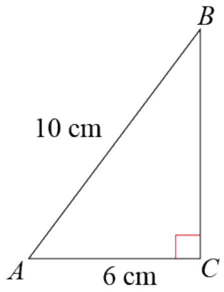
Exercise 3G

1



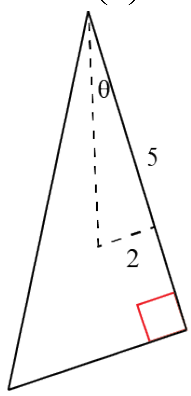
Let A be the origin and let AC be the positive x-axis.
By Pythagoras' theorem;

$BC^2 = 10^2 - 6^2$
$= 64$
$BC = 8$ cm

$$24\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix} = 10\begin{pmatrix}3\\4\end{pmatrix} + 6\begin{pmatrix}3\\0\end{pmatrix} + 8\begin{pmatrix}6\\4\end{pmatrix}$$

$$\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix} = \frac{1}{24}\begin{pmatrix}96\\72\end{pmatrix}$$

$$= \begin{pmatrix}4\\3\end{pmatrix}$$

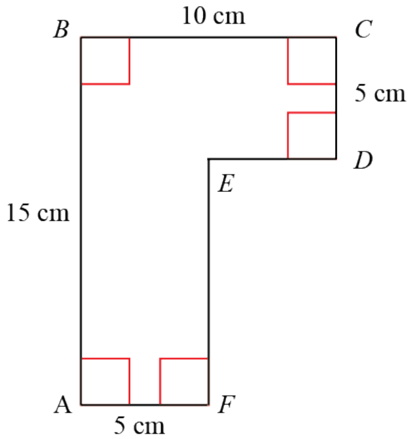


$$\tan\theta = \frac{2}{5}$$

$\theta = 21.801\ldots$

$= 21.8°$ (3 s.f.)

2



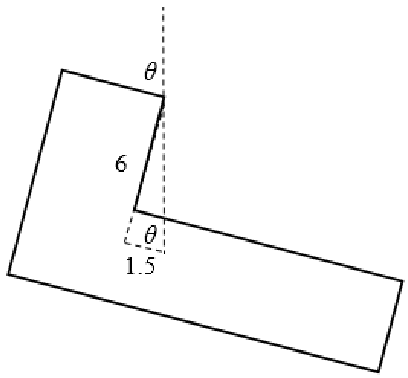
Let A be the origin and let AF be the positive x-axis.

$$50\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}=15\begin{pmatrix}0\\7.5\end{pmatrix}+10\begin{pmatrix}5\\15\end{pmatrix}+5\begin{pmatrix}10\\12.5\end{pmatrix}+5\begin{pmatrix}7.5\\10\end{pmatrix}+10\begin{pmatrix}5\\5\end{pmatrix}+5\begin{pmatrix}2.5\\0\end{pmatrix}$$

$$=\begin{pmatrix}0\\112.5\end{pmatrix}+\begin{pmatrix}50\\150\end{pmatrix}+\begin{pmatrix}50\\62.5\end{pmatrix}+\begin{pmatrix}37.5\\50\end{pmatrix}+\begin{pmatrix}50\\50\end{pmatrix}+\begin{pmatrix}12.5\\0\end{pmatrix}$$

$$\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}=\frac{1}{50}\begin{pmatrix}200\\425\end{pmatrix}$$

$$=\begin{pmatrix}4\\8.5\end{pmatrix}$$

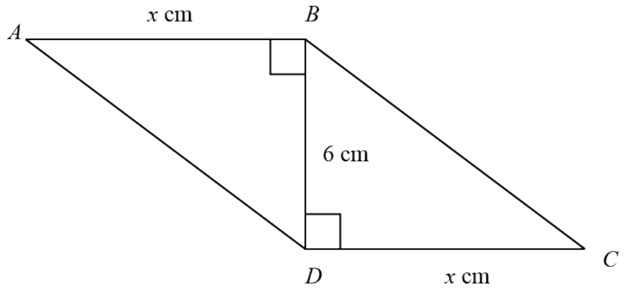


$$\tan\theta=\frac{6}{1.5}$$

$\theta = 75.963\ldots$

$= 76.0°$ (3 s.f.)

3 a



Since the Let D be the origin and let DC be the positive x-axis.

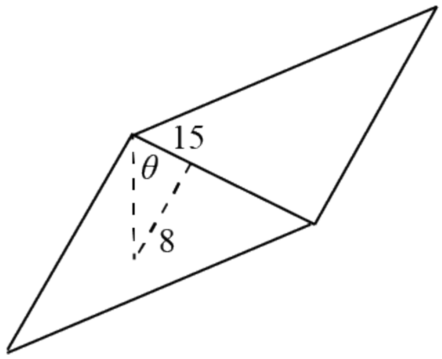
$\tan\theta = \frac{\bar{y}}{\bar{x}} = \frac{3}{8}$ so $\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix} = \begin{pmatrix}8\\ 3\end{pmatrix}$

Therefore $x = 8$

b $(M + kM)\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix} = M\begin{pmatrix}0\\ 3\end{pmatrix} + kM\begin{pmatrix}-8\\ 6\end{pmatrix}$

$\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix} = \frac{1}{1+k}\begin{pmatrix}-8k\\ 3+6k\end{pmatrix}$

$= \begin{pmatrix}\frac{-8k}{1+k}\\ \frac{3+6k}{1+k}\end{pmatrix}$



$\tan\theta = \frac{8}{15}$

$\frac{\frac{8k}{1+k}}{6 - \frac{3+6k}{1+k}} = \frac{8}{15}$

$\frac{\frac{8k}{1+k}}{\frac{6+6k-3-6k}{1+k}} = \frac{8}{15}$

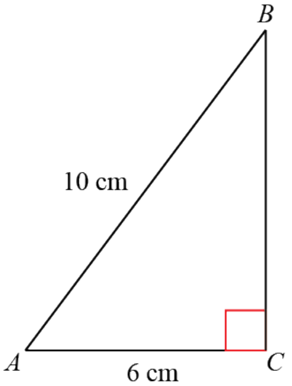
$\frac{8k}{3} = \frac{8}{15}$

$k = \frac{1}{5}$

4 From question 1:

$$\begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$$

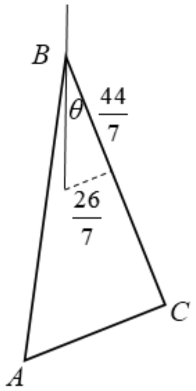
Where A is the origin and AC is the positive x-axis.



$$1.75M\begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix} = M\begin{pmatrix} 4 \\ 3 \end{pmatrix} + 0.75M\begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix} = \frac{1}{1.75}\begin{pmatrix} 4 \\ 3 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{16}{7} \\ \frac{12}{7} \end{pmatrix}$$



$$\tan\theta = \frac{\frac{26}{7}}{\frac{44}{7}}$$

$$\theta = 30.579...$$
$$= 30.6° \text{ (3 s.f.)}$$

5 From question 2:

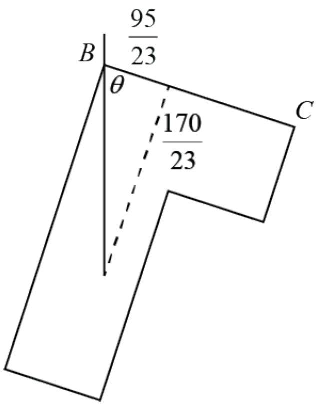
$$\begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix} = \begin{pmatrix} 4 \\ 8.5 \end{pmatrix}$$

Let A be the origin and let AF be the positive x-axis.

$$1.15M\begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix} = M\begin{pmatrix} 4 \\ 8.5 \end{pmatrix} + 0.15M\begin{pmatrix} 5 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix} = \frac{1}{1.15}\begin{pmatrix} 4.75 \\ 8.5 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{95}{23} \\ \frac{170}{23} \end{pmatrix}$$

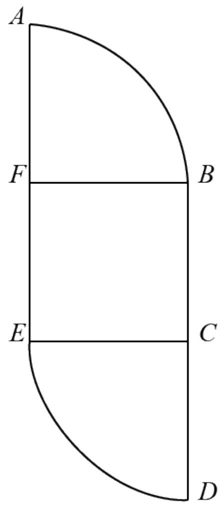


$$\tan\theta = \frac{\frac{170}{23}}{\frac{95}{23}}$$

$\theta = 60.802...$

$= 60.1°$ (3 s.f.)

6

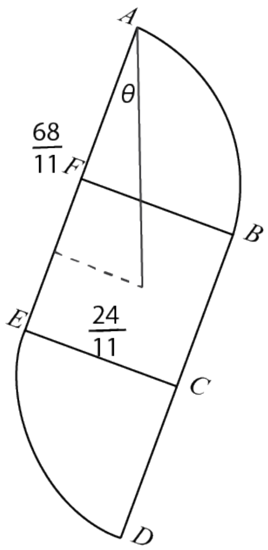


Let E be the origin and let EC lie along the positive x-axis.
By symmetry the centre of mass of the lamina is at the point (2, 2).

$$1.1M\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix} = M\begin{pmatrix}2\\ 2\end{pmatrix} + 0.1M\begin{pmatrix}4\\ 0\end{pmatrix}$$

$$\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix} = \frac{1}{1.1}\begin{pmatrix}2.4\\ 2\end{pmatrix}$$

$$= \begin{pmatrix}\frac{24}{11}\\ \frac{20}{11}\end{pmatrix}$$

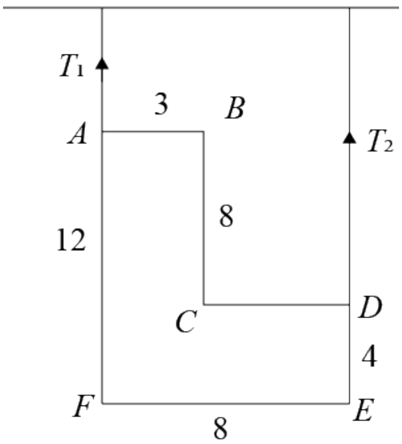


$$\tan\theta = \frac{\frac{24}{11}}{\frac{68}{11}}$$

$$\theta = 19.440...$$

$$= 19.4° \text{ (3 s.f.)}$$

7 a



Let F be the origin and let FE lie on the positive x-axis.

$W\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix}=\frac{12}{40}W\begin{pmatrix}0\\6\end{pmatrix}+\frac{3}{40}W\begin{pmatrix}1.5\\12\end{pmatrix}+\frac{8}{40}W\begin{pmatrix}3\\8\end{pmatrix}+\frac{5}{40}W\begin{pmatrix}5.5\\4\end{pmatrix}+\frac{4}{40}W\begin{pmatrix}8\\2\end{pmatrix}+\frac{8}{40}W\begin{pmatrix}4\\0\end{pmatrix}$

$\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix}=\frac{1}{40}\begin{pmatrix}120\\200\end{pmatrix}$

$=\begin{pmatrix}3\\5\end{pmatrix}$

Res(↑) $T_1+T_2=W$ (1)

Taking moments about the centre of mass gives:

$3T_1=5T_2$

$T_1=\frac{5}{3}T_2$ (2)

Substituting (2) into (1) gives:

$\frac{5}{3}T_2+T_2=W$

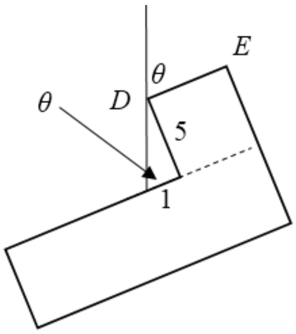
$\frac{8}{3}T_2=W$

$T_2=\frac{3}{8}W$

Therefore:

$T_1=\frac{5}{8}W$

7 b

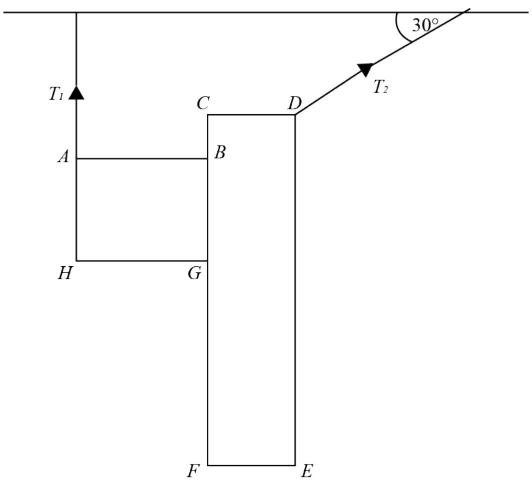


$\tan\theta = \frac{5}{1}$

$\theta = 78.690...$

$= 78.7°$ (3 s.f.)

8 a



Let A be the origin and let AB lie on the positive x-axis.

$$W\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix} = \frac{6}{42}W\begin{pmatrix}3\\0\end{pmatrix} + \frac{3}{42}W\begin{pmatrix}7.5\\2\end{pmatrix} + \frac{10}{42}W\begin{pmatrix}9\\-3\end{pmatrix} + \frac{3}{42}W\begin{pmatrix}7.5\\-8\end{pmatrix} + \frac{10}{42}W\begin{pmatrix}6\\-3\end{pmatrix} + \frac{6}{42}W\begin{pmatrix}3\\-4\end{pmatrix} + \frac{4}{42}W\begin{pmatrix}0\\-2\end{pmatrix}$$

$$\begin{pmatrix}\bar{x}\\ \bar{y}\end{pmatrix} = \begin{pmatrix}\frac{231}{42}\\ -\frac{110}{42}\end{pmatrix} = \begin{pmatrix}\frac{11}{2}\\ -\frac{55}{21}\end{pmatrix}$$

Res(↑) $T_1 + T_2\sin 30 = W$

$T_1 + \frac{1}{2}T_2 = W$

Taking moments about D gives:

$9\times T_1 = \left(9 - \frac{11}{2}\right)\times W$

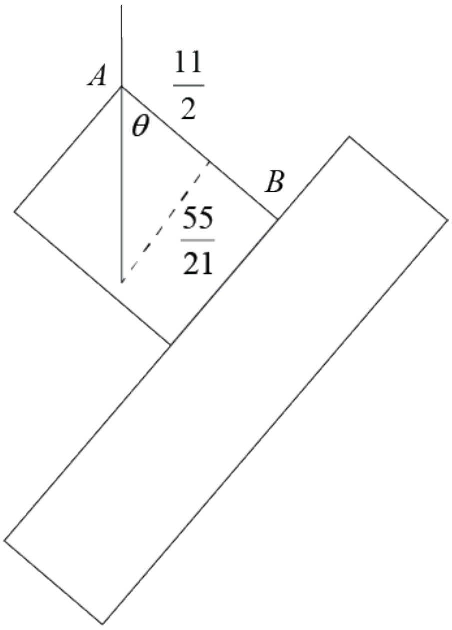
$9T_1 = \frac{7}{2}W$

$T_1 = \frac{7}{18}W$

Therefore:

$T_2 = \frac{11}{9}W$

8 b



$$\tan\theta = \frac{\frac{55}{21}}{\frac{11}{2}}$$

$$\theta = 25.463...$$
$$= 25.5° \text{ (3 s.f.)}$$