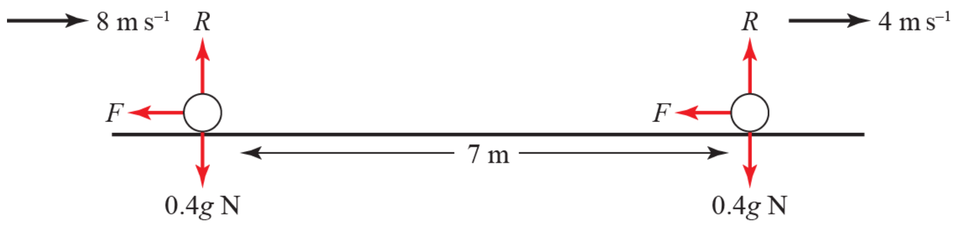
$5^2 = 2.5^2 + 2\left(\frac{5}{3}\right)s$

$25 = 6.25 + \frac{10}{3}s$

$s = 5.625\ \text{m}$

$s = 5.63\ \text{m (3 s.f.)}$

4



a K.E. lost $= \frac{1}{2}mu^2 - \frac{1}{2}mv^2$

$= \frac{1}{2} \times 0.4 \times 8^2 - \frac{1}{2} \times 0.4 \times 4^2$

$= 9.6$

The K.E. lost by the particle is 9.6 J

b The work done against friction is 9.6 J ← Work done = change in energy

c Resolving perpendicular to the surface: $R = 0.4g$

Friction is limiting: $F = \mu R$

$F = 0.4g \times \mu$

Work done $= Fs$

$9.6 = 0.4g \times \mu \times 7$

$\mu = \frac{9.6}{0.4 \times 9.8 \times 7} = 0.3498\ldots$

The coefficient of friction is 0.350 (3 s.f.)

5

$\mu = 0.4$

a K.E. lost $= \frac{1}{2}mu^2 - \frac{1}{2}mv^2$

$= \frac{1}{2} \times 3 \times 6^2 - 0$

$= 54$

The kinetic energy lost by the box is 54 J

b The work done against friction is 54 J

c Resolving perpendicular to the floor: $R = 3g$

Friction is limiting: $F = \mu R$

$F = 0.4 \times 3g$

Work done $= Fs$

$54 = 0.4 \times 3g \times s$

$s = \frac{54}{0.4 \times 3g} = 4.591\ldots$

The distance AB is 4.59 m (3 s.f.)

6 P.E. lost $= mgh$

$= 0.8 \times 9.8 \times 5$

$= 39.2$

K.E. gained $=$ P.E. lost

$= 39.2$

K.E. gained $= \frac{1}{2}mv^2 - \frac{1}{2}mu^2$

$39.2 = \frac{1}{2} \times 0.8v^2 - 0$

$v^2 = \frac{39.2 \times 2}{0.8}$

$v = 9.899\ldots$

The particle hits the ground at a speed of 9.90 m s^{-1} (3 s.f.)

7 K.E. gained $= \frac{1}{2}mv^2 - \frac{1}{2}mu^2$

$= \frac{1}{2} \times 0.3 \times 20^2 - 0$

$= 60$

P.E. lost $=$ K.E. gained

$= 60$

P.E. lost $= mgh$

$60 = 0.3 \times 9.8 \times h$

$h = \frac{60}{0.3 \times 9.8}$

$h = 20.40\ldots$

The cliff is 20.4 m high (3 s.f.)

8 P.E. gained $= mgh$

$= 0.3 \times 9.8 \times 5$

K.E. lost = initial K.E. − final K.E.

$= \frac{1}{2} \times mu^2 - 2.1$

$= \frac{1}{2} \times 0.3u^2 - 2.1$

K.E. lost = P.E. gained

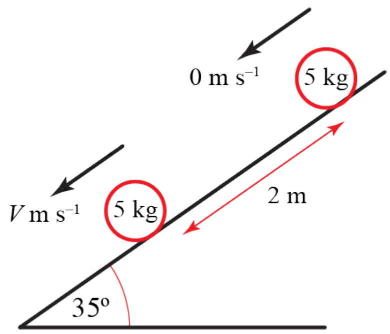
$\frac{1}{2} \times 0.3u^2 - 2.1 = 0.3 \times 9.8 \times 5$

$u^2 = \frac{0.3 \times 9.8 \times 5 + 2.1}{\frac{1}{2} \times 0.3}$

$u = 10.58\ldots$

The value of u is 10.6 (3 s.f.)

9



a P.E. lost = mgh

$$= 5 \times 9.8 \times (2\sin 35°)$$
$$= 56.21\ldots$$

The P.E. lost is 56.2 J (3 s.f.)

b The K.E. gained is 56.2 J

c K.E. gained $= \frac{1}{2}mv^2 - \frac{1}{2}mu^2$

$$56.21 = \frac{1}{2} \times 5 \times v^2 - 0$$
$$v^2 = \frac{56.21 \times 2}{5}$$
$$v = 4.741\ldots$$

The final speed of the package is 4.74 m s^{-1} (3 s.f.)

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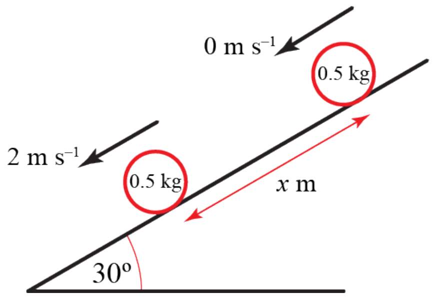

$$\text{K.E. gained} = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$
$$= \frac{1}{2} \times 0.5 \times 2^2 - 0$$
$$= 1$$

P.E. lost $= mgh = 0.5 \times 9.8 \times (x \sin 30°)$

P.E. lost $=$ K.E. gained

$$0.5 \times 9.8 \times (x \sin 30°) = 1$$
$$x = \frac{1}{0.5 \times 9.8 \times \sin 30°}$$
$$= 0.4081\ldots$$

The value of x is 0.408 (3 s.f.)

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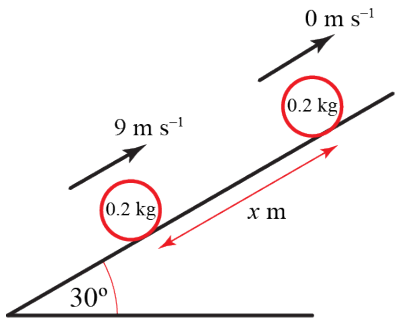


$$\text{K.E. lost} = \frac{1}{2}mu^2 - \frac{1}{2}mv^2$$
$$= \frac{1}{2} \times 0.2 \times 9^2 - 0$$

$$\text{P.E. gained} = mgh$$
$$= 0.2 \times 9.8 \times (x \sin 30°)$$

P.E. gained = K.E. lost

$$0.2 \times 9.8 \times (x \sin 30°) = \frac{1}{2} \times 0.2 \times 9^2$$
$$x = \frac{\frac{1}{2} \times 0.2 \times 9^2}{0.2 \times 9.8 \sin 30°}$$
$$= 8.265\ldots$$

The value of x is 8.27 (3 s.f.)

12 $$\text{K.E. lost} = \frac{1}{2}mu^2 - \frac{1}{2}mv^2$$
$$= \frac{1}{2} \times 0.6u^2 - 0$$

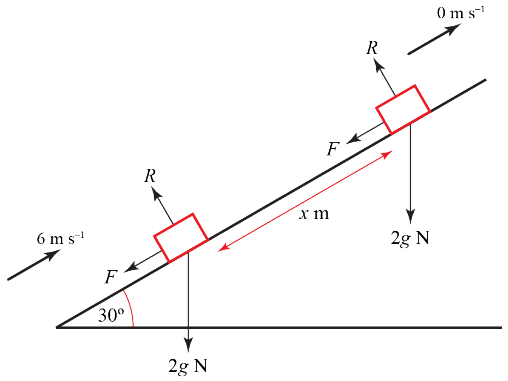
$$\text{P.E. gained} = mgh$$
$$= 0.6 \times 9.8 \times (5 \sin 40°)$$

K.E. lost = P.E. gained

$$\frac{1}{2} \times 0.6u^2 = 0.6 \times 9.8 \times 5 \sin 40°$$
$$u^2 = \frac{0.6 \times 9.8 \times 5 \sin 40°}{\frac{1}{2} \times 0.6}$$
$$u = 7.936\ldots$$

The speed of projection is 7.94 m s^{-1} (3 s.f.)

13



$$\text{K.E. lost} = \frac{1}{2}mu^2 - \frac{1}{2}mv^2$$
$$= \frac{1}{2}\times 2\times 6^2 - 0$$
$$= 36$$

$$\text{P.E. gained} = mgh$$
$$= 2\times 9.8\times (x\sin 30°)$$
$$= 9.8x$$

Resolving perpendicular to the plane: $R = 2g\cos 30°$

Friction is limiting: $F = \mu R$

$$F = \frac{1}{3}\times 2g\cos 30° = \frac{2}{3}g\cos 30°$$

Work done against friction $= Fx = \frac{2}{3}gx\cos 30°$

K.E. lost = P.E. gained + work done against friction

$$\Rightarrow 36 = 9.8x + \frac{2}{3}gx\cos 30°$$
$$36 = 9.8x\left(1+\frac{2}{3}\cos 30°\right)$$
$$x = \frac{36}{9.8\left(1+\frac{2}{3}\cos 30°\right)} = 2.328\ldots$$

The particle moves 2.33 m up the plane (3 s.f.)

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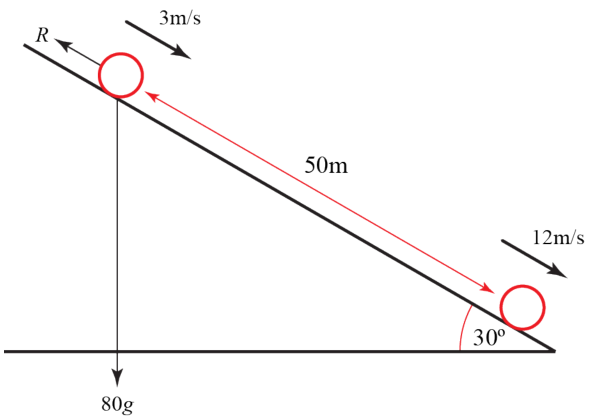

a Work done by resistive forces on the skier = change in total energy of the skier

Loss in P.E. = mgh

Increase in K.E. = $\frac{1}{2}mv^2 - \frac{1}{2}mu^2$

Total loss of energy = P.E. lost – K.E. gained

$= mgh + \frac{1}{2}mu^2 - \frac{1}{2}mv^2$

Force × distance = $mgh + \frac{1}{2}mu^2 - \frac{1}{2}mv^2$

$50R = (80 \times 9.8 \times 50\sin 30°) + \left(\frac{1}{2} \times 80 \times 3^2\right) - \left(\frac{1}{2} \times 80 \times 12^2\right)$

$50R = 14\ 200$

$R = 284$

The value of R is 284.

b The resistive force may not be constant, and could depend on speed, for example.

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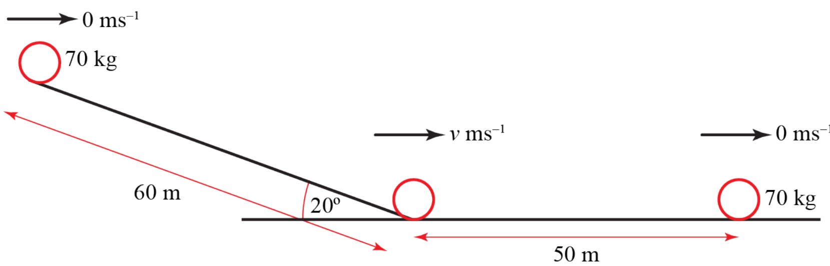


Consider energy changes from start to end – do not divide the motion into two parts.

Change in K.E. $= \frac{1}{2}mv^2 - \frac{1}{2}mu^2$

$= 0 - 0$

Loss of P.E. = mgh

$= 70 \times 9.8 \times (60\sin 20°)$

Work done against resistance $= Fs$

$= R \times (60 + 50)$

$= 110R$

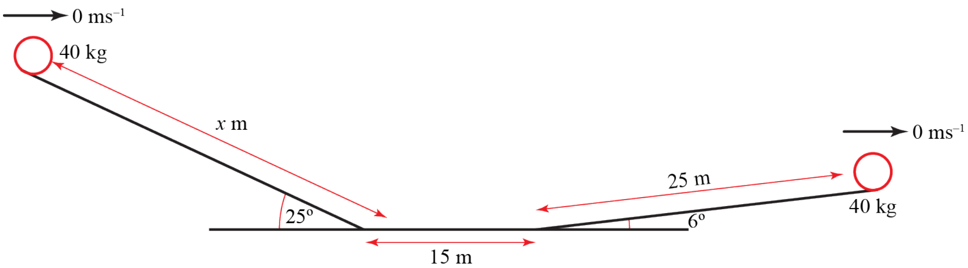
Work done against resistance = loss of P.E.

$110R = 70 \times 9.8 \times (60\sin 20°)$

$R = \frac{70 \times 9.8 \times 60\sin 20°}{110}$

$R = 127.9\ldots$

The value of R is 128 (3 s.f.)

16 

Consider energy changes from start to end – do not divide the motion into three parts.

Loss of P.E. $= mgh$

$= 40 \times 9.8 \times (x \sin 25° - 25 \sin 6°)$

Change in K.E. $= \frac{1}{2}mv^2 - \frac{1}{2}mu^2$

$= 0 - 0$

Work done against resistance $= Fs$

$= 18 \times (x + 15 + 25)$

$= 18 \times (x + 40)$

Work done against resistance = loss of P.E.

$$18x + 18 \times 40 = 40 \times 9.8 \times x \sin 25° - 40 \times 9.8 \times 25 \sin 6°$$

$$(40 \times 9.8 \sin 25° - 18)x = 18 \times 40 + 40 \times 9.8 \times 25 \sin 6°$$

$$x = \frac{18 \times 40 + 40 \times 9.8 \times 25 \sin 6°}{40 \times 9.8 \sin 25° - 18}$$

$$x = 11.81\ldots$$

The girl travels 11.8 m down the slope.

Challenge

Let the mass of a hydrogen molecule $= m$

So the mass of an oxygen molecule $= 8m$

Consider the average kinetic energy of the oxygen molecules:

$\frac{1}{2}mv^2 = \frac{1}{2} \times 8m \times 400^2 = \frac{3}{2}kT$

Consider the average kinetic energy of the hydrogen molecules:

Average K.E. $= \frac{3}{2}kT = \frac{1}{2} \times 8m \times 400^2 = \frac{1}{2}mv^2$

So $\frac{1}{2} \times 8m \times 400^2 = \frac{1}{2}mv^2$

$8 \times 400^2 = v^2$

$v = \sqrt{1\,280\,000}$

$= 1131.3\ldots$

The average speed of the hydrogen molecules is 1130 m s^{-1}