		РМТ
une 2005		_
	Sub	

Q 1		mark		Sub
(a) (i)	240 i N s $\rightarrow$	B1		1
(ii) (A)	240 <b>i</b> = 70 <b>i</b> +50 <b>v</b> so <b>v</b> = 3.4 <b>i</b> m s <sup>-1</sup>	M1 A1	Equating to <b>their</b> 240 <b>i</b> in this part FT 240 <b>i</b>	
(B)	240 <b>i</b> = 70 <i>u</i> <b>i</b> – 50 <i>u</i> <b>i</b>	M1	Must have <i>u</i> in both RHS terms and opposite signs	
	u = 12 so $v = -12$ i m s <sup>-1</sup>	A1	FT 240 i	
(C)	240 $\mathbf{i} = 280(\mathbf{i} + \mathbf{j}) + 50\mathbf{v}_{B}$	M1	FT 240 i Must have all terms present	
	so $\mathbf{v}_{\rm B} = (-0.8  \mathbf{i} - 5.6  \mathbf{j})  \mathrm{m  s}^{-1}$	A1	cao	6
(b) (i)	before $4 \text{ m s}^{-1}$ $2 \text{ m s}^{-1}$ after $v_1$ $v_2$ $v_2$ $v_1$ $v_2$ $v_2$ $v_1$ $v_2$ $v_2$ $v_1$ $v_2$ $v_2$ $v_2$ $v_1$ $v_2$	M1	NEL	
	$so v_2 - v_1 = 3$	A1	Any form	
	PCLM $8-6 = 2v_1 + 3v_2$ Solving $v_2 = 1.6$ so $1.6 \text{ m s}^{-1} \rightarrow v_1 = -1.4$ so $1.4 \text{ m s}^{-1} \leftarrow$	M1 A1 A1 A1	PCLM Any form Direction must be clear (accept diagram) Direction must be clear (accept diagram). [Award A1 A0 if $v_1 & v_2$ correct but directions not clear]	6
(ii)	1.6 m s <sup><math>-1</math></sup> at 60° to the wall (glancing angles both 60°)	B1 B1	FT <b>their</b> 1.6	
	No change in the velocity component parallel to the wall as no impulse No change in the velocity component perpendicular to the wall as perfectly elastic	E1 E1	Must give reason Must give reason	
	total	17		4

Q 2		mark		Sub
(i)	We need $\frac{mgh}{t} = \frac{850 \times 9.8 \times 60}{20} = 24990$ so approx 25 kW	M1 E1	Use of $\frac{mgh}{t}$ Shown	2
(ii)	Driving force – resistance = 0 25000 = 800v so $v = 31.25$ and speed is $31.25$ m s <sup>-1</sup>	B1 M1 A1	May be implied Use of $P = Fv$	3
(iii)	Force is $\frac{25000}{10} = 2500$ N N2L in direction of motion 2500 - 800 = 850a a = 2 so 2 m s <sup>-2</sup>	B1 M1 A1	Use of N2L with all terms	3
(iv)	$0.5 \times 850 \times 20^{2} = 0.5 \times 850 \times 15^{2}$ +25000 × 6.90 -800 x x = 122.6562  so  123  m  (3  s. f.)	M1 B1 B1 B1 A1 A1	W-E equation with KE and power term One KE term correct Use of <i>Pt</i> .Accept wrong sign WD against resistance. Accept wrong sign All correct cao	6
(v)	either $0.5 \times 850 \times v^2 = 0.5 \times 850 \times 20^2$ $-850 \times 9.8 \times \frac{105}{20}$ $-800 \times 105$ $v^2 = 99.452 \text{ so } 9.97 \text{ m s}^{-1}$ or N2L + ve up plane $-(800 + 850g \times 0.05) = 850a$ a = -1.43117 $v^2 = 20^2 + 2 \times (-1.43117) \times 105$ $v^2 = 99.452 \text{ so } 9.97 \text{ m s}^{-1}$	M1 M1 A1 B1 A1 M1 A1 M1 A1 A1	<ul> <li>W-E equation inc KE, GPE and WD</li> <li>GPE term with attempt at resolution</li> <li>Correct. Accept expression. Condone wrong sign.</li> <li>WD term. Neglect sign.</li> <li>cao</li> <li>N2L. All terms present. Allow sign errors.</li> <li>Accept ±</li> <li>Appropriate <i>uvast</i>. Neglect signs.</li> <li>All correct including consistent signs. Need not follow sign of <i>a</i> above.</li> <li>cao</li> </ul>	5
		19		

PMT

Q 3		mark		Sub
(i)	$28\left(\frac{\overline{x}}{\overline{y}}\right) = 16\left(\frac{2}{2}\right) + 2\left(\frac{5}{0}\right) + 2\left(\frac{6}{1}\right) + 2\left(\frac{5}{2}\right)$ $+ 2\left(\frac{0}{5}\right) + 2\left(\frac{1}{6}\right) + 2\left(\frac{2}{5}\right)$ $\overline{x} = 2.5$ $\overline{y} = 2.5$	M1 B1 B1 A1 A1	Complete method Total mass correct 3 c. m. correct (or 4 <i>x</i> - or <i>y</i> -values correct) [Allow A0 A1 if only error is in total mass] [If $\overline{x} = \overline{y}$ claimed by symmetry and only one component worked replace final A1, A1 by B1 explicit claim of symmetry A1 for the 2.5]	5
(ii)				5
	$\overline{x} = \overline{y}$ $28\overline{x} = 16 \times 2 + 6 \times 4 + 2 \times 0 + 2 \times 1 + 2 \times 2$ $\overline{x} = \frac{31}{14}  (2.21428)$ $8 \times (-1) + 4 \times (-2) \qquad 4$	B1 M1 A1 A1	Or by direct calculation Dealing with 'folded' parts for $\overline{x}$ or for $\overline{z}$ At least 3 terms correct for $\overline{x}$	
	$\bar{z} = \frac{8 \times (-1) + 4 \times (-2)}{28} = -\frac{4}{7} (-0.57142)$	A1	All terms correct allowing sign errors	
		A1		
	Distance is $\sqrt{\left(\frac{31}{14}\right)^2 + \left(\frac{31}{14}\right)^2 + \left(\frac{4}{7}\right)^2}$	M1	Use of Pythagoras in 3D on their c.m.	
	= 3.18318. so $3.18  m (3  s. f.)$	F1		
(iii)				8
(111)		M1	c.m. clearly directly below A	
	A 3.18318 centre of mass	B1	Diagram showing $\alpha$ and known lengths (or equivalent). FT their values. Award if final answer follows <b>their</b> values.	
	$\sin \alpha = \frac{4}{7}/3.18318$	M1	Appropriate expression for $\alpha$ . FT <b>their</b> values.	
	so $\alpha = 10.3415$ so $10.3^{\circ}$ (3 s. f.)	A1	cao	4
	total	17		

**Mark Scheme** 

Q 4		mark		Sub
(a)	Moments c.w. about A			
(i)	2R = 5L so $R = 2.5L$	E1		
	Resolve $\rightarrow U = 0$	E1		
	Resolve $\uparrow$ $V + R = L$	M1	Resolve vertically or take moments about B (or C)	
	so $V = -1.5L$	E1		
				4
(ii)	$A \bigcirc T_{AC} \rightarrow T_{AC}$			
	45°			
		M1	Equilibrium at a pin-joint	
	$15 I$ $T_{AB}$			
		141		
	For equilibrium at A	MI	Attempt at equilibrium at A or C including resolution	
	↑ T		with correct angle	
	$I_{AB}\cos 45 + 1.5L = 0$			
	so $T_{L} = -\frac{3\sqrt{2L}}{3\sqrt{2L}}$ so $\frac{3\sqrt{2L}}{2}$ N (C) in AB	A1	(2.12L(3.s.f))	
	$\begin{array}{c} 30 \ T_{AB} = \\ 2 \\ 2 \\ 2 \\ 2 \end{array}$			
	$\rightarrow T_{AC} + T_{AB} \cos 45 = 0$			
	T = 3L = 3L $M = T$	<b>F</b> 1	(1.51)	
	so $I_{AC} = \frac{1}{2}$ so $\frac{1}{2}$ N (1) in AC	FI	(1.5 <i>L</i> )	
	At C $\downarrow L + T_{\rm BC}\cos\theta = 0$	M1	Must include attempt at angle	
	$\tan \theta = 3/2 \implies \cos \theta = 2/\sqrt{13}$	B1		
	$\frac{1}{12}I = \frac{1}{12}I$	ы		
	so $T_{\rm BC} = -\frac{\sqrt{15L}}{2}$ so $\frac{\sqrt{15L}}{2}$ N (C) in BC	A1	(1.80 <i>L</i> (3 s. f.))	
	2 2	F1	Award for $T/C$ correct from their internal forces	
		11	Do not award without calcs	8
(b)	F D			
(i)	$\bigwedge^{I} \mathcal{I}^{R}$			
	G S			
	A	B1	All forces present with arrows and labels.	
	B		Angles and distances not required.	
	$W \bigvee - \frac{\theta}{2}$			
				1
(ii)	c.w.moments about B			
、 <i>/</i>	$R \times 3 - W \times 1 \cos \theta = 0$	M1	If moments about other than B, then need to resolve	
			perp to plank as well	
		A1	Correct	
	so $R = \frac{1}{2}W\cos\theta$	Δ1		
	$30 \text{ K} = \frac{-1}{3} \text{ (030)}$	AI		
				3
(iii)	Resolve parallel to plank			
	$F = W \sin \theta$	B1		
	$\mu = \frac{F}{F} = \frac{W \sin \theta}{W \sin \theta} = 3 \tan \theta$			
	$r^{\mu} = \frac{1}{R} = \frac{1}{1 - W \cos \theta}$	M1	Use of $F = \mu R$ and their F and R	
	3			
		A1	Accept any form.	
		10		3
	total	19		

Q 1		mark		Sub
(i)	16 = 0.4v so 40 m s <sup>-1</sup>	M1 A1	Use of $I = \Delta m v$	2
(ii)	PCLM $\uparrow$ + ve $0.4 \times 32 - 0.6u = 0.4v_{p} + 0.6 \times 4$ NEL $\uparrow$ +ve $\frac{4 - v_{p}}{-u - 32} = -0.1$ Solving u = 18	M1 A1 M1 A1 E1	Use of PCLM Any form Use of NEL. Allow sign errors. Any form Must be obtained from a pair of correct equations. If given $u = 18$ used then $v_P = -1$ must be obtained from 1 equation and both values tested in the second equation	
	$v_{\rm P} = -1$ so 1 m s <sup>-1</sup> downwards	A1 A1	cao. Accept use of given $u = 18$ cao	7
(iii )	Considering the momenta involved $0.5 \begin{pmatrix} -3.6 \\ 5.2 \end{pmatrix} = 0.2 \begin{pmatrix} 3 \\ 4 \end{pmatrix} + 0.3 \mathbf{v}_{\mathrm{D}}$ $\mathbf{v}_{\mathrm{D}} = \begin{pmatrix} -8 \\ 6 \end{pmatrix} \text{ so } a = -8 \text{ and } b = 6$ Gradients of the lines are $\frac{4}{3}$ and $\frac{6}{-8}$ Since $\frac{4}{3} \times \frac{6}{-8} = -1$ , they are at 90°	M1 B1 A1 A1 A1 A1 M1 E1	PCLM applied. May be implied. LHS momentum of C correct Complete equation. Accept sign error. cao cao Any method for the angle Clearly shown	8
				17

Q 2		mark		Sub
(i)	Moments about C $240 \times 2 = 3R_D$ $R_D = 160$ so 160 N Resolve vertically	M1 A1	Moments about C or equivalent. Allow 1 force omitted Resolve vertically or moments about D or	
	$R_{\rm c} + R_{\rm p} = 240$ $R_{\rm c} = 80 \text{ so } 80 \text{ N}$	MI F1	equivalent. All forces present. FT from <b>thei</b> r $R_{\rm D}$ only	4
(ii) (A)	Moments about D 240×1 = $4T \sin 40$ T = 93.343 so 93.3 N (3 s. f.)	M1 M1 A1 A1	Moments about D or equivalent Attempt at resolution for RHS RHS correct	4
(ii) (B)	In equilibrium so horizontal force needed to balance cpt of $T$ . This must be friction and cannot be at C.		Need reference to horizontal force that must come from friction at D.	1
(iii ) (A)	Moments about B $3 \times 240 \times \cos 30 = 6P$ $P = 60\sqrt{3}$ (103.92) <i>P</i> inclined at 30° to vertical Resolve horizontally. Friction force <i>F</i> $F = P \sin 30$ so $F = 30\sqrt{3}$ (51.961)	M1 E1 B1 M1 A1	All terms present, no extras. Any resolution required attempted. Accept decimal equivalent Seen or equivalent or implied in (iii) (A) or (B). Resolve horizontally. Any resolution required attempted Any form	
				5

(iii ) (B)	Resolve vertically. Normal reaction $R$ $P\cos 30 + R = 240$	M1	Resolve vertically. All terms present.and resolution attempted	
	Using $F = \mu R$	A1 M1		
	$\mu = \frac{30\sqrt{3}}{240 - 60\sqrt{3} \times \frac{\sqrt{3}}{2}}$	A1	Substitute <b>their expressions</b> for <i>F</i> and <i>R</i>	
	$=\frac{30\sqrt{3}}{240-90}=\frac{\sqrt{3}}{5}=0.34641 \text{ so } 0.346 \text{ (3)}$	A1	cao. Any form. Accept 2 s. f. or better	
	5. Ly			5
				19

Mark Scheme

January

Q 3		mark		Sub
(a) (i)	$80\left(\frac{\overline{x}}{\overline{y}}\right) = 48\left(\frac{6}{2}\right) + 12\left(\frac{1}{-3}\right) + 20\left(\frac{11}{9}\right)$ $80\left(\frac{\overline{x}}{\overline{y}}\right) = \left(\frac{520}{240}\right)$	M1 B1 B1	Correct method for c.m. Total mass correct One c.m. on RHS correct [If separate components considered, B1 for 2 correct]	
	$\overline{x} = 6.5$ $\overline{y} = 3$	E1 A1	cao	5
(ii)	Consider x coordinate $520 = 76 \times 6.4 + 4x$	M1 B1 A1	Using additive principle o. e. on $x$ cpts Areas correct. Allow FT from masses from (i)	3
(iii )	y coordinate is 1 so we need $240 = 76\overline{y} + 4 \times 1$ and $\overline{y} = 3.10526$ so 3.11 (3 s. f.)	B1 M1 A1	Position of centre of square cao	3
(b) (i)	Moments about C $4R = 120 \times 3 + 120 \times 2$ so $4R = 600$ and $R = 150$	M1 E1	Moments equation. All terms present	2
(ii)	$150 \text{ N}$ $T_{AE}$ $T_{EB}$ $T_{DB}$ $T_{DC}$ $T_{BC}$ $T_{BC}$ $T_{BC}$ $T_{BC}$ $T_{BC}$ $T_{BC}$	B1		
	A <sup>↑</sup> 150 + $T_{AE} \cos 30 = 0$ $T_{AE} = -100\sqrt{3} \text{ so } 100\sqrt{3} \text{ N (C)}$ E $\downarrow$ 120 + $T_{AE} \cos 30 + T_{EB} \cos 30 = 0$ $T_{EB} = 20\sqrt{3} \text{ so } 20\sqrt{3} \text{ N (T)}$	M1 A1 M1 F1 F1	Equilibrium at a pin-joint Any form. Sign correct. Neglect (C) Equilibrium at E, all terms present Any form. Sign follows working. Neglect (T). T/C consistent with answers	

				6
(iii )	Consider $\rightarrow$ at E, using (ii) gives ED as thrust	E1	Clearly explained. Accept 'thrust' correctly deduced from wrong answers to (ii).	1
				20

Q 4		mark		Sub
(i)	$\frac{0.5 \times 20 \times 8^2 - 0.5 \times 20 \times 5^2 + 510}{6}$ = 150 W	M1 B1 A1 A1	Use of $P = WD/t$ $\triangle$ KE. Accept ±390 soi All correct including signs	4
(ii) (A)	$20g \times \frac{3}{5}x - 5gx$ $7gx (68.6x) \text{ gain}$	M1 B1 A1 A1	Use of <i>mgh</i> on both terms Either term (neglecting signs) $\pm 7gx$ in any form. cao	4
(B)	11gx	B1		1
(C)	$0.5 \times 25 \times 4^2 = 7gx + 11gx = 18gx$ x = 1.13378 so 1.13 m (3 s. f.)	M1 B1 A1	Use of work-energy equation. Allow 1 RHS term omitted. KE term correct cao. Except follow wrong sign for 7gx only.	3
(iiii )	either $0.5 \times 35 \times v^2 - 0.5 \times 35 \times 16$ $= 15g \times 0.5 - 11g \times 0.5 - 12g \times 0.5$ $v^2 = 13.76 \text{ so } v = 3.70944$ so $3.71 \text{ m s}^{-1}$ (3 s. f.) or 15g - T = 15a $T - 12g - 11g = 20aso a = -2.24v^2 = 4^2 + 2 \times (-2.24) \times 0.5so 3.71 \text{ m s}^{-1} (3 s. f.)$	M1 B1 A1 A1 M1 A1 M1 A1	Use of work-energy. KE, GPE and WD against friction terms present. △ GPE correct inc sign (1.5g J loss) All correct cao N2L in 1 or 2 equations. All terms present cao Use of appropriate (sequence of) <i>uvast</i> cao	4
				16

Q 1		mark		Sub
(a) (i) (A)	PCLM $\rightarrow$ +ve $2 \times 4 - 6 \times 2 = 8v$ v = -0.5 so 0.5 m s <sup>-1</sup> in opposite direction to initial motion of P	M1 A1 A1	Use of PCLM and correct mass on RHS Any form Direction must be negative and consistent or clear. Accept use of a diagram.	3
(B)	$0.5 \times 2 \times 4^{2} + 0.5 \times 6 \times 2^{2} - 0.5 \times 8 \times (-0.5)^{2}$ = 27 J	M1 A1	Use of KE. Must sum initial terms. Must have correct masses FT <b>their</b> (A) only	2
(ii) (A)	PCLM $\rightarrow$ +ve $2 \times 4 - 6 \times 2 = 2v_{p} + 6v_{Q}$ $v_{p} + 3v_{Q} = -2$ NEL $\rightarrow$ +ve $\frac{v_{Q} - v_{p}}{-2 - 4} = -\frac{2}{3}$ $v_{Q} - v_{p} = 4$ $v_{Q} = 0.5$ so 0.5 m s <sup>-1</sup> in orig direction of P $v_{p} = -3.5$ so 3.5 m s <sup>-1</sup> in opp to orig dir of P	M1 A1 M1 A1 A1 A1	Use of PCLM Any form NEL Any form cao. Direction need not be made clear. cao. Direction must be negative and consistent or clear (e.g diag)	6
(B)	$\rightarrow$ +ve 2×-3.5-2×4 = -15 N s so 15 N s in opp to orig direction	M1 A1	Use of change in momentum with correct mass. FT (A). Dir must be clear (e.g. diag)	2
(b)	Let $\alpha = \arcsin(12/13)$ and $\beta = \arcsin(3/5)$ Parallel: $26 \cos \alpha = u \cos \beta$ so $26 \times \frac{5}{13} = u \times \frac{4}{5}$ and $u = 12.5$ Perp: $e = \frac{u \sin \beta}{26 \sin \alpha}$ $= \frac{12.5 \times \frac{3}{5}}{26 \times \frac{12}{13}} = \frac{5}{16}$	M1 A1 A1 F1 F1	PCLM parallel to plane attempted. At least one resolution correct NEL on normal components attempted. FT <b>their</b> <i>u</i> FT <b>their</b> <i>u</i>	6
				19

Q 2		mark		Sub
(i)	Diagrams	B1	Internal force at B must be shown	
	cw moments about A $2 \times 90 - 3R_{\rm B} = 0$ $R_{\rm B} = 60$ so 60 N upwards	M1 A1	1 <sup>st</sup> moments equation attempted for either force. Accept direction not specified	
	cw moments about R: $T \downarrow$ $75 \times 1 + 3T - 60 \times 0.5 = 0$	M1 A1	2 <sup>nd</sup> moments equation for other force. All forces present. No extra forces. Allow only sign errors	
	T = -15 so 15 N upwards	A1	Direction must be clear (accept diag)	6
(ii)	cw moments about A $90 \times 2\cos 30 - V \times 3\cos 30 - U \times 3\cos 60 = 0$	M1	Moments equation with resolution. Accept terms missing	0
	giving $60\sqrt{3} = U + V\sqrt{3}$	E1	Clearly shown	3
(iii)	Diagram	B1	U and $V$ correct with labels and arrows	1
(iv)	ac moments about C $75 \times 2\cos 30 + 3.5V \cos 30 - 3.5U \cos 60 = 0$	M1 B1	Moments equation with resolution. Accept term missing	
	$\frac{300}{7}\sqrt{3} = U - V\sqrt{3}$	A1	Accept any form	
	Solving for <i>U</i> and <i>V</i>	M1	Any method to eliminate one variable	
	$U = \frac{360\sqrt{3}}{7} \ (= 89.0768)$	A1	Accept any form and any reasonable accuracy	
	$V = \frac{60}{7}$ ( = 8.571428)	F1	Accept any form and any reasonable accuracy	
	Resolve $\rightarrow$ on BC F = U	M1	[Either of <i>U</i> and <i>V</i> is cao. FT the other]	
	so frictional force is $\frac{360\sqrt{3}}{7}$ N	F1		
	( = 89.1 N (3 s. f.))			
				8 18

Q 3		mark		Sub
(a)	$20000 = (R + 900g \times 0.1) \times 16$	M1 B1 A1	Use of $P = Fv$ , may be implied. Correct weight term All correct	
	<i>R</i> = 368 so 368 N	A1		4
(b) (i)	$F_{\rm max} = \mu mg \cos \alpha$ Force down slope is weight cpt $mg \sin \alpha$	B1 B1	Correct expression for $F_{max}$ or wt cpt down slope (may be implied and in any form) Identifying $\sin \alpha$ as $\frac{5}{13}$ or equivalent	
	Require $\mu mg \cos \alpha \ge mg \sin \alpha$			
	so $\mu \ge \tan \alpha = \frac{5}{12}$	E1	Proper use of $F \leq \mu R$ or equivalent.	
			$[\mu = \tan \alpha \text{ used WW; SC1}]$	3
(ii)	either $0.5 \times 11 \times v^2$	M1	Use of work energy with at least three required terms attempted	
	$=11g \times 1.5 \times \frac{5}{13} + 0.2 \times 11g \times 1.5 \times \frac{12}{13} + 9$	B1	Any term RHS. Condone sign error.	
		B1 A1	Another term RHS. Condone sign error. All correct . Allow if trig consistent but wrong	
	$v^2 = 18.3717$ v = 4.2862 so 4.29 m s <sup>-1</sup> (3 s. f.) or + ve up the slope	A1	сао	5
	$-11g \times \frac{5}{12} - 0.2 \times 11g \times \frac{12}{12} - 6 = 11a$	M1	Use of N2L	
	13 13	B1	Any correct term on LHS	
	$a = -6.1239 \text{ m s}^2$ $v^2 = -3a$	Al M1	use of appropriate <i>uvast</i>	
	$v = 4.286 \text{ m s}^{-1}$	A1	c.a.o.	
(iii)	continued overleaf			

**Mark Scheme** 

(iii) either Extra GPE balances WD against resistances $mgx \sin \alpha$ $= 6(x+3)+0.2 \times 11g \times \cos \alpha(x+3)$ $x = 4.99386 \ so \ 4.99 \ m \ (3 \ s. \ f.)$ or $(1.5+x) \times 11g \times \frac{5}{13} - 6(1.5+x)$ $= (1.5+x) \times 11g \times \frac{5}{13} - 6(1.5+x)$ $-(1.5+x) \times 0.2 \times 11g \times \frac{12}{13}$ $x = 4.99386 \ so \ 4.99 \ m \ (3 \ s. \ f.)$ or $(1.5+x) \times 0.2 \times 11g \times \frac{12}{13}$ $x = 4.99386 \ so \ 4.99 \ m \ (3 \ s. \ f.)$ $(1.5+x) \times 0.2 \times 11g \times \frac{12}{13}$ $x = 4.99386 \ so \ 4.99 \ m \ (3 \ s. \ f.)$ $(1.5+x) \times 0.2 \times 11g \times \frac{12}{13}$ $(1.5+x) \times 0.2 \times 11g \times \frac{12}{13}$ (1.5+x)	3	continued			
(iii) Entrer Extra GPE balances WD against resistances $m_{gx \sin \alpha}$ = $6(x+3)+0.2 \times 11g \times \cos \alpha(x+3)$ X = 4.99386 so 4.99 m (3 s. f.) Or equivalent B1 Another of 1 <sup>st</sup> three terms on RHS correct A1 All correct. FT their v if used. cao. 6 Or $0.5 \times 11 \times 18.3717$ = $(1.5+x) \times 11g \times \frac{5}{13} - 6(1.5+x)$ = $(1.5+x) \times 0.2 \times 11g \times \frac{12}{13}$ A1 x = 4.99386 so 4.99 m (3 s. f.) Or $-(1.5+x) \times 0.2 \times 11g \times \frac{12}{13}$ B1 WD against friction A1 All correct cao. A1 All correct cao. A1 All correct x = 4.99386 so 4.99 m (3 s. f.) Or + ve down the slope $11g \times \frac{5}{13} - 0.2 \times 11g \times 1\frac{12}{13} - 6 = 11a$ a = 1.4145 m s <sup>-2</sup> $4.286^2 = 2a(1.5+x)$ M1 Or equivalent Or equivalent Or equivalent A1 All correct Cao. A1 N2L with all terms present all correct except condone sign errors A1 W1 use of appropriate uvast	(:::)	-:4h			
$mgx \sin \alpha$ = $6(x+3)+0.2 \times 11g \times \cos \alpha(x+3)$ B1 B1 B1 Another of $1^{st}$ three terms on RHS correct Another of $1^{st}$ 3 terms on RHS correct All correct. FT their v if used. cao.6or $0.5 \times 11 \times 18.3717$ = $(1.5+x) \times 11g \times \frac{5}{13} - 6(1.5+x)$ M1 B1 B1 B1Allow 1 term missing B1 Use of $1.5 + x$ (may be below)6or $-(1.5+x) \times 0.2 \times 11g \times \frac{12}{13}$ B1 B1 B1Use of $1.5 + x$ (may be below)6or $-(1.5+x) \times 0.2 \times 11g \times \frac{12}{13}$ B1 B1 M1WD against friction A1 A1 All correct cao.6or $x = 4.99386$ so $4.99$ m (3 s. f.) or $+ ve$ down the slope $11g \times \frac{5}{13} - 6 = 11a$ M1 A1 A1 A1 A1 A1 Correct except condone sign errors A1 A1 all correct except condone sign errors A1 A1 all correct except condone sign errorsM1 A1 A1 all correct except condone sign errors A1 all correct except condone sign errors $a = 1.4145m s^{-2}$ $4.286^2 = 2a(1.5+x)$ M1 M1 M1 use of appropriate uvastM1 and bar	(111)	eitner Extra GPE balances WD against resistances	M1	Or equivalent	
$ \begin{array}{c c c c c c c c c c c c c c c c c c c $		$mgx\sin\alpha$	B1		
$x = 4.99386$ so $4.99 \text{ m} (3 \text{ s. f.})$ A1cao.6or $0.5 \times 11 \times 18.3717$ $= (1.5 + x) \times 11g \times \frac{5}{13} - 6(1.5 + x)$ $-(1.5 + x) \times 0.2 \times 11g \times \frac{12}{13}$ B1Allow 1 term missing KE. FT their $v$ 6B1Use of $1.5 + x$ (may be below)B1WD against friction $x = 4.99386$ so $4.99 \text{ m} (3 \text{ s. f.})$ or $+ ve down the slope11g \times \frac{5}{13} - 0.2 \times 11g \times \frac{12}{13} - 6 = 11aB1WD against frictiona = 1.4145\text{ m s}^{-2}4.286^2 = 2a(1.5 + x)M1N2L with all terms presentall correct except condone sign errorsx = 4.99386 so 4.99 \text{ m} (3 \text{ s. f.})or+ ve down the slope11g \times \frac{5}{13} - 0.2 \times 11g \times 12/13 - 6 = 11aM1x = 1.4145\text{ m s}^{-2}4.286^2 = 2a(1.5 + x)M1N2L with all terms presentall correct except condone sign errors$		$= 6(x+3) + 0.2 \times 11g \times \cos \alpha (x+3)$	B1 B1 A1	One of 1 <sup>st</sup> three terms on RHS correct Another of 1 <sup>st</sup> 3 terms on RHS correct All correct, FT <b>their</b> v if used.	
or $0.5 \times 11 \times 18.3717$ $= (1.5 + x) \times 11g \times \frac{5}{13} - 6(1.5 + x)$ M1 B1Allow 1 term missing B1 $-(1.5 + x) \times 11g \times \frac{12}{13}$ B1Use of $1.5 + x$ (may be below) $-(1.5 + x) \times 0.2 \times 11g \times \frac{12}{13}$ B1WD against friction $x = 4.99386$ so $4.99$ m (3 s. f.) or $+$ ve down the slope $11g \times \frac{5}{13} - 0.2 \times 11g \times \frac{12}{13} - 6 = 11a$ M1All correct cao. $a = 1.4145m$ s <sup>-2</sup> $4.286^2 = 2a(1.5 + x)$ M1N2L with all terms present all correct except condone sign errorsM1M1with all terms present all correct except condone sign errors		<i>x</i> = 4.99386 so 4.99 m (3 s. f.)	A1	cao.	6
$= (1.5+x) \times 11g \times \frac{5}{13} - 6(1.5+x)$ $-(1.5+x) \times 0.2 \times 11g \times \frac{12}{13}$ x = 4.99386  so  4.99  m  (3  s. f.) or +  ve down the slope $11g \times \frac{5}{13} - 0.2 \times 11g \times \frac{12}{13} - 6 = 11a$ $a = 1.4145\text{ m s}^{-2}$ $4.286^2 = 2a(1.5+x)$ B1 Use of 1.5 + x (may be below) B1 WD against friction A1 A1 A1 A1 Cao. M1 A1 A1 N2L with all terms present A1 A1 A1 WD against friction A1 A1 A1 A1 A1 A1 A1 A1 A1 A1		<b>or</b> 0.5×11×18.3717	M1 B1	Allow 1 term missing KE. FT <b>their</b> <i>v</i>	
$-(1.5+x) \times 0.2 \times 11g \times \frac{12}{13}$ B1WD against friction $x = 4.99386$ so $4.99 \text{ m}$ (3 s. f.)A1All corrector+ ve down the slopeA1A1 $11g \times \frac{5}{13} - 0.2 \times 11g \times \frac{12}{13} - 6 = 11a$ M1N2L with all terms present $a = 1.4145\text{ m s}^{-2}$ A1A1 $4.286^2 = 2a(1.5+x)$ M1use of appropriate uvast		$= (1.5+x) \times 11g \times \frac{5}{13} - 6(1.5+x)$	B1	Use of 1.5 + <i>x</i> (may be below)	
$x = 4.99386$ so $4.99 \text{ m}$ (3 s. f.)A1 A1 A1 Cor + ve down the slope $11g \times \frac{5}{13} - 0.2 \times 11g \times \frac{12}{13} - 6 = 11a$ A1 A1 Cao.A1 Cao. $a = 1.4145\text{ m s}^{-2}$ $4.286^2 = 2a(1.5+x)$ A1 		$-(1.5+x) \times 0.2 \times 11g \times \frac{12}{13}$	B1	WD against friction	
or + ve down the slope $11g \times \frac{5}{13} - 0.2 \times 11g \times \frac{12}{13} - 6 = 11a$ M1N2L with all terms present all correct except condone sign errors $a = 1.4145m s^{-2}$ A1 $4.286^2 = 2a(1.5+x)$ M1		x = 4.99386 so 4.99 m (3 s. f.)	A1 A1	All correct cao.	
$\begin{array}{c c} 11g \times 5/13^{-0.2 \times 11g \times 12/13^{-6} = 11a} \\ a = 1.4145m \ s^{-2} \\ 4.286^2 = 2a(1.5+x) \end{array}$ $\begin{array}{c c} M1 \\ A1 \\ M1 \\ M1 \\ M1 \\ M1 \\ M1 \\ M1 \\$		or + ve down the slope			
$a = 1.4145m s^{-2}$ A1 A1 A1 A1all correct except condone sign errors A1 use of appropriate uvast $4.286^2 = 2a(1.5+x)$ M1		$11g \times \frac{5}{13} - 0.2 \times 11g \times \frac{12}{13} - 6 = 11a$	M1	N2L with all terms present	
$4.286^{2} = 2a(1.5+x)$ $M1$ use of appropriate <i>uvast</i>		$a = 1.4145 \text{ m s}^{-2}$	A1	all correct except condone sign errors	
		$4286^2 = 2a(1.5+x)$	M1	use of appropriate uvast	
<b>B1</b> for $(15 + x)$ (may be implied)			R1	for $(1.5 + x)$ (may be implied)	
x = 4.99 A1 c.a.o.		<i>x</i> = 4.99	A1	C.a.o.	
					18

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Q 4		mark		Sub
(i)	$100\left(\frac{\overline{x}}{\overline{y}}\right) = 10\left(\frac{5}{0}\right) + 30\left(\frac{10}{15}\right) + 30\left(\frac{20}{15}\right) + 30\left(\frac{25}{30}\right)$	M1 B1 B1	Correct method for c.m. Total mass correct One c.m. on RHS correct [If separate components considered, B1 for 2 correct]	
	$100\left(\frac{\overline{x}}{\overline{y}}\right) = \begin{pmatrix} 1700\\1800 \end{pmatrix}$ $\overline{x} = 17\\\overline{y} = 18$	A1 A1	cao cao. [Allow SC 4/5 for $\overline{x} = 18$ and $\overline{y} = 17$ ]	5
(ii)	(17,18,20)	B1 B1	<i>x-</i> and <i>y-</i> coordinates. FT from (i). <i>z</i> coordinate	2
(iii)	cw moments about horizontal edge thro' D x component $P \times 20 - 60 \times (20 - 17) = 0$ P = 9	M1 B1 B1 A1	Or equivalent with all forces present One moment correct (accept use of mass or length) correct use of <b>their</b> $\overline{x}$ in a distance FT only <b>their</b> $\overline{x}$	4
(iv)	Diagram	B1	Normal reaction must be indicated acting vertically upwards at edge on Oz and weight be in approximately the correct place.	1
(v)	On point of toppling ac moments about edge along Oz $30 \times Q - 60 \times 17 = 0$ Q = 34 Resolving horizontally $F = Q$ As $34 > 30$ , slips first	M1 B1 F1 B1 B1	Or equivalent with all forces present Any moment correct (accept use of mass or length) FT only <b>their</b> $\overline{x}$ FT <b>their</b> Q correctly argued.	5 17

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Q 1		mark		sub
(i)	before after $v_2 \text{ m s}^{-1}$ $v_2 \text{ m s}^{-1}$ $v_1 \text{ m s}^{-1}$ $v_1 \text{ m s}^{-1}$ $v_1 \text{ m s}^{-1}$ $\frac{v_1 - v_2}{0 - 10} = -0.8$	M1 A1 M1	PCLM and two terms on RHS All correct. Any form. NEL	
	$v_1 = 0.3$ so $V_1 = 0.3$ $v_2 = -7.7$ so $V_2 = 7.7$ m s <sup>-1</sup> in opposite to original direction	A1 A1 A1 F1	Any form Speed. Accept ±. Must be correct interpretation of clear working	7
(ii) (A)	$10 \times 0.5 = 30V$ so $V = \frac{1}{6}$	M1 A1 A1	PCLM and coalescence All correct. Any form. Clearly shown. Accept decimal equivalence. Accept no direction.	3
(B)	Same velocity No force on sledge in direction of motion	E1 E1	Accept speed	2
(iii)	before after $2 \text{ m s}^{-1} \qquad 2 \text{ m s}^{-1}$ $0.5 \text{ kg}$ $u$	B1		
	$2 \times 40 = 0.5u + 39.5V$ u - V = 10 Hence $V = 1.875$	M1 A1 B1 A1 17	PCLM, masses correct Any form May be seen on the diagram. Accept no reference to direction.	5

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Q 2		mark	comment	sub
(i)	$X = R \cos 30$ (1) $Y + R \sin 30 = L$ (2)	B1 M1 A1	Attempt at resolution	3
(ii)	ac moments about A $R - 2L = 0$	B1		
	Subst in $(1)$ and $(2)$	M1	Subst <b>their</b> $R = 2L$ into <b>their</b> (1) or (2)	
	$X = 2L\frac{\sqrt{3}}{2} \text{ so } X = \sqrt{3}L$	E1	Clearly shown	
	$Y + 2L \times \frac{1}{2} = L$ so $Y + L = L$ and $Y = 0$	E1	Clearly shown	4
(iii)	(Below all are taken as tensions e. g. $T_{AB}$ in AB)	B1 B1	Attempt at all forces (allow one omitted) Correct. Accept internal forces set as tensions or thrusts or a mix	2
(iv)	$\downarrow A  T_{AD} \cos 30 \ (-Y) = 0$ so $T_{AD} = 0$	M1 E1	Vert equilibrium at A attempted. $Y = 0$ need not be explicit	2
(v)	Consider the equilibrium at pin-joints	M1	At least one relevant equilib attempted	
	A $\rightarrow$ $T_{AB} - X = 0$ so $T_{AB} = \sqrt{3}L$ (T)	B1	(T) not required	
	$C \downarrow L + T_{CE} \cos 30 = 0$	B1	Or equiv from <b>their</b> diagram	
	so $T_{\rm CE} = \frac{-2L}{\sqrt{3}}$ so $\frac{2L}{\sqrt{3}} \left( = \frac{2L\sqrt{3}}{3} \right)$ (C)	B1	Accept any form following from their	
	$C \leftarrow T_{BC} + T_{CE} \cos 60 = 0$	B1	equation. (C) not required. Or equiv from <b>their</b> diagram	
	so $T_{\rm BC} = -\left(-\frac{2\sqrt{3}L}{3}\right) \times \frac{1}{2} = \frac{\sqrt{3}L}{3}$ (T)	B1	FT <b>their</b> $T_{CE}$ or equiv but do not condone inconsistent signs even if right answer	
		F1	obtained. (T) not required. T and C consistent with <b>their</b> answers and <b>their</b> diagram	
				7
(vi)	↓ B $T_{BD} \cos 30 + T_{BE} \cos 30 = 0$ so $T_{DD} = -T_{DE}$ so mag equal and opp sense	M1 E1	Resolve vert at B A statement required	
				2
		20		

Q 3		mark		sub
(i)	(10, 2, 2.5)	B1		1
(ii)	By symmetry $\overline{x} = 10,$ $\overline{y} = 2$ $(240 + 80)\overline{z} = 80 \times 0 + 240 \times 2.5$ so $\overline{z} = 1.875$	B1 B1 B1 M1 A1	Total mass correct Method for c.m. Clearly shown	5
(iii)	$\overline{x} = 10 \text{ by symmetry}$ $(320 + 80) \begin{pmatrix} \overline{x} \\ \overline{y} \\ \overline{z} \end{pmatrix} = 320 \begin{pmatrix} 10 \\ 2 \\ 1.875 \end{pmatrix} + 80 \begin{pmatrix} 10 \\ 4 \\ 3 \end{pmatrix}$ $\overline{y} = 2.4$ $\overline{z} = 2.1$	E1 M1 B1 E1 E1	Could be derived Method for c.m. y coord c.m. of lid z coord c.m. of lid shown shown	
(iv)	$\frac{2.4 \text{ cm}}{1.2.1 \text{ cm}} \frac{5 \text{ cm}}{30^{\circ}} \frac{5 \text{ cm}}{40 \text{ N}}$ c.w moments about X $40 \times 0.024 \cos 30 - 40 \times 0.021 \sin 30$ = 0.41138 so 0.411 N m (3 s. f.)	B1 B1 B1 E1	Award for correct use of dimensions 2.1 and 2.4 or equivalent 1 <sup>st</sup> term o.e. (allow use of 2.4 and 2.1) 2 <sup>nd</sup> term o.e. (allow use of 2.4 and 2.1) Shown [Perpendicular method: M1 Complete method: A1 Correct lengths and angles E1 Shown]	6
(v)	0.41138 0.05 <i>P</i> = 0 <i>P</i> = 8.22768 so 8.23 (3 s. f.)	M1 A1	Allow use of 5 Allow if cm used consistently	2
		18		

Q 4		mark		sub
(i)	$F_{\text{max}} = \mu R$ $R = 2g \cos 30$ $\sin E_{\text{max}} = 0.75 \times 2 \times 9.8 \times \cos 30 = 12.730$	M1 B1	Must have attempt at $R$ with $mg$ resolved	
	so $P_{\text{max}} = 0.75 \times 2 \times 9.8 \times \cos 50 = 12.750$ so 12.7 N (3 s. f.)	A1	[Award 2/3 retrospectively for limiting friction seen below]	
	either Weight cpt down plane is $2gsin 30 = 9.8$ N so no as $9.8 < 12.7$ or	B1 E1	The inequality must be properly justified	
	Slides if $\mu < \tan 30$ But 0.75 > 0.577 so no	B1 E1	The inequality must be properly justified	5
(ii) (A)	Increase in GPE is $2 \times 9.8 \times (6 + 4 \sin 30) = 156.8 \text{ J}$	M1 B1 A1	Use of <i>mgh</i> 6 + 4 sin 30	3
(B)	WD against friction is $4 \times 0.75 \times 2 \times 9.8 \times \cos 30 = 50.9222$ J	M1 A1	Use of WD = $Fd$	2
(C)	Power is 10×(156.8 + 50.9222)/60	M1	Use $P = WD/t$	
	= 34.620 so 34.6 W (3 s. f.)	A1		2
(iii)	$0.5 \times 2 \times 9^2$ $= 2 \times 9.8 \times (6 + x \sin 30)$	M1	Equating KE to GPE and WD term. Allow sign errors and one KE term omitted. Allow 'old' friction as well.	
	+ $0.5 \times 2 \times 4^2$ -90	B1 A1 A1	Both KE terms. Allow wrong signs. All correct but allow sign errors All correct, including signs.	
	so x = 3.8103so 3.82 (3 S. I.)	AI		5
		17		

speed is $\frac{18}{6} = 3 \text{ m s}^{-1}$ .	B1 B1		2
PCLM $\rightarrow$ 3×6-1×2=8v v = 2 so 2 m s <sup>-1</sup> in orig direction of A	M1 A1 E1	Use of PCLM + combined mass RHS All correct Must justify direction (diag etc)	3
$\rightarrow 2 \times 2 - 2 \times -1 = 6$ N s	M1 A1	Attempted use of <i>m</i> <b>v</b> - <i>m</i> <b>u</b> for 6 N s dir specified (accept diag)	2
$2 \text{ ms}^{-1} \qquad 1.8 \text{ m s}^{-1}$ $AB \qquad C$ $v \text{ ms}^{-1} \qquad 1.9 \text{ m s}^{-1}$	B1	Accept masses not shown	1
PCLM $\rightarrow$ 2×8+10×1.8 = 8v+10×1.9 v = 1.875	M1 A1 A1	PCLM. All terms present Allow sign errors only	3
NEL $\frac{1.9 - 1.875}{1.8 - 2} = -e$ so $e = 0.125$	M1 A1 F1	Use of NEL with <b>their</b> $v$ Any form. FT <b>their</b> $v$ FT <b>their</b> $v$ (only for $0 < e \le 1$ )	3
Using $v^2 = u^2 + 2as$ $v = \sqrt{2 \times 10 \times 9.8} = 14$ rebounds at $14 \times \frac{4}{7}$ $= 8 \text{ m s}^{-1}$ No change to the horizontal component Since both horiz and vert components are $8 \text{ m s}^{-1}$ the angle is $45^{\circ}$	B1 M1 F1 B1 A1	Allow ±14 Using <b>their</b> <i>vertical</i> component FT from <b>their</b> 14. Allow ± Need not be explicitly stated cao	5
	speed is $\frac{18}{6} = 3 \text{ m s}^{-1}$ . PCLM $\rightarrow$ $3 \times 6 - 1 \times 2 = 8v$ $v = 2 \text{ so } 2 \text{ m s}^{-1}$ in orig direction of A $\rightarrow 2 \times 2 - 2 \times -1 = 6 \text{ N s}$ $2 \text{ ms}^{-1}$ $1.8 \text{ m s}^{-1}$ $\overrightarrow{AB}$ $\overrightarrow{C}$ $v \text{ ms}^{-1}$ $1.9 \text{ m s}^{-1}$ PCLM $\rightarrow$ $2 \times 8 + 10 \times 1.8 = 8v + 10 \times 1.9$ v = 1.875 NEL $\frac{1.9 - 1.875}{1.8 - 2} = -e$ so $e = 0.125$ Using $v^2 = u^2 + 2as$ $v = \sqrt{2 \times 10 \times 9.8} = 14$ rebounds at $14 \times \frac{4}{7}$ $= 8 \text{ m s}^{-1}$ No change to the horizontal component Since both horiz and vert components are $8 \text{ m s}^{-1}$ the angle is $45^{\circ}$	speed is $\frac{18}{6} = 3 \text{ m s}^{-1}$ . B1 PCLM $\rightarrow$ $3 \times 6 - 1 \times 2 = 8\nu$ M1 $\mu = 2 \text{ so } 2 \text{ m s}^{-1}$ in orig direction of A M1 $\rightarrow 2 \times 2 - 2 \times -1 = 6 \text{ N s}$ M1 41 $2 \text{ ms}^{-1}$ $1.8 \text{ m s}^{-1}$ B1 $2 \text{ ms}^{-1}$ $1.8 \text{ m s}^{-1}$ B1 41 $2 \text{ ms}^{-1}$ $1.9 \text{ m s}^{-1}$ B1 PCLM $\rightarrow$ $2 \times 8 + 10 \times 1.8 = 8\nu + 10 \times 1.9$ M1 $\lambda = 1.875$ A1 NEL $\frac{1.9 - 1.875}{1.8 - 2} = -e$ M1 so $e = 0.125$ F1 Using $\nu^2 = u^2 + 2as$ $\nu = \sqrt{2 \times 10 \times 9.8} = 14$ B1 rebounds at $14 \times \frac{4}{7}$ M1 $= 8 \text{ m s}^{-1}$ F1 No change to the horizontal component Since both horiz and vert components are $8 \text{ m s}^{-1}$ the angle is $45^{\circ}$ A1	speed is $\frac{18}{6} = 3 \text{ m s}^{-1}$ .B1PCLM $\rightarrow$ $3 \times 6 - 1 \times 2 = 8\nu$ $\nu = 2 \text{ so } 2 \text{ m s}^{-1} \text{ in orig direction of A}M1E1Use of PCLM + combined mass RHSAll correctMust justify direction (diag etc)\rightarrow 2 \times 2 - 2 \times -1 = 6 \text{ N s}M1E1Attempted use of mv - mufor 6 N s dir specified (accept diag)2 \times 2 - 2 \times -1 = 6 \text{ N s}M1Attempted use of mv - mufor 6 N s dir specified (accept diag)2 \times 2 - 2 \times -1 = 6 \text{ N s}M1Attempted use of mv - mufor 6 N s dir specified (accept diag)2 \times 8 - 10 \times 1.8 = 8\nu + 10 \times 1.9\nu \text{ ms}^{-1}B1Accept masses not shownPCLM \rightarrow2 \times 8 + 10 \times 1.8 = 8\nu + 10 \times 1.9\nu = 1.875M1A1NEL\frac{1.9 - 1.875}{1.8 - 2} = -e1.8 - 2M1A1NEL\frac{1.9 - 1.875}{1.8 - 2} = -e1.8 - 2M1A1V = \sqrt{2 \times 10 \times 9.8} = 14\nu = 8 \text{ m s}^{-1}B1Allow \pm 14Using their vertical component= 8 \text{ m s}^{-1}No change to the horizontal component since both horiz and vert components are8 \text{ m s}^{-1} the angle is 45^\circB1A1Need not be explicitly statedcaoCao$

Q 2				
(i)	$\theta = \frac{\pi}{2}$	B1		
	gives CG = $\frac{8\sin\frac{\pi}{2}}{\frac{\pi}{2}} = \frac{16}{\pi}$	E1		
	$\left(-\frac{16}{\pi},8\right)$ justified	E1		3
(ii)	$(8\pi + 72) \left(\frac{\overline{x}}{\overline{y}}\right) = 8\pi \left(-\frac{16}{\pi}\right) + 72 \left(\frac{36}{0}\right)$ $\left(\frac{\overline{x}}{\overline{y}}\right) = \left(\frac{25.3673}{2.06997}\right) = \left(\frac{25.37}{2.07}\right) (4 \text{ s. f.})$	M1 B1 A1 A1 E1 E1	Method for c.m. Correct mass of 8 or equivalent 1 <sup>st</sup> RHS term correct 2 <sup>nd</sup> RHS term correct [If separate cpts award the A1s for <i>x</i> - and <i>y</i> - cpts correct on RHS]	6
(iii)	$A = \begin{bmatrix} 13.93 \\ 13.93 \\ 13.93 \\ 25.37 \\ 25.37 \end{bmatrix}$	B1	General position and angle (lengths need not be shown)	
	$\tan \alpha = \frac{13.93}{25.37}$	M1 M1 A1	Angle or complement attempted. arctan or equivalent. Attempt to get 16 – 2.0699 Obtaining 13.93 cao Accept use of 2.0699 but not 16.	
	<i>α</i> = 28.7700 so 28.8° (3 s. f.)	A1	cao	5
(iv)	c. w. moments about A $12 \times 13.93 - 16F = 0$	M1 A1	[FT use of 2.0699] Moments about any point, all forces present	
	so <i>F</i> = 10.4475	A1	(1.5525 if 2.0699 used)	3
		17		

03				
(i)	Moments c.w. about B $200 \times 0.6 - 0.8R_A = 0$ $R_A = 150$ so 150 N Resolve or moments $R_B = 50$ so 50 N	M1 A1 M1 F1	Accept about any point. Allow sign errors.	4
(ii)	Moments c.w. about D $-0.8R_{\rm c} + 1.2 \times 200 = 0$ $R_{\rm c} = 300 \uparrow$ Resolve or moments $R_{\rm D} = 100 \downarrow$	M1 A1 M1 A1 E1	Or equiv. Accept about any point. All terms present. No extra terms. Allow sign errors. Neglect direction Or equiv. All terms present. No extra terms. Allow sign errors. Neglect direction Both directions clearly shown (on diag)	5
(iii)	Moments c.w. about P $0.4 \times 200 \cos \alpha - 0.8R_Q = 0$ $R_Q = 96 \text{ so } 96 \text{ N}$ resolve perp to plank $R_P = 200 \cos \alpha + R_Q$ $R_P = 288 \text{ so } 288 \text{ N}$	M1 A1 A1 M1 A1 A1	Or equiv. Must have some resolution. All terms present. No extra terms. Allow sign errors. Correct [No direction required but no sign errors in working] Or equiv. Must have some resolution. All terms present. No extra terms. Allow sign errors. Correct [No direction required but no sign errors in working]	6
(iv)	Need one with greatest normal reaction So at P Resolve parallel to the plank $F = 200 \sin \alpha$ so $F = 56$ $\mu = \frac{F}{R}$ $= \frac{56}{288} = \frac{7}{36} (= 0.194 (3 \text{ s. f.}))$	B1 B1 M1 A1	FT their reactions Must use <b>their</b> <i>F</i> and <i>R</i> cao	4

June 2007

Q 4				
(i)	either $0.5 \times 20 \times 0.5^{2} + 20 \times 9.8 \times 4$ = 786.5 J or $a = \frac{1}{22}$	M1 B1 B1 A1 B1	KE or GPE terms KE term GPE term cao	
	$T - 20g = 20 \times \frac{1}{32}$ T = 196.625 WD is $4T = 786.5$ so 786.5 J	M1 A1 A1	N2L. All terms present.	4
(ii)	$20g \times 0.5 = 10g \text{ so } 98 \text{ W}$	M1 A1 A1	Use of $P = Fv$ or $\Delta WD / \Delta t$ All correct	3
(iii)	GPE lost is $35 \times 9.8 \times 3 = 1029$ J KE gained is $0.5 \times 35 \times (3^2 - 1^2) = 140$ J so WE gives WD against friction is 1029 - 140 = 889 J	B1 M1 A1 M1 A1	Δ KE The 140 J need not be evaluated Use of WE equation cao	5
(iv)	either $0.5 \times 35 \times 3^{2} + 35 \times 9.8 \times 0.1x = 150x$ x = 1.36127 so $1.36$ m (3 S. F.) or $35g \times 0.1 - 150 = 35a$ a = -3.3057 0 = 9 - 2ax x = 1.36127 so $1.36$ m (3 S. F.)	M1 B1 B1 A1 A1 M1 A1 A1 M1 A1	<ul> <li>WE equation. Allow 1 missing term. No extra terms.</li> <li>One term correct (neglect sign)</li> <li>Another term correct (neglect sign)</li> <li>All correct except allow sign errors cao</li> <li>Use of N2L. Must have attempt at weight component. No extra terms.</li> <li>Allow sign errors, otherwise correct cao</li> <li>Use of appropriate <i>uvast</i> or sequence cao</li> </ul>	5
		17		-

# Mark Scheme

January 2008

V	a	ĸ	3	cr	۱e	m	

Q1		Mark	Comment	Sub
<b>(a)</b> (i)	either In direction of the force I = Ft = mv so $1500 \times 8 = 4000v$ giving $v = 3$ so $3$ m s <sup>-1</sup> or	M1 A1 A1	Use of <i>Ft</i> = <i>mv</i>	
	N2L gives $a = \frac{1500}{4000}$ $v = 0 + \frac{1500}{4000} \times 8$ giving $v = 3$ so 3 m s <sup>-1</sup>	M1 A1 A1	Appropriate use of N2L <b>and</b> <i>uvast</i>	3
(ii)	$ \begin{array}{c}                                     $			
	<b>PCLM</b> $12000 = 4000V_{R} + 500V_{S}$ so $24 = 8V_{R} + V_{C}$	M1 A1	Appropriate use of PCLM	
	NEL $\frac{V_{\rm s} - V_{\rm R}}{0 - 3} = -0.2$	M1	Appropriate use of NEL	
	so $V_{\rm s} - V_{\rm R} = 0.6$ Solving	A1	Any form	
	$V_{\rm R} = 2.6$ , $V_{\rm S} = 3.2$ so ram 2.6 m s <sup>-1</sup> and stone 3.2 m s <sup>-1</sup>	A1 F1	Either value	
				6
(iii)	$0.5 \times 4000 \times 3^2 - 0.5 \times 4000 \times 2.6^2 - 0.5 \times 500 \times 3.2^2$	M1 B1	Change in KE. Accept two terms Any relevant KE term correct (FT their	
	= 1920 J	A1	speeds) cao	3
(b)	see over			

		Maula	0	0
1		Mark	Comment	Sub
(b) (i)	72i N s 8(9 cos 60i + 9 sin 60j) = (36i + 36 $\sqrt{3}$ j) N s	B1 E1	Neglect units but must include direction Evidence of use of 8 kg , 9 m s <sup>-1</sup> and $60^{\circ}$	2
(ii)	72 <b>i</b> + (36 <b>i</b> + 36 $\sqrt{3}$ <b>j</b> ) = 12( <i>u</i> <b>i</b> + <i>v</i> <b>j</b> ) Equating components 72 + 36 = 12 <i>u</i> so <i>u</i> = 9 36 $\sqrt{3}$ = 12 <i>v</i> so <i>v</i> = 3 $\sqrt{3}$	M1 M1 A1	PCLM. Must be momenta both sides Both	3
(iii)	either $4 \times 18 = 8 \times 9$ so equal momenta so $60/2 = 30^{\circ}$ or $\arctan\left(3\sqrt{3}/{9}\right) = \arctan\left(\frac{1}{\sqrt{3}}\right) = 30^{\circ}$	M1 A1 M1 A1	Must be clear statements cao FT <b>their</b> <i>u</i> and <i>v</i> . cao	2

Q 2		Mark	Comment	Sub
(i) (A)	$0.5 \times 80 \times 3^2 = 360 \text{ J}$	M1 A1	Use of KE	2
				2
(B)	$360 = F \times 12$ so $F = 30$ so 30 N	M1 F1	<i>W</i> = <i>Fd</i> attempted FT <b>their</b> WD	2
(ii)	Using the WE equation	M1	Attempt to use the WE equation. Condone one missing term	
	$0.5 \times 80 \times 10^2 - 0.5 \times 80 \times 4^2$	M1	$\Delta$ KE attempted	
	$= 80 \times 9.8 \times h - 1600$ h = 6.32653 so 6.33 (3 s. f.)	B1 A1 A1	1600 with correct sign All terms present and correct (neglect signs)	
	11 - 0.52055 30 0.55 (5 3. 1.)			5
(iii) (A)	We have driving force $F = 40$ so $200 = 40v$ and $v = 5$ so 5 m s <sup>-1</sup>	B1 M1 A1	May be implied Use of $P = Fv$	
				3
(B)	From N2L, force required to give accn is $F-40 = 80 \times 2$ so $F = 200$ $P = 200 \times 0.5 = 100$ so 100 W	M1 A1 A1 M1 A1	Use of N2L with all terms present (neglect signs) All terms correct correct use of P = Fv cao	
		,,,,		5
		17		

Q 3		Mark	Comment	Sub
(i)	For $\overline{z}$ $(2 \times 20 \times 100 + 2 \times 50 \times 120)\overline{z}$ $= 2 \times 2000 \times 50 + 2 \times 6000 \times 60$ so $\overline{z} = 57.5$ and $\overline{y} = 0$	M1 B1 B1 A1 B1	Method for c.m. Total mass of 16000 (or equivalent) At least one term correct NB This result is given below. NB This result is given below. Statement (or proof) required. N.B. If incorrect axes specified, award max 4/5	5
(ii)	$\overline{y}$ and $\overline{z}$ are not changed with the folding For $\overline{x}$ $100 \times 120 \times 0 + 2 \times 20 \times 100 \times 10 = 16000\overline{x}$ so $\overline{x} = \frac{40000}{16000} = 2.5$	E1 M1 B1 E1	A statement, calculation or diagram required. Method for the c.m. with the folding Use of the 10 Clearly shown	4
(iii)	Moments about AH. Normal reaction acts through this line c.w. $P \times 120 - 72 \times (20 - 2.5) = 0$ so $P = 10.5$	M1 B1 B1 A1 A1	May be implied by diagram or statement $20-2.5$ or equivalent All correct cao	5
(iv)	$F_{\text{max}} = \mu R$ so $F_{\text{max}} = 72\mu$ For slipping before tipping we require $72\mu < 10.5$ so $\mu < 0.1458333$ ( $7_{48}$ )	M1 A1 M1 A1	Allow $F = \mu R$ Must have clear indication that this is max F Accept $\leq$ . Accept <b>their</b> $F_{max}$ and $R$ . cao	4

Q 4		Mark	Comment	Sub
(i)	Centre of CE is 0.5 m from D a.c. moment about D $2200 \times 0.5 = 1100$ so 1100 N m c.w moments about D $R \times 2.75 - 1100 = 0$ R = 400 so 400 N	B1 M1 E1 M1 B1 A1	Used below correctly Use of <b>their</b> 0.5 0.5 must be clearly established. Use of moments about D in an equation Use of 1100 and 2.75 or equiv	6
(ii)	c.w moments about D $W \times 1.5 - 1100 - 440 \times 2.75 = 0$ so $W = 1540$	M1 A1 E1	Moments of all relevant forces attempted All correct Some working shown	3
(iii) ( <i>A</i> )	c.w. moments about D $1.5 \times 1540 \cos 20 - 1.75T$ $-1100 \cos 20 - 400 \times 2.75 \cos 20 = 0$ T = 59.0663 so 59.1 N (3 s. f.)	M1 M1 A1 B1 A1 A1	Moments equation. Allow one missing term; there must be some attempt at resolution. At least one res attempt with correct length Allow sin $\leftrightarrow$ cos Any two of the terms have cos 20 correctly used (or equiv) 1.75 <i>T</i> All correct cao Accept no direction given	
(iii) ( <i>B</i> )	either Angle required is at 70° to the normal to CE so $T_1 \cos 70 = 59.0663$ so $T_1 = 172.698$ so 173 N (3 s.f.) or $400\cos 20 \times 2.75 + 1100\cos 20$ $= 1540\cos 20 \times 1.5 - T \sin 20 \times 1.75$ T = 172.698 so 173 N (3s.f.)	B1 M1 A1 M1 A1 A1 A1	FT (iii) (A) Moments attempted with all terms present All correct (neglect signs) FT(iii)(A)	3
		18		

Q1		mark	k comment	
(a) (i)	In i direction: $6u - 12 = 18$ so $u = 5$ i.e. 5i m s <sup>-1</sup>	M1 E1	Use of I-M Accept $6u - 12 = 18$ as total working. Accept 5 instead of 5i.	
	In i direction: $0.5v + 12 = 0.5 \times 11$ v = -13 so $-13$ i m s <sup>-1</sup>	M1 B1 A1	Use of I-M Use of + 12i or equivalent Accept direction indicated by any means	
	$6 \times 5 + 0.5 v = 6 \times 3 + 0.5 \times 11$ v = -13 so $-13i \text{ m s}^{-1}$	M1 A1 A1	PCLM Allow only sign errors Accept direction indicated by any means	5
(ii)	Using NEL: $\frac{11-3}{-13-5} = -e$	M1	Use of NEL. Condone sign errors but not reciprocal expression	
	$e = \frac{4}{9} (0.\dot{4})$	F1 F1	FT only <b>their</b> $-13$ (even if +ve) FT only <b>their</b> $-13$ and only if -ve (allow 1 s.f. accuracy)	3
(iii)	In i direction: $-2 \times 7 = 0.5v - 0.5 \times 11$ v = -17 so $-17$ i m s <sup>-1</sup>	M1 M1 A1 A1	Use of <b>I</b> = <b>F</b> <i>t</i> Use of <b>I</b> = <i>m</i> ( <b>v</b> – <b>u</b> ) For ±17 cao. Direction (indicated by any means)	
	$ \begin{array}{l} -2 \ \mathbf{i} = 0.5 \ \mathbf{a} \\ \mathbf{so} \ \mathbf{a} = -4 \ \mathbf{i} \ \mathbf{m} \ \mathbf{s}^{-2} \\ \mathbf{v} = 11 \ \mathbf{i} - 4 \ \mathbf{i} \ \mathbf{x} \ 7 \\ \mathbf{v} = -17 \ \mathbf{so} \ -17 \ \mathbf{i} \ \mathbf{m} \ \mathbf{s}^{-1} \end{array} $	M1 A1 M1 A1	Use of $F = ma$ For $\pm 4$ Use of <b>uvas</b> <i>t</i> cao. Direction (indicated by any means)	4
(b)	$u\mathbf{i} + ev\mathbf{j}$ $\tan \alpha = \frac{v}{u}, \ \tan \beta = \frac{ev}{u}$	B1 B1 M1	For <i>u</i> For <i>ev</i> Use of tan. Accept reciprocal argument. Accept use of <b>their</b> components	
	$\tan \beta = e\left(\frac{v}{u}\right) = e \tan \alpha$	B1 E1	Both correct. Ignore signs. Shown. Accept signs not clearly dealt with.	5
		17		-

## Mark Scheme

June 2008

PMT

Q 2		mark	comment	sub
(i)	$(2+3\times6)\left(\frac{\overline{x}}{\overline{y}}\right) = 6\binom{3}{0} + 6\binom{6}{3} + 6\binom{3}{6} + 2\binom{0}{7}$	M1 B1	Method for c.m. Total mass correct	
	$20\left(\frac{x}{\overline{y}}\right) = \left(\frac{18+36+18}{18+36+14}\right) = \left(\frac{72}{68}\right)$	B1	For any of the 1 <sup>st</sup> 3 RHS terms	
	$\overline{x} = 3.6$ $\overline{y} = 3.4$	B1 E1 A1	For the 4 <sup>th</sup> RHS term cao [If separate cpts, award the 2 <sup>nd</sup> B1 for 2 <i>x</i> - terms correct and 3 <sup>rd</sup> B1 for $2 \times 7$ in <i>y</i> term]	6
(ii)	$\operatorname{arctan}\left(\frac{3.6}{2 + (6 - 3.4)}\right) = \operatorname{arctan}\left(\frac{3.6}{4.6}\right)$ so 38.047 so 38.0° (3 s. f.)	B1 B1 M1 B1 A1	Diagram showing G vertically below D 3.6 and <b>their</b> 3.4 correctly placed (may be implied) Use of arctan on <b>their</b> lengths. Allow reciprocal of argument. Some attempt to calculate correct lengths needed 2 + (6 - <b>their</b> 3.4) seen cao	5
(iii)	moments about D $5 \times 3.6 = 6 \times T_{BP}$ so tension in BP is 3 N Resolve vert: $3 + T_{DQ} = 5$ so tension in DQ is 2 N	M1 F1 M1 F1	moments about D. No extra forces FT <b>their</b> values if calc 2nd Resolve vertically or moments about B. FT <b>their</b> values if calc 2nd	4
(iv)	We require <i>x</i> -cpt of c.m. to be zero either $(20+L)\overline{x} = 20 \times 3.6 - \frac{1}{2}L^2$ or $2 \times 6 \times (0.5 \times 6) + 6 \times 6 - 0.5 \times L^2 = 0$	M1	A method to achieve this with all cpts	
	L = 12	B1 A1 A1	For the $0.5 \times L^2$ All correct	4
		19		

PMT

(a) (b) (c) (c) (c) (c) (c) (c) (c) (c	Q 3		mark	comment	sub
(ii) (ii) A $\uparrow$ $T_{AD} \sin 30 - L = 0$ so $T_{AD} = 2L$ so $2L$ N (T) A $\rightarrow$ $T_{AB} + T_{AD} \cos 30 = 0$ so $T_{AB} = -\sqrt{3}L$ so $\sqrt{3}L$ N (C) B $\uparrow$ $T_{BD} \sin 60 - 3L = 0$ so $T_{BD} = 2\sqrt{3}L$ so $2\sqrt{3}L$ N (C) B $\rightarrow$ $T_{BC} + T_{BD} \cos 60 - T_{AB} = 0$ so $T_{BC} = -2\sqrt{3}L$ so $2\sqrt{3}L$ N (C) E1 All T/C consistent [SC 1 all T/C correct WWW] 9 (b) Leg QR with frictional force $F \leftarrow$ moments c.w. about R $U \times 2I \sin 60 - WI \cos 60 = 0$ Horiz equilibrium for QR F = U Hence $\frac{1}{2}W = \sqrt{3}F$ and so $F = \frac{\sqrt{3}}{4}W$ At the content of the second equation explicitly derived Correct use of $2^{nd}$ equation with the moments equation for the second equation of the second equation the moments equation attempted At the second correct equation of horizontal or vertical equilibrium to eliminate a force (U or reaction at foot) [Award if correct moments equation explicitly derived Correct use of $2^{nd}$ equation with the moments equation the moments (b) $C = \frac{1}{2}W = \sqrt{3}F$ and so $F = \frac{\sqrt{3}}{2}W$ (c) $C = \frac{1}{2}W = \sqrt{3}F$ $C = \frac{\sqrt{3}}{2}W$ $C = \frac{\sqrt{3}}$	(a) (i)	$\begin{array}{c} & & & \\ & & & \\ A \\ & & & & \\ & & & \\ & & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\$	B1 B1	Internal forces all present and labelled All forces correct with labels and arrows (Allow the internal forces set as tensions, thrusts or a mixture)	2
(b)Leg QR with frictional force $F \leftarrow$ moments c.w. about R $U \times 2l \sin 60 - Wl \cos 60 = 0$ Accept only 1 leg considered (and without comment)M1Suitable moments equation. Allow 1 force omitted a.c. moments C.w. momentsHoriz equilibrium for QR $F = U$ M1A second correct equation for horizontal or vertical equilibrium to eliminate a force (U or reaction at foot) [Award if correct moments equation containing only W and F]Hence $\frac{1}{2}W = \sqrt{3}F$ E1* This second equation explicitly derived Correct use of $2^{nd}$ equation with the moments equation	(ii)	A $T_{AD} \sin 30 - L = 0$ so $T_{AD} = 2L \operatorname{so} 2L \operatorname{N}$ (T) A $\rightarrow T_{AB} + T_{AD} \cos 30 = 0$ so $T_{AB} = -\sqrt{3}L$ so $\sqrt{3}L \operatorname{N}$ (C) B $\uparrow T_{BD} \sin 60 - 3L = 0$ so $T_{BD} = 2\sqrt{3}L \operatorname{so} 2\sqrt{3}L \operatorname{N}$ (T) B $\rightarrow$ $T_{BC} + T_{BD} \cos 60 - T_{AB} = 0$ so $T_{BC} = -2\sqrt{3}L$ so $2\sqrt{3}L \operatorname{N}$ (C)	M1 A1 F1 M1 A1 M1 F1 E1	Equilibrium equation at a pin-joint attempted 1 <sup>st</sup> ans. Accept + or –. Second equation attempted 2 <sup>nd</sup> ans. FT any previous answer(s) used. Third equation attempted 3 <sup>rd</sup> ans. FT any previous answer(s) used. Fourth equation attempted 4 <sup>th</sup> ans. FT any previous answer(s) used. All T/C consistent [SC 1 all T/C correct WWW]	9
	(b)	Leg QR with frictional force $F \leftarrow$ moments c.w. about R $U \times 2l \sin 60 - Wl \cos 60 = 0$ Horiz equilibrium for QR F = U Hence $\frac{1}{2}W = \sqrt{3}F$ and so $F = \frac{\sqrt{3}}{6}W$	M1 A1 M1 E1 E1	Accept only 1 leg considered (and without comment) Suitable moments equation. Allow 1 force omitted a.c. moments c.w. moments A second correct equation for horizontal or vertical equilibrium to eliminate a force (U or reaction at foot) [Award if correct moments equation containing only <i>W</i> and <i>F</i> ] * This second equation explicitly derived Correct use of 2 <sup>nd</sup> equation with the moments equation Shown. CWO but do not penalise * again.	7

## Mark Scheme

Q 4		mark	comment	sub
(a) (i)	Tension is perp to the motion of the sphere (so WD, $Fd \cos \theta = 0$ )	E1		1
(ii)	Distance dropped is $2-2\cos 40 = 0.467911$ GPE is <i>mgh</i> so $0.15 \times 9.8 \times 0.467911 = 0.687829 J$	M1 E1 M1 B1	Attempt at distance with resolution used. Accept $sin \leftrightarrow cos$ Accept seeing $2-2cos40$ Any reasonable accuracy	4
(iii)	$0.5 \times 0.15 \times v^2 = 0.687829$ so $v = 3.02837$ so 3.03 m s <sup>-1</sup> (3 s. f.)	M1 F1	Using KE + GPE constant FT <b>their</b> GPE	2
(iv)	$\frac{1}{2} \times 0.15 \left( v^2 - 2.5^2 \right)$	M1	Use of W-E equation (allow 1 KE term or GPE term omitted)	
		B1	KE terms correct	
	$= 0.687829 0.6 \times \frac{40}{260} \times 2\pi \times 2$	M1	WD against friction	
	v = 2.06178 so 2.06 m s <sup>-1</sup> (3 s. f.)	A1 A1	WD against friction correct (allow sign error) cao	5
(b)	N2L down slope: $3g \sin 30 - F = 3 \times \frac{1}{8}g$ so $F = \frac{9g}{8}$ (= 11.025) $R = 3g \times \frac{\sqrt{3}}{2}$ (= 25.4611) $\mu = \frac{F}{R} = \frac{\sqrt{3}}{4}$ (= 0.43301)	M1 A1 A1 B1 M1 E1	Must have attempt at weight component Allow sign errors. Use of $F = \mu R$ Must be worked precisely	
		18		0

# 4762 Mechanics 2

Q 1		Mark		Sub
(i)	either			
		M1	Use of $I = Ft$	
	$m \times 2u = 5F$	Al	Must have reference to direction. A court diamon	
	so $F = 0.4mu$ in direction of the velocity	AI	Must have reference to direction. Accept diagram.	
	01	M1	Use of suvat and N2L	
	$a = \frac{2u}{5}$	A1	May be implied	
	so $F = 0.4mu$ in direction of the velocity	A1	Must have reference to direction. Accept diagram.	
				3
(ii)		MI		
	$\mathbf{DCIM} \rightarrow 2um + 2um - mu + 2mu$	MI	For 2 equins considering PCLM, NEL or Energy	
	$PCLM \rightarrow 2um + sum = mv_p + smv_Q$			
	$\text{NEL} \rightarrow v_Q - v_P = 2u - u = u$			
	Energy $\frac{1}{2}m \times (2u)^2 + \frac{1}{2}(3m) \times u^2$			
	$=\frac{1}{2}m \times v_{\rm P}^2 + \frac{1}{2}(3m) \times v_{\rm O}^2$			
		A1	One correct equation	
		A1	Second correct equation	
	Solving to get both velocities	M1	Dep on 1 <sup>st</sup> M1. Solving pair of equations.	
	$v_o = \frac{3u}{2}$	E1	If Energy equation used, allow 2 <sup>nd</sup> root discarded	
	~ 2		without comment	
	u		without comment.	
	$v_p = \frac{1}{2}$	A1		
			[If AG subst in one equation to find other velocity,	
			and no more, max SC3]	
(iii)	aithar			6
(111)	either 3eu			
	After collision with barrier $v_{\rm Q} = \frac{36\pi}{2}$ $\leftarrow$	B1	Accept no direction indicated	
	2			
	so $\rightarrow m \frac{u}{a} - 3m \frac{3eu}{a} = -4m \frac{u}{a}$	M1	PCLM	
	2 2 4		LUS Allow sign among Allow and of 2	
		AI	LETS Allow sign errors. Allow use of $3mv_Q$ .	
	1	AI	KHS Allow sign errors	
	so $e = \frac{1}{3}$	A1		
	At the barrier the impulse on Q is given by $(2ii - 1, 2ii)$			
	$\rightarrow 3m\left(-\frac{3u}{2}\times\frac{1}{3}-\frac{3u}{2}\right)$	M1	Impulse is $m(v - u)$	
		F1	$+\frac{3u}{2}\times\frac{1}{2}$	
		1.1	÷ 2 ^ 3	
	so impulse on Q is $-6mu \rightarrow$	F1	Allow $\pm$ and direction not clear. FT only <i>e</i> .	
	so impulse on the barrier is $6mu \rightarrow$	A1	cao. Direction must be clear. Units not required.	0
		18		9

1		1		1
Q 1	continued	mark		sub
(iii)	or After collision with barrier $v_{\rm Q} = \frac{3eu}{2} \leftarrow$	B1		
	Impulse – momentum overall for Q			
	$\rightarrow 2mu + 3mu + I = -4m \times \frac{u}{4}$	M1	All terms present	
	I = -6mu	A1 A1	All correct except for sign errors	
	so impulse of $6mu$ on the barrier $\rightarrow$	A1	Direction must be clear. Units not required.	
	Consider impact of Q with the barrier to give speed $v_{Q}$ after impact			
	$\rightarrow \frac{3u}{2} \times 3m - 6mu = 3mv_Q$	M1	Attempt to use I - M	
		F1		
	so $v_{\rm Q} = -\frac{u}{2}$	F1		
	$e = \frac{u}{2} \div \frac{3u}{2} = \frac{1}{3}$	A1	cao	
				9

Q 2		Mark		Sub
(i)				
	$R = 80g\cos\theta$ or $784\cos\theta$	B1	Seen	
	$F_{\rm max} = \mu R$	M1		
	so $32g\cos\theta$ or $313.6\cos\theta$ N	A1		
		711		3
(ii)				
	Distance is 1.25	D1		
	Distance is $\frac{1}{\sin \theta}$	DI		
	WD is $F_{max}d$	M1		
	so $32g\cos\theta \times \frac{1.25}{2}$	E1	Award for this or equivalent seen	
	$\sin \theta$			
	$=\frac{392}{}$			
	$\tan  heta$			3
(iii)	<u> </u>			5
(111)	$\Delta$ GPE is mgh	M1		
	so $80 \times 9.8 \times 1.25 = 980$ J	A1	Accept 100g J	
				2
(iv)				
	either D Fr	MI		
	P = FV so (80 a sin 35 + 32 a cos 35) × 1.5		W/-:-let to me	
	$50(000 \sin 55 + 520 \cos 55) \times 1.5$		All connect	
	-1050.85 so $1060  W (3  s  f)$		All correct	
	- 1059.85 S0 1000 W (5 S. 1.)	AI	cau	
	WD			
	$P = \frac{1}{\Delta t}$	M1		
	392			
	$980 + \frac{1}{\tan 35}$	B1	Numerator FT <b>their</b> GPE	
	so $\frac{1.25}{1.25}$	B1	Denominator	
	$\left(\frac{1}{\sin 35}\right)^{\pm 1.5}$			
	= 1059.85 so 1060 W (3 s. f.)	A1	cao	
	、 <i>/</i>			4
(v)	either			
	Using the W-E equation	M1	Attempt speed at ground or dist to reach required	
	(		speed. Allow only init KE omitted	
	$0.5 \times 80 \times v^2 - 0.5 \times 80 \times \left(\frac{1}{2}\right)^2 = 980 - \frac{392}{25}$	B1	KE terms. Allow sign errors. FT from (iv).	
	(2) tan 35	D1	Dath WD against friction of LODE (comp. All	
		ы	sign errors ET from parts above	
		A1	All correct	
	v = 3.2793. so yes	A1	CWO	
	or			
	N2L down slope	M1	All forces present	
	<i>a</i> = 2.409973	A1		
	distance slid, using <i>uvast</i> is 1.815372	Al	valid composicon	
	vertical distance is $1.8135/2 \times 1000$		CWO	
	= 1.0 + 12 < 1.25  so yes	ЛІ		5
		17		

Q 3		Mark		Sub
(i)	$\overline{y}:  250 \times 4 + 125 \left(8 + \frac{30}{2} \cos \alpha\right) = 375 \overline{y}$ $\overline{y} = \frac{28}{3} = 9\frac{1}{3}$ $\overline{z}:  (250 \times 0 + 125 \times \frac{30}{5} \sin \alpha = 375 \overline{z}$	Mark M1 B1 M1 B1 B1 E1 E1	Correct method for $\overline{y}$ or $\overline{z}$ Total mass correct $15 \cos \alpha$ or $15 \sin \alpha$ attempted either part $\left(8 + \frac{30}{2} \cos \alpha\right)$ $250 \times 4$ Accept any form	Sub
	$\overline{z} = 3$	E1		8
(ii)	Yes. Take moments about CD. c.w moment from weight; no a.c moment from table	E1 E1	[Award E1 for $9\frac{1}{2} > 8$ seen or 'the line of action	5
			of the weight is outside the base]	2
(iii)	c.m. new part is at (0, 8 + 20, 15)	M1 M1	Either y or z coordinate correct Attempt to 'add' to (i) or start again. Allow mass error.	
	$375 \times \frac{28}{3} + 125 \times 28 = 500 \overline{y}$ so $\overline{y} = 14$	E1		
	$375 \times 3 + 125 \times 15 = 500\overline{z}$ so $\overline{z} = 6$	E1		4
(iv)	Diagram Angle is $\arctan \frac{6}{14}$	B1 B1 M1	Roughly correct diagram Angle identified (may be implied) Use of tan. Allow use of 14/6 or equivalent.	
	$= 23.1985 \text{ so } 23.2^{\circ} (3 \text{ s. f.})$	A1	cao	4
		18		

Q 4		mark		sub
(a) (i)	Let the $\uparrow$ forces at P and Q be $R_{\rm p}$ and $R_{\rm Q}$ c.w. moments about P $2 \times 600 - 3R_{Q} = 0$ so force of 400 N $\uparrow$ at Q a.c. moments about Q or resolve $R_{\rm p} = 200$ so force of 200 N $\uparrow$ at P	M1 A1 M1 A1	Moments taken about a named point.	4
(ii)	$R_{\rm p} = 0$ c.w. moments about Q $2L - 1 \times 600 = 0$ so $L = 300$	B1 M1 A1	Clearly recognised or used. Moments attempted with all forces. Dep on $R_p = 0$ or $R_p$ not evaluated.	3
(b) (i)	$\cos \alpha = \frac{15}{17}$ or $\sin \alpha = \frac{8}{17}$ or $\tan \alpha = \frac{8}{15}$ c.w moments about A $16 \times 340 \cos \alpha - 8R = 0$ so $R = 600$	B1 M1 A1 E1	Seen here or below or implied by use. Moments. All forces must be present and appropriate resolution attempted. Evidence of evaluation.	4
(ii)	Diagram (Solution below assumes all internal forces set as tensions)	B1 B1	Must have 600 (or <i>R</i> ) and 340 N and reactions at A. All internal forces clearly marked as tension or thrust. Allow mixture. [Max of B1 if extra forces present]	2
(iii)	B ↓ 340 cos $\alpha$ + $T_{BC}$ cos $\alpha$ = 0 so $T_{BC}$ = -340 (Thrust of) 340 N in BC C → $T_{BC}$ sin $\alpha$ - $T_{AC}$ sin $\alpha$ = 0 so $T_{AC}$ = -340 (Thrust of) 340 N in AC B ← $T_{AB}$ + $T_{BC}$ sin $\alpha$ - 340 sin $\alpha$ = 0 so $T_{AB}$ = 320 (Tension of) 320 N in AB Tension/ Thrust all consistent with working	M1 A1 F1 M1 A1 F1	Equilibrium at a pin-joint Method for $T_{AB}$ [Award a max of 4/6 if working inconsistent with diagram]	6
		19		
### 4762 Mechanics 2

Q 1		mark	comment	sub
(a) (i)	before <u>u</u>	B1		1
	$ \begin{pmatrix} P \\ m  \mathrm{kg} \end{pmatrix} \begin{pmatrix} Q \\ km  \mathrm{kg} \end{pmatrix} $			
	after $\frac{u}{v}$ $\frac{u}{u_3}$			
(ii)				
	$mu - kmu = mv + km\frac{u}{3}$	M1	PCLM applied	
		A1	Either side correct (or equiv)	
	$v = \left(1 - \frac{4k}{3}\right)u$	E1	Must at least show terms grouped	
(;;;)				3
(11)	Need <i>v</i> < 0	E1	Accept $\frac{4k}{3} > 1$ without reason	
	<b>SO</b> $k > \frac{3}{4}$	B1	-	
			[SC1: $v = 0$ used and inequality stated without reason]	
				2
(iv)				
	$\frac{\frac{u}{3} - v}{-u - u} = -\frac{1}{2}$	M1	Use of NEL	
	u u <i>L</i>	A1		
	SO $v = -\frac{2u}{3}$	E1		
	$-\frac{2u}{3} = u\left(1 - \frac{4k}{3}\right)$	M1		
	so <i>k</i> = 1.25	A1	сао	5
(b)				
(i)	$9\binom{1}{-2} + 5\binom{3}{2} = 8\mathbf{V}$	M1	Use of PCLM	
		B1 M1	Use of mass 8 in coalescence Use of $I = Ft$	
	$\mathbf{V} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$	E1		
	· · ·			4
(ii)	· · 1			
	I cpt $3 \rightarrow -3 \times \frac{1}{2}$	M1	Allow wrong sign	

	7	c	2
- 4	1	υ	4

PMT

	j cpt unchanged	B1	May be implied	
	new velocity $\begin{pmatrix} -1.5\\ -1 \end{pmatrix}$ m s <sup>-1</sup>	A1	$\begin{bmatrix} 1\\ 0 \end{bmatrix}$	
			(0)	3
		18		0
Q 2		mark	comment	sub
(a) (i) (A)	Yes. Only WD is against conservative forces.	E1	Accept only WD is against gravity or no work done against friction.	
(B)	Block has no displacement in that direction	E1		2
(ii)	$0.5 \times 50 \times 1.5^2 = 20gx - 5gx$	M1	Use of WE with KE. Allow $m = 25$ .	
		B1 M1 A1	Use of 50 At least 1 GPE term GPE terms correct signs	
	x = 0.38265 so 0.383 m (3 s. f.)	A1	сао	_
				5
(iii)				
(,	$0.5 \times 50 \times V^2 - 0.5 \times 50 \times 1.5^2$	M1	WE equation with WD term. Allow GPE terms missing	
	$= 2 \times 20g - 2 \times 5g - 180$	B1 B1 B1	Both KE terms. Accept use of 25. Either GPE term	
	V = 2.6095 so 2.61 m s <sup>-1</sup>	A1	cao	
				5
(b)	Force down the slope is			
			Both terms Allow mass not	
	$2000 + 450g \sin 20$	M1	weight	
	Usina $P = Fv$	В1 M1	Weight term correct	
	$P = (2000 + 450g\sin 20) \times 2.5$	F1	FT their weight term	
	<i>P</i> = 8770.77 so 8770 W (3 s. f.)	A1	сао	
	••			5
		17		

PMT

Q 3		mark	comment	sub
(i)				00.00
	c.w. moments about A	M1	Moments equation.	
	$5R_{\rm B} - 3 \times 85 = 0$ so $R_{\rm B} = 51$ giving	A1	Accept no direction given	
	51 N ↑		5	
	Either a.c. moments about B or	M1		
	$R_{\star} = 34 \text{ so } 34 \text{ N} \uparrow$	F1	Accept no direction given	
		• •	Accept the direction given	4
(ii)				
	c.w. moments about A	M1	Moments with attempt to resolve	
			at least	
	$85 \times 3\cos \alpha - 27.2 \times 5\sin \alpha = 0$	B1	Weight term	
	65 x 5 c 65 a 27.2 x 5 sm a 6	B1	horiz force term	
	<b>SO</b> $\tan \alpha = \frac{3 \times 85}{3 \times 85} = \frac{15}{3}$	⊑1	Must see some arrangement of	
	$27.2 \times 5^{-8}$	<b>L</b> 1	terms	
			or equiv	4
(iii)				-
( )				
	$A \searrow S$			
	34 N			
		B1	All forces present and labelled	
	$F \downarrow \alpha$			
	85 N B			
			Moments with attempt to resolve	
	a.c. moments about B	M1	forces	
			and all relevant forces present	
	$85 \times 2 \times \cos \alpha + 34 \times 2.5 - 5S \times \sin \alpha = 0$	B1	$34 \times 2.5$	
		A1	All other terms correct. Allow sign	
	S = 37.4	A1	All correct	
	Resolving horizontally and	M1	Either attempted	
	$\rightarrow$ S-F-34 sin $\alpha = 0$ SO F = 7.4	F1		
			R = 101 need not be evaluated	
	$T R - 85 - 34 \cos \alpha = 0$	A1	here	
			[Allow A1 for the two expressions	
			IT correct other than a cost	
	Using $F = \mu R$	M1	conset other than $s \leftrightarrow c$	
	7.4 0.07000 0.0700			
	$\mu = \frac{1}{101} = 0.07326 \text{ so } 0.0733$	A1	сао	
	(3 s. f.)			
		10		10
		18		

58

Q 4		mark	comment	sub
(i)	Taking a <i>v</i> -axis vert downwards			
	from O		Allow areas used as masses	
	$2\pi\sigma \times 8^2 \times 4 + 2\pi\sigma \times 8 \times k \times \frac{k}{2}$	M1	Method for c.m.	
	_	B1	'4' used	
		B1 B1	k/2 used	
	$= \left(2\pi\sigma \times 8^2 + 2\pi\sigma \times 8k\right)\overline{y}$	B1	Masses correct	
	$\mathbf{SO}  \overline{y} = \frac{64 + k^2}{16 + 2k}$	E1	Must see some evidence of simplification Need no reference to axis of symmetry	
				6
(ii)	k = 12 gives OG as 5.2 and	D4	Allow for either Allow - 1	
	mass as $320\pi\sigma$	BI	Allow for either. Allow $\sigma = 1$	
	$320\pi\sigma \times 5.2 + \pi\sigma \times 8^2 \times 12$	M1	or starting again	
		B1 B1	One term correct Second term correct	
	$= (320\pi\sigma + 64\pi\sigma)\overline{y}$	DI		
	$\overline{y} = 6\frac{1}{2}$	E1	Some simplification shown	
	3			5
(iii)				
	$\begin{array}{c} 0 \\ 12 \\ \theta \\ \theta \end{array}$	B1 B1 B1	G above edge of base $12-6\frac{1}{3}=5\frac{2}{3}$ seen here or below 8 seen here or below	
	$\tan\theta = \frac{8}{5^{\frac{2}{3}}}$	M1	Accept $\frac{5\frac{2}{3}}{8}$ or attempts based on	
	$\theta = 54.6887$ so 54.7° (3 s.f.)	Δ1	$6\frac{1}{3}$ and 8.	
	0 - 04.0007 30 04.7 (0 3.1.)		000	5
(iv)	Slips when $\mu = \tan \theta$	M1	Or	
	<u></u>	R1		
	$5\frac{2}{3}$	Δ1	There must be a rosson	
			ווופול ווועטו של מ ולמטטוו	3
		19		

### 4762 Mechanics 2

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1 (a) (i)	Let vel of Q be $v \rightarrow 6 \times 1 = 4v + 2 \times 4$ $v = -0.5 \text{ so } 0.5 \text{ m s}^{-1}$ in opposite direction to R	M1 A1 A1 A1	Use of PCLM Any form Direction must be made clear. Accept – 0.5 only if + ve direction clearly shown	4
(ii)	Let velocities after be R: $v_R \rightarrow$ ; S: $v_S \rightarrow$ PCLM +ve $\rightarrow 4 \times 2 - 1 \times 3 = 2v_R + 3v_S$ $2v_R + 3v_S = 5$ NEL +ve $\rightarrow$ $\frac{v_S - v_R}{-1 - 4} = -0.1$ so $v_S - v_R = 0.5$ Solving gives $v_R = 0.7 \rightarrow$ $v_S = 1.2 \rightarrow$	M1 A1 M1 A1 A1 A1	PCLM Any form NEL Any form Direction not required Direction not required	
(iii)	R and S separate at 0.5 m s <sup>-1</sup> Time to drop <i>T</i> given by $0.5 \times 9.8T^2 = 0.4$ so $T = \frac{2}{7}$ (0.28571) so distance is $\frac{2}{7} \times 0.5 = \frac{1}{7}$ m (0.142857m)	M1 B1 A1	Award cao for 1 vel and FT second FT <b>their</b> result above. Either from NEL or from difference in final velocities cao	63
(b)	before after $v \rightarrow u$ $u \rightarrow u$ $v \rightarrow (-)ev$ KE loss is $\frac{1}{2}m(u^2 + v^2) - \frac{1}{2}m(u^2 + e^2v^2)$ $= \frac{1}{2}mu^2 + \frac{1}{2}mv^2 - \frac{1}{2}mu^2 - \frac{1}{2}me^2v^2$ $= \frac{1}{2}mv^2(1-e^2)$	B1 B1 M1 E1	Accept $v \rightarrow ev$ Attempt at difference of KEs Clear expansion and simplification of correct expression	4
				4

2(i)	GPE is 1200 × 9.8 × 60 = 705 600 Power is (705 600 + 1 800 000) ÷ 120 = 20 880 W = 20 900 W (3 s. f.)	B1 M1 B1 A1	Need not be evaluated power is WD ÷ time 120 s cao	4
(ii)	Using $P = Fv$ . Let resistance be $R$ N 13500 = 18 $F$ so $F = 750$ As $v$ const, $a = 0$ so $F - R = 0$	M1 A1	Use of $P = Fv$ .	
	Hence resistance is 750 N	E1	Needs some justification	
	We require $750 \times 200 = 150\ 000\ J$	M1	Use of $WD = Fd$ or $Pt$	
	(= 150 kJ)	F1	FT their F	5
(iii)	$\frac{1}{2} \times 1200 \times (9^2 - 18^2)$ = 1200 \times 9.8 \times x \sin 5 - 1500x	M1 B1 M1 A1	Use of W-E equation with 'x' 2 KE terms present GPE term with resolution GPE term correct All correct	
	Hence $145800 = 475.04846x$ so $x = 306.91$ so $307 \text{ m} (3 \text{ s, f,})$	A1	cao	6
(iv)	P = Fv and N2L gives $F - R = 1200a$ Substituting gives P = (R + 1200a)v If $a \neq 0$ , v is not constant. But P and R are constant so a cannot be constant.	B1 B1 E1 E1	Shown	
				4
3 (i) (A)	Let force be P a.c. moments about C $P \times 0.125 - 340 \times 0.5 = 0$ P = 1360 so 1360 N	M1 A1 A1	Moments about C. All forces present. No extra forces. Distances correct cao	3
(i) ( <i>B</i> )	Let force be P c.w. moments about E $P \times 2.125 - 340 \times (2 - 0.5) = 0$ P = 240 so 240 N	M1 A1 A1	Moments about E. All forces present. No extra forces. Distances correct cao	3

(ii)	$Q\sin\theta \times 2.125 + Q\cos\theta \times 0.9$ $= \frac{25.5Q}{13} + \frac{4.5Q}{13}$ $= \frac{30Q}{13} \text{ so } \frac{30Q}{13} \text{ N m}$	M1 B1 E1	Moments expression. Accept $s \leftrightarrow c$ . Correct trig ratios <b>or</b> lengths Shown	3
(iii)	We need $\frac{30Q}{13} = 340 \times 1.5$ so $Q = 221$ Let friction be <i>F</i> and normal reaction <i>R</i> Resolve $\rightarrow$ $221\cos\theta - F = 0$ so $F = 85$ Resolve $\uparrow$ $221\sin\theta + R = 340$ so $R = 136$ $F < \mu R$ as not on point of sliding so $85 < 136\mu$	M1 E1 M1 A1 M1 A1 M1 A1	Moments equn with all relevant forces Shown Accept $\leq$ or = Accept $\leq$ . FT <b>their</b> <i>F</i> and <i>R</i>	
	so $\mu > \frac{5}{8}$	E1		
				9
4 (i)	$4000\left(\frac{\overline{x}}{\overline{y}}\right) = 4800\left(\frac{30}{40}\right) - 800\left(\frac{50}{20}\right)$	M1 A1	Any complete method for c.m. Either one RHS term correct or one component of both RHS terms correct	10
	so $\overline{x} = 26$ $\overline{y} = 44$	E1 A1	[SC 2 for correct $\overline{y}$ seen if M 0]	4
(ii)	$250\left(\frac{\overline{x}}{\overline{y}}\right) = 110\left(\frac{0}{55}\right) + 40\left(\frac{20}{0}\right) + 40\left(\frac{40}{20}\right) + 20\left(\frac{50}{40}\right) + 40\left(\frac{60}{60}\right)$	M1 B1	Any complete method for c.m. Any 2 edges correct mass and c.m. <b>or</b> any 4 edges correct with mass and <i>x</i> or <i>y</i> c.m. coordinate correct.	
	$\overline{x} = 23.2$ $\overline{y} = 40.2$	B1 E1 A1	At most one consistent error	5

(iii)	$\begin{array}{c} \begin{array}{c} & \\ 110 - 40.2 \\ \hline \\ 40.2 \\ \hline \\ 0 \\ \end{array} \\ \begin{array}{c} \\ \\ \\ \end{array} \\ \begin{array}{c} \\ \\ \\ \end{array} \\ \begin{array}{c} \\ \end{array} \\ \begin{array}{c} \\ \end{array} \\ \end{array} \\ \begin{array}{c} \\ \end{array} \\ \begin{array}{c} \\ \end{array} \\ \begin{array}{c} \\ \end{array} \\ \end{array} \\ \begin{array}{c} \\ \end{array} \\ \begin{array}{c} \\ \end{array} \\ \end{array} \\ \begin{array}{c} \\ \end{array} \\ \begin{array}{c} \\ \end{array} \\ \end{array} \\ \end{array} \\ \begin{array}{c} \\ \end{array} \\ \end{array} \\ \end{array} \\ \begin{array} \\ \end{array} \\ \end{array} \\ \end{array} \\ \end{array} \\ \begin{array}{c} \\ \end{array} \\ $	В1	Indicating c.m. vertically below Q	
	Angle is $\arctan\left(\frac{23.2}{110-40.2}\right)$ = 18.3856 so 18.4° (3 s. f.)	B1 M1 A1	Clearly identifying correct angle (may be implied) and lengths Award for $\arctan\left(\frac{b}{a}\right)$ where $b = 23.2$ and $a = 69.8$ or 40.2 or where $b = 69.8$ or 40.2 and $a = 23.2$ . Allow use of <b>their</b> value for <i>y</i> only. cao	4
(iv)	$10\left(\frac{\overline{x}}{\overline{y}}\right) = 2 \times 1.5 \times \binom{26}{44} + 7\binom{23.2}{40.2}$ $\overline{x} = 24.04 \text{ so } 24.0 \text{ (3 s.f.)}$ $\overline{y} = 41.34 \text{ so } 41.3 \text{ (3 s.f.)}$	M1 B1 A1 A1 F1	Combining the parts using masses Using both ends All correct cao FT <b>their</b> y values only.	5





# Mathematics (MEI)

Advanced GCE 4762

Mechanics 2

## Mark Scheme for June 2010

01		mark		sub
<b>V</b> <sup>1</sup>		murit		Sub
(i)	For P $200 \times 5 + 250 = 200v_{\rm p}$ $v_{\rm p} = 6.25 \text{ so } 6.25 \text{ i m s}^{-1}$ For O	M1 E1	Award for I-M Accept no i and no units	
	$250 \times 5 - 250 = 250v_Q$ $v_Q = 4 \text{ so } 4\mathbf{i} \text{ m s}^{-1}$	M1 A1	Must have impulse in opposite sense Must indicate direction. Accept no units.	4
(ii)	i direction positive PCLM: $2250 = 200 \times 4.5 + 250w_Q$ $w_Q = 5.4 \text{ so } 5.4i \text{ m s}^{-1}$ NEL: $\frac{w_Q - 4.5}{4 - 6.25} = -e$ e = 0.4	M1 F1 E1 M1 A1 A1	PCLM used. Allow error in LHS FT from (i) Any form. FT only from (i) NEL . Allow sign errors Signs correct. FT only from (i) cao	6
(iii)	i direction positive Suppose absolute vel of object is $-Vi$ $200 \times 4.5 = -20V + 180 \times 5.5$ V = 4.5 speed of separation is $5.5 + 4.5 = 10$ m s <sup>-1</sup>	M1 B1 A1 F1	Applying PCLM. All terms present. Allow sign errors. Correct masses All correct (including signs) FT <b>their</b> V.	5
(iv)	$180 \times 5.5 + 250 \times 5.4 = 430W$ W = 5.4418 so 5.44 <b>i</b> m s <sup>-1</sup> (3 s. f.)	M1 A1	Using correct masses and velocities cao	2

		mark		sub
(i)	$20\left(\frac{\overline{x}}{\overline{y}}\right) = 15\left(\frac{20}{0}\right) + 3\left(\frac{0}{100}\right) + 2\left(\frac{25}{200}\right)$ $\overline{x} = 17.5$ $\overline{y} = 35$	M1 B1 A1 A1 A1	Method to obtain at least 1 coordinate '100' or '25' correct Either one RHS term correct or one component of two RHS terms correct	5
(ii)	$25\left(\frac{\overline{x}}{\overline{y}}\right) = \left(\frac{350}{700}\right) + 5\left(\frac{40}{200}\right)$ so $\overline{x} = 22$ , $\overline{y} = 68$	M1 E1	Using (i) or starting again Clearly shown.	2
(iii)	We need the edge that the $\overline{x}$ position is nearest $\overline{x} = 22$ ; distances are 22 to PQ, 18 to SR 15 to QR so edge QR	M1 B1 B1 A1	This may be implied One distance correct All distances correct	4
(iv)	Moments about RS In sense $xOy$ $T \sin 50 \times 200 - T \cos 50 \times 40$ $-20g \times (40 - 17.5) = 0$ T = 34.5889 so 34.6 N (3 s. f.)	M1 B1 M1 A1 B1 A1 A1	Moments about RS attempted Use of weight not mass below. FT mass from here Attempt to find moment of <i>T</i> about RS, including attempt at resolution. May try to find perp dist from G to line of action of the force. 40 - 17.5 All correct allowing sign errors cao (except for use of mass)	7

Q 3		mark		sub
(i)	a.c. moments about A $1 \times T - 2 \times 300 = 0$ so $T = 600$ Resolving $\rightarrow X = 0$ $\uparrow T - Y = 300$ so $Y = 300$	E1 B1 M1 A1	Justified	4
(ii)	Diagram The working below sets all internal forces as tensions; candidates need not do this.	B1 B1	All external forces marked consistent with (i) All internal forces with arrows and labels	2
(iii)	Let angle DAB be $\theta$ . $\cos \theta = \frac{1}{2}$ , $\sin \theta = \frac{\sqrt{3}}{2}$ A $\uparrow -300 - T_{AB} \sin \theta = 0$ so $T_{AB} = -200\sqrt{3}$ so force is $200\sqrt{3}$ (C) A $\rightarrow T_{AD} + T_{AB} \cos \theta = 0$ so $T_{AD} = 100\sqrt{3}$ so force is $100\sqrt{3}$ (T) C $\uparrow T_{CD} \sin \theta - 300 = 0$ so $T_{CD} = 200\sqrt{3}$ so force is $200\sqrt{3}$ (T) C $\leftarrow T_{BC} + T_{CD} \cos \theta = 0$ so $T_{BC} = -100\sqrt{3}$ so force is $100\sqrt{3}$ (C) B $\uparrow T_{AB} \sin \theta + T_{BD} = 0$ so $T_{BD} = 300$ so force is $300$ (T)	<ul> <li>B1</li> <li>M1</li> <li>M1</li> <li>A1</li> <li>F1</li> <li>F1</li> <li>F1</li> <li>F1</li> <li>F1</li> <li>F1</li> <li>F1</li> <li>F1</li> <li>F1</li> </ul>	Or equivalent seen Attempt at equilibrium at pin-joints 1 equilib correct, allowing sign errors All T/C consistent with their calculations and diagrams	9
(iv)	AD, AB, BC, CD 300 N, X and Y not changed. Equilibrium equations at A and C are not altered B $\uparrow T_{AB} \sin \theta + T'_{BD} + 600 = 0$ so $T'_{BD} = -300$ so force is 300 (C)	B1 E1 M1 A1	C not needed. [If 300 N (C) given WWW, award SC1 (NB it must be made clear that this is a compression)]	4

-		1		1
Q 4		mark		sub
(i)	Let friction be <i>F</i> N and normal reaction <i>R</i> N $F_{max} = 58\cos 35$ $R = 16g + 58\sin 35$ $F_{max} = \mu R$ so $\mu = 0.249968$ about 0.25	B1 M1 A1 M1 E1	Need not be explicit Both terms required.	5
(ii)	WD is 70 cos 35 × 3 = 210 cos 35 so 172.0219 = 172 J (3 s. f.) Average power is WD/time so 34.4043 = 34.4 W (3 s. f.)	M1 A1 M1 A1	Use of WD = $Fd$ . Accept cos 35 omitted. cao	4
(iii)	Using the constant acceleration result $s = \frac{1}{2}(u+v)t$ with $s = 3$ , $u = 0$ , $v = 1.5$ and $t = 5$ we see that $3 \neq \frac{1}{2}(0+1.5) \times 5 = 3.75$	M1 E1	Attempt to substitute in <i>suvat</i> (sequence) Conclusion clear	2
(iv)	172.0219 = $\frac{1}{2} \times 16 \times 1.5^2$ +0.25×(16g + 70 sin 35)×3 + WD so WD by S is 6.30916 so 6.31 J (3 s. f.)	M1 M1 A1 M1 A1 A1 A1	Using W-E equn, allow 1 missing term KE term attempted correct Attempt at using new F in $F_{max} = \mu R$ All correct cao	7 18





# **Mathematics (MEI)**

Advanced GCE Unit **4762:** Mechanics 2

## Mark Scheme for January 2011

January 2011

Q 1		mark	notes
(i)	Let normal reaction be R $\sin \alpha = \sqrt{1-0.8^2} = 0.6$ $R = 2.5 \times 9.8 \times 0.8$ $F_{\text{max}} = 0.85 \times R = 16.66$ Wt cpt down slope is $2.5 \times 9.8 \times 0.6 = 14.7$ 16.66 > 14.7 so at rest	B1 M1 B1 F1 B1 E1	Accept any form and implied Use of $F_{max} = \mu R$ Expression for $R$ ; may be implied FT their $R$ FT if their $F$ and weight component show given result If $g$ omitted, allow B1M1B0F1B0E1, so 4/6 [Award as follows for use of $\tan \alpha < \mu$ : B1 $\tan \alpha = \frac{3}{4}$ E1 $\tan \alpha < \mu$ shown]
(ii)	Let the speeds down the plane be $v_A$ and $v_B$ . PCLM down the plane $1.5 \times 16 = 2.5v_A + 1.5v_B$ so $5v_A + 3v_B = 48$ NEL +ve down the plane $\frac{v_A - v_B}{0 - 16} = -0.4$ $v_A - v_B = 6.4$ $v_A = 8.4$ so $8.4$ m s <sup>-1</sup> down plane $v_B = 2$ so $2$ m s <sup>-1</sup> down plane	M1 A1 M1 A1 E1 F1 6	PCLM Any form NEL. Allow sign errors Any form Condone direction not clear if +8.4 seen Condone direction not clear if +2 seen. SC1 if 2 equations obtained and 8.4 substituted into one to obtain answer 2 (instead of E1F1)
(iii)	$1.5 \times (2 - 16)$ down plane = -21 N s down the plane so 21 Ns up the plane	M1 A1 A1 3	Use of $m(\mathbf{v} - \mathbf{u})$ If impulse on <i>A</i> found, treat as MR unless final answer relates this to impulse on <i>B</i> $\pm 21$ N s Direction explicitly commented on

#### Mark Scheme

Q 1		mark	notes
(iv)	either		
	$(2.5 \times 9.8 \times 0.6 - F_{\text{max}}) \times t = 2.5(0 - 8.4)$	M1	Using Impulse-momentum (must use 8.4). sufficient to consider one term on LHS
		B1	Either side correct
		A1	Allow only sign errors
	so <i>t</i> = 10.7142 10.7 s (3 s. f.)	A1	cao
	or		
	Using N2L down the plane	M1	Using N2L ; sufficient to consider one force term
	a = -0.784	A1	Allow sign errors
		M1	Using appropriate <i>suvat</i> must use <i>a</i> or- <i>a</i> found by use of N2L and $u = 8.4$
	using $v = u + at$ , $t = 10.7142 10.7 \text{ s} (3 \text{ s. f.})$	A1	cao
	or		
	$0.5 \times 2.5 \times 8.4^2 + (14.7 - 16.66)x = 0$	M1	Use energy with 8.4, sufficient to consider one non-KE term
	<i>x</i> = 45	A1	
		M1	Using appropriate suvat
	T = 10.7142 10.7 (3  s. f.)	A1	cao
		4	
		19	

Q 2		mark	notes
(a)	$v \text{ m s}^{-1}$ $V \text{ m s}^{-1}$ $\mathbf{i}$		
	C 0.004 kg 🛛 🔤 B 0.060 kg		
	Energy: $\frac{1}{2} \times 0.004 \times v^2 + \frac{1}{2} \times 0.060 \times V^2 = 0.8$	M1	Use of KE in two terms in an equation.
	$v^2 + 15V^2 = 400$	A1	Any form
	PCLM in <b>i</b> direction: $0.06V - 0.004v = 0$ v = 15V Solving $(15V)^2 + 15V^2 = 400$ so $V^2 = \frac{400}{5}$ and $V = \sqrt{5}i$	M1 A1 M1	PCLM. Accept sign errors. Any form Valid method for elimination of $v$ or $V$ from a linear and a quadratic
	$\mathbf{v} = -15\sqrt{\frac{5}{3}}\mathbf{i} \ (= -\sqrt{375}\mathbf{i})$	F1 A1 8	Accept – 19.3649i Accept no direction Second answer follows from first (Relative) directions indicated - accept diagram. Both speeds correct.
(b) (i)	W is work done by resistances on car		
	$\frac{1}{2} \times 800 \times (12^{2} - 30^{2}) = -800 \times 9.8 \times 20 + W$	M1 B1 A1	Use of WE. Must have KE, W and GPE. Allow -W Both KE terms. Accept sign error All correct with W or -W
	$W = -145\ 600$ so 145 600 J done by car against resistances	A1	cao
		+	

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Q 2		mark	notes
(ii)	either The slope is $18 \times 25 = 450$ m long $\frac{800 \times 9.8 \times 20 + 750 \times 450}{25}$	B1 M1	Use of $P = (Work done) / (elapsed time)$ used for at least one work done term
	= 19 772 W or The angle of the slope is $\arcsin(1/22 5)$	M1 A1 A1 B1	WD is force $\times$ distance used for at least one force Allow only sign errors both terms cao.
	$\left(800 \times 9.8 \times \frac{1}{22.5} + 750\right) \times 18$ = 19 772 W	M1 M1 A1 A1 5	Use of $P = Fv$ used for at least one term Attempt at weight component Allow only sign errors both terms cao.
		17	

Q 3		mark	notes
(i)	Horizontal $X - 50 = 0$ Vertical: $R - Y - 45 = 0$	B1 B1 2	Any form Any form
(ii)	a. c. moments about A $1 \times R = 3 \times 45$ so $R = 135$ so $135 - Y - 45 = 0$ and $Y = 90$	M1 E1 E1 3	Clearly shown Shown
(iii)	In analysis below all internal forces are taken as tensions	B1 B1 2	Correct arrow pairs for all internal forces Correct labels

#### Mark Scheme

Q 3		mark	notes
(iv)		M1 M1 M1	Equilibrium attempted at a pin-joint Equilibrium attempted at a 2 <sup>nd</sup> pin-joint Either Equilibrium equation for 2 <sup>nd</sup> direction at a pin-joint or 3 <sup>rd</sup> pin-joint considered
	At C	B1	At least 3 equations of resolution correct or follow through
	$\uparrow T_{\rm CD} \cos 30 - 45 = 0$ so $T_{\rm CD} = 30\sqrt{3}$		
	and force in CD is $30\sqrt{3}$ N (T)	A1	
	$\leftarrow T_{\rm BC} + T_{\rm CD} \cos 60 = 0 \text{ so } T_{\rm BC} = -15\sqrt{3}$		
	and force in BC is $15\sqrt{3}$ N (C)	F1	
	At D $T = \cos 20 + T = \cos 20 = 0$		
	$\int_{BD} \cos 30 + I_{CD} \cos 30 = 0$		
	so $T_{BD} = -50\sqrt{3}$ and force in BD is $30\sqrt{3}$ N (C)	F1	
		11	
	$\leftarrow T_{\rm AD} + T_{\rm BD} \cos 60 - T_{\rm CD} \cos 60 - 50 = 0$		
	so $T_{AD} = 50 + 30\sqrt{3}$		
	and the force in AD is $50 + 30\sqrt{3}$ N (T)	F1	
	At A $T_{-} \cos 30 + 90 = 0$ so $T_{-} = -60 \sqrt{3}$		
	and the force in AB is $60\sqrt{3}$ N (C)	F1	
	and the force in AB is 6005 in (C)	B1	At least 4 T/C correct
		10	
(v)	The equilibria at C depend only on the		Resolve in two directions at C and obtain same results as in (iv) M1A1
	framework geometry and the 45 N.	E1	
	These are not changed so forces in CB and CD are not changed	E1	
		2	
		19	

Q 4		mark	notes
(i)	(2, 2.5)	B1 1	Condone writing as a vector
(ii)	By symmetry, $\overline{y} = 2.5$ For $\overline{x}$ : $\left(5h + \frac{1}{2} \times 5 \times 6\right)\overline{x} = 5h \times \left(-\frac{h}{2}\right) + \frac{1}{2} \times 5 \times 6 \times 2$ so $(5h+15)\overline{x} = -2.5h^2 + 30$ so $5(h+3)\overline{x} = 2.5(12-h^2)$	B1 M1 A1 A1 A1	Some justification needed These next 4 marks may be obtained from correct FT of their "2" from (i) 1 <sup>st</sup> term RHS correct (allow sign error) Either other term correct All correct
	and $\bar{x} = \frac{12 - h^2}{2(h+3)}$	E1 6	Clearly shown, including signs.
(iii)	Need $\bar{x} > 0$ So $\frac{12 - h^2}{2(h+3)} > 0$ Hence $12 - h^2 > 0$ Since $h > 0, \ 0 < h < 2\sqrt{3}$	M1 B1 A1 3	Allow $\overline{x} \ge 0$ or $= 0$ $2\sqrt{3}$ or $-2\sqrt{3}$ oe seen Accept only +ve root mentioned. WWW for signs Accept $h < 2\sqrt{3}$ as answer strict inequality for final A mark

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Q 4		mark	notes
Q4 (iv)	<b>continued</b> When $h = 3$ , $\bar{x} = 0.25$ Let mag of vert force be $T$ N a.c moments about axis thro' O $T \times 6 - 15 \times 0.25 = 0$ so $T = 0.625$ so $0.625$ N	B1 M1 A1 3	Could be scored in (v) If moments about another point need all relevant forces. Allow sign errors. Condone use of 15g cao
(v)	Let magnitude of force be U N a.c. moments about axis thro' D $U \cos 30 \times 5 - 15 \times (3 + 0.25) = 0$ U = 11.25833 so 11.3 N (3 s. f.)	M1 B1 A1 A1 4	Each term must be a moment. If moments about another point need all relevant forces. Condone use of $15g$ . moment of $U$ (5 $U$ cos30 or) oe (3 + 0.25) oe cao
		17	





# Mathematics (MEI)

Advanced GCE Unit **4762:** Mechanics 2

## Mark Scheme for June 2011

Q 1		mark	notes
(a) (i)	13T = 10(4.75 - (-1.75)) so T = 5. So 5 s. OR: 13 = 10a $T = \frac{4.75 - (-1.75)}{1.3} = 5$	M1 A1 A1 B1 M1 A1 3	Use of $I = Ft$ . Allow sign errors Signs correct on RHS cao N2L Use of <i>suvat</i> cao
(ii)	PCLM: $10 \times 4.75 - 15 \times 0.5 = 25v_{P+Q}$ $v_{P+Q} = 1.6$ so $1.6$ m s <sup>-1</sup> in +ve direction	M1 A1 2	PCLM with combined mass. Allow sign errors No need for reference to direction
(iii)	PCLM: $10 \times 4.75 - 15 \times 0.5 = 10 \times 1 + 15v_Q$ Hence $v_Q = 2$ and Q has velocity 2 m s <sup>-1</sup> NEL: $\frac{v_Q - 1}{-0.5 - 4.75} = -e$ so $e = 0.19047$ so $0.190$ (3 s. f.)	M1 A1 A1 M1 A1 A1 A1 6	PCLM with all correct terms. Allow sign errors Any form Accept no direct reference to direction NEL. Accept <b>their</b> $v_{\rm Q}$ and any sign errors. Fraction must be correct way up Any form. FT <b>their</b> $v_{\rm Q}$ . cao accept 0.19, 4/21 accept 0.2 only if 0.19 seen earlier

(b)	Initial vert cpt is $14\sin 30 = 7$	B1	
(-)	$1^{\text{st}}$ hits ground at v given by		
	$v^2 = 7^2 + 2 \times 9.8 \times 3.125$	M1	Appropriate <i>suvat</i> . Allow $\pm 9.8$ etc Condone $u = 14$
	<i>v</i> = 10.5	A1	
	Vert cpt after 2 <sup>nd</sup> bounce		
	$10.5  imes 0.6^2$	M1	<b>their</b> $10.5 \times 0.6^n$ for $n = 1, 2$ or 3 Condone use of their initial vertical component. Do not award if horiz component is also multiplied by 0.6
		B1	use of $\times 0.6^2$ or attempt at two bounces with 0.6 used each time
	Horiz cpt is unchanged throughout (14 cos 30)	B1	Award even if value wrong or not given
	Angle is $\arctan\left(\frac{10.5 \times 0.6^2}{14\cos 30}\right) = 17.31586$	M1	FT their horiz and vert components. oe. Fraction must be for correct angle.
	so 17.3° (3 s. f.)	A1 8	cao SC answer of 11.7 will usually earn 5/8
		19	

4	7	6	2
_		v	_

Q 2		mark	notes
(i)	) cw moments about A Let force be S $600 \times 0.8 - S \times 2 = 0$ M1 Mo		Penalise answers to fewer than 4sf only once Moments. All forces. No extras
	S = 240 so 240 N vertically upwards	A1 A1 3	Need statement of direction or diagram
(ii)	cw moments about A Let tension be T $600 \times 0.8 - T \sin 50 \times 0.3 = 0$	M1	Moments. All forces. No extras. Attempt at moment of $T$ (need not be resolved) Note that mmts about $B$ needs forces at hinge.
		M1 A1 A1	Correct method for moment of <i>T</i> . Allow length errors and $s \leftrightarrow c$ Moment of <i>T</i> correct (allow sign error) All correct
	$T = 2088.65 (\frac{1600}{\sin 50})$ so 2089 N (4 s. f.)	A1 5	cao
(iii)	Resolve $\rightarrow X - T\cos 50 = 0$ so $X = 1342.55$	M1	Resolving horiz. Allow sign error. T must be resolved, allow $s \leftrightarrow c$
	= 1343 (4 s. f.)	F1	FT their T only. Allow 1600cot50
	Resolve $\downarrow Y - T\sin 50 + 600 = 0$	M1	NB other methods possible
	so $Y = 1000$	F1	FT their T only
	Method for either <i>R</i> or $\alpha$	MI	M dependent on attempts at X and Y using moments/resolution
	$R = \sqrt{1600^2 \cot^2 50 + 1000^2} = 1674.05$		
	so 1674 (4 s. f.)	F1	FT their X and Y Numerical value only
	$\alpha = \arctan \frac{1000}{1600 + 50}$		
	$\alpha = 36.6804$ so $36.68^{\circ}$ (4 s. f.)	F1 7	FT <b>their</b> <i>X</i> and <i>Y</i> Numerical value only Accept 36.67
(iv)	Angle GAP is $\alpha$ above so 36.68° (4 s. f.) Weight, <i>T</i> and <i>R</i> are the only forces acting on	B1 E1	Must be clear
	the beam which is in equilibrium. Hence they are concurrent. Or geometrical calculation	2	
		17	

Q 3		mark	notes
(i)	$10\left(\frac{\overline{x}}{\overline{y}}\right) = 4\left(\frac{-\frac{1}{2}}{2}\right) + 2\left(\frac{1}{2}}{3}\right) + \left(\frac{1\frac{1}{2}}{3\frac{1}{2}}\right) + 3\left(\frac{2\frac{1}{2}}{2\frac{1}{2}}\right)$ $= \left(\frac{-2+1+1\frac{1}{2}+7\frac{1}{2}}{8+6+3\frac{1}{2}+7\frac{1}{2}}\right) = \left(\frac{8}{25}\right)$ so $\left(\frac{\overline{x}}{\overline{y}}\right) = \left(\frac{0.8}{2.5}\right)$ and c.m. is (0.8, 2.5)	M1 B1 E1 E1 4	Correct method clearly indicated for <i>x</i> or <i>y</i> component. If 2D method, at least 1 mass + cm correct for a region. If separate cpts, at least 2 mass + cm correct for one of the cpts Working shown. Either expression shown oe Both
(ii)	c.w. moments about J $3.2 \times 1.8 - T_{\rm H} \times 4 = 0$ so $T_{\rm H} = 1.44$ and the force at H is 1.44 N Resolving $\uparrow$ force at J is $3.2 - 1.44 = 1.76$ N	B1 M1 A1 M1 F1 5	Use of 1.8 oe A moments equation with all relevant forces. Allow use of 10 instead of 3.2 Or moments again Only FT if positive final answer
(iii)	below		

		18	
		4	
	so angle is 20.323 so 20.3° (3 s. f.)	A1	Do NOT isw
	$\tan\theta = \frac{1}{2.7}$	M1	Accept $\tan \theta = \frac{2.7}{1}$ oe
	Let angle IOG be $\theta$	B1	Correctly identifying the angle
(17)		DI	Recognising that chi is vertically below O (may be implied)
(iv)		B1	Recognising that cm is vertically below Q (may be implied)
	so $\left(\frac{\overline{x}}{\overline{y}}\right) = \begin{pmatrix} 1\\ 2.7\\ 0 \end{pmatrix}$ and c.m. is (1, 2.7, 0)	E1 5	All three components
	$= \begin{pmatrix} 0+1+4+5\\ 8+6+7+6\\ 2+0+0-2 \end{pmatrix} = \begin{pmatrix} 10\\ 27\\ 0 \end{pmatrix}$	E1	Working shown. Either expression shown oe
		B1 B1	Dealing correctly with one folded part Dealing with the other folded part
(iii)	$10\left(\frac{\overline{x}}{\overline{y}}\right) = 4\left(\begin{array}{c}0\\2\\\frac{1}{2}\end{array}\right) + 2\left(\begin{array}{c}1\\2\\0\end{array}\right) + 2\left(\begin{array}{c}2\\3\\\frac{1}{2}\\0\end{array}\right) + 2\left(\begin{array}{c}2\\3\\\frac{1}{2}\\0\end{array}\right) + 2\left(\begin{array}{c}2\\3\\-1\end{array}\right)$	M1	Dealing with 3D

#### June 2011

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Q 4		mark	notes
(a)	$\frac{1}{2} \times 80 \times (6^{2} - V^{2})$ = 80 × 9.8 × 1600 - 1300000 so V = 34.29285 so 34.3 m s <sup>-1</sup> , (3 s. f.)	M1 B1 B1 A1 A1 5	WE equation. Allow GPE OR init KE term omitted or wrong. Allow sign errors. There must be 3 terms one of which is the WD term KE terms correct (accept $40 \times (V^2 - 6^2)$ ) GPE term. Allow sign error All terms present. Accept only sign errors, but not the 1300000 and 80x9.8x1600 terms with same sign Cao accept $14\sqrt{6}$
(b) (i)	N2L up the slope. Driving force is <i>S</i> N $S - 1150 - 800 \times 9.8 \times 0.1 = 800 \times 0.25$ S = 2134 Power is 2134 × 8 = 17072 so 17.1 kW (3 s. f.)	M1 B1 M1 A1 E1 M1 A1 7	N2L. Allow either resistance or weight cpt omitted. Allow weight not resolved and sign errors. RHS correct Attempt at weight cpt ( $800g\sin\theta$ is sufficient) Allow missing $g$ Weight cpt correct (numerical) May be implied Use of $P = Fv$
(ii)	Let resistance on sledge be F N N2L up slope for sledge $900 - F - 300 \times 9.8 \times 0.1 = 300 \times 0.25$ so $F = 531$ normal reaction is $300g\cos\theta$ Use $\cos\theta = \sqrt{0.99}$ or $\cos 5.7$ $\mu = \frac{531}{300 \times 9.8 \times \sqrt{0.99}}$ = 0.181522 so 0.182 (3 s. f.)	M1 A1 B1 B1 M1 A1 6	Need non-zero accn, correct mass and 900. Allow weight missing or unresolved and allow sign errors. Do not award if 2134 included In context Use of $F = \mu R$ for any <i>F</i> and <i>R</i> but not F=900 cao

C	luesti	ion	Answer	Marks	Guidance
1	(a)	(i)	KE change: $\frac{1}{2} \times 0.6 \times (7.5^2 - 5.5^2)$	M1	Difference of two KE terms
1	(2)	(ii)	= 7.8 J GPE change: $0.6 \times 9.8 \times 1.5 = 8.82$ J	A1 B1 [3]	Allow –8.82J
1	(a)	(11)	7.8 = 8.82 – $W$ so $W = 1.02$ J	M1 A1 [2]	W–E all terms. Allow sign errors FT (i) only. Also FT only if mod (their KE) < mod (their PE) -1.02 gets M1A0; 16.62 gets M1A0
1	(a)	(iii)	Average resistance is $F$ so $F \times 1.5 = 1.02$ so $F = 0.68$ Power is $0.68 \times 5.5$ = 3.74 so $3.74$ W	M1 A1 M1 A1 [4]	Use of WD = $Fs$ OR find $a = 8.667$ and use F = $0.6g - 0.6 \times 8.667$ May be implied. FT (ii) Use of $P = Fv$ any calculated F cao
1	(b)	(i)	$R = mg\cos 40$ $F_{max} = mg\sin 40$ $F_{max} = \mu R$ so $\mu = \frac{mg\sin 40}{mg\cos 40} = \tan 40$	B1 B1 M1 E1 [4]	Seen or implied Seen or implied Use of $F = \mu R$ : substitute <i>F</i> and <i>R</i> This is the minimum amount of working needed to earn the E1 Must see explicit evidence of method Note: <i>g</i> omitted, treat as MR
1	(b)	(ii)	EITHER $\tan 40 \times 0.8 \times 9.8 \times \cos 20$ $\times 3 (= 18.545)$ (+)0.8 $\times 9.8$ $\times 3 \sin 20 (= 8.044)$ = 26.5897 so 26.6 J (3 s.f.)	B1 M1 B1 M1 A1	Use of $F_{\text{max}} = \mu R$ with tan 40 and cos 20 Use of WD = <i>Fs</i> NOTE: This mark may be awarded here or for use in PE term Use of <i>mgh</i> Allow sin $\leftrightarrow$ cos interchange Two relevant terms added Cao Allow 26.7 Allow 27 Omission of <i>g</i> can get B0M1B1M1A0

PMT

Question		Answer	Marks	Guidance
		OR		
		$\tan 40 \times 0.8 \times 9.8 \times \cos 20 \ (= 6.182)$	B1	Use of $F_{\text{max}} = \mu R$ with tan 40 and cos 20
		$(+) 0.8 \times 9.8 \times \sin 20 (= 2.68)$	B1	Allow $\sin \leftrightarrow \cos$ interchange
		(= 8.8632444)	M1	Two relevant forces added
		WD is 3 × 8.8632444	M1	Use of WD = $Fs$ (for at least one of forces)
		= 26.5897 so 26.6 J (3 s.f.)	A1	cao Omission of a con got POP1M1M1A0
2	(i)	a.c. moments about B	[5]	
4	(1)	$10T_{C} - 15 \times 2 = 0$	M1	Moments with all forces present, no extra forces.
		so $T_{\rm C} = 3$ . Tension at C is 3 N	A1	
		$\uparrow T_{\rm C} + T_{\rm B} - 15 = 0$	M1	May take moments again
		so $T_{\rm B} = 12$ . Tension at B is 12 N	F1	
			[4]	
2	(ii)	a.c. moments about A		
		$25T\sin 30 - 15 \times 17 = 0$	M1	Attempt at moments with resolution; allow $\cos \leftrightarrow \sin$ error. All forces present, no extra forces
		so $T = 20.4$	A1	cao
		At A Let force $\uparrow$ be <i>Y</i> N		
		$\uparrow Y + T \sin 30 - 15 = 0$ so $Y = 4.8$	B1	FT (can take moments about C)
		$\rightarrow X = T \cos 30 = 17.6669 \text{ N}$	B1	FT Need not be evaluated
		$\sqrt{4.8^2 + (T\cos 30)^2}$	M1	
		= 18.3073755 so 18.3 N (3 s.f.)	A1	cao
2	(iii)	L et force he P	[0]	
2	(111)	a c moments about D		
		$8 \times 15 - 12 \times P = 0$	M1	Moments about D with all forces present, no extra forces
		so $P = 10$ on point of tipping	A1	cao
		Using $F_{\text{max}} = \mu R$ on point of slipping	M1	
		with $R = 15$	B1	
		gives $F_{\text{max}} = 0.65 \times 15 = 9.75$	A1	cao
		so slips first	E1	cao and WWW
		-	[6]	

C	Question		Answer	Marks	Guidance
3	(a)	(i)		B1	Correctly identifying the position of the c.m of triangle EFH (10, –4)
			$300\left(\frac{\overline{x}}{\overline{y}}\right) = 72\left(\frac{-6}{3}\right) + 192\left(\frac{4}{-6}\right) + 36\left(\frac{10}{-4}\right)$	M1	A systematic method for at least 1 cpt
				B1	<i>Either</i> all <i>x</i> or all <i>y</i> values correct <i>or</i> 2 vector terms correct <i>or</i> allow one common error in both components, e.g. one wrong mass, misunderstanding of c.m. of triangle
			$\begin{pmatrix} \overline{x} \\ \overline{y} \end{pmatrix} = \begin{pmatrix} 696 \\ -1080 \end{pmatrix}$		
			so $\bar{x} = 2.32$	A1	
			$\overline{y} = -3.6$	A1	Allow FT for either if only error is common to both
				[5]	
5	(a)	(11)	$\begin{array}{c c} B & 12 & C & 2.32 \\ \hline \alpha & & & \\ 0 & & & \\ 0 & & & \\ 0 & & & \\ G & & \\ \end{array}$		
			centre of mass is at G		
				M1*	Identifying correct angle. May be implied
			top a 96	BI	At least 1 relevant distance found. FT (1) Use of erroton $\frac{96}{2}$ or erroton $\frac{1432}{2}$ of
			$\tan \alpha - \frac{1}{14.32}$	MIdep*	$\frac{1}{9.6}$ 0.e.
			$so \ \alpha = 55.85/0 \ so \ 55.8^{\circ} \ (5 \ S.1.)$	[4]	cao or $180^{\circ} - 53.8^{\circ}$
3	(b)	(i)	Marking given tension and thrust	B1	Each labelled with magnitude and correct direction
			Marking all other forces internal to rods $A = B = a d C (a \pi T = a \pi C)$	B1	Need ALL forces at A, B and C. Need pairs of arrows on AB, AC and BC
			acting on A, D and C (as 1 or C)	[2]	
				L-3	

Question		on	Answer	Marks	Guidance
3	(b)	(ii)	Equilibrium at A ↑		
			$T_{\rm AB} \cos 30 - 18 = 0$	M1	Equilibrium at one pin-joint
			$T_{\rm AB} = 12\sqrt{3}$ . Force in AB: $12\sqrt{3}$ N (T)	A1	20.8 Sign consistent with tension on their diagram
			$A \leftarrow$		
			$T_{\rm AC} + T_{\rm AB} \cos 60 + 5 = 0$	M1	
			$T_{\rm AC} = -(5 + 6\sqrt{3})$ .		-15.39
			Force in AC: $(5 + 6\sqrt{3}) N(C)$	F1	FT their $T_{AB}$
			At B in direction AB		
			$T_{\rm BR}\cos 60 - T_{\rm AB} = 0$		
			so $T_{pp} = 24\sqrt{3}$	M1	Allow FT Other methods are possible, but award this M1 only for a $T_{\rm M}$
			At D in dimetion DC		complete method that would lead to $I_{\rm BC}$
			$T_{ra} - T_{ra} \cos 30 = 0$		
			$T_{BC} = 36$ Force in BC: 36 N (T)	F1	
				A1	cao WWW T/C all correct
				[7]	
4	(i)		$26t = 3 \times 13$	M1	Use of $Ft = m(v - u)$ or N2L to find $a (= 26/3)$ and use $v = u + at$
			t = 1.5 so 1.5 s	A1	cao
4	(ii)		PCI M		
1	(11)		$10 \times 0 + 3 \times 13 = 10v_0 + 3v_P$	M1	Use of PCLM
			$39 = 10v_{\rm Q} + 3v_{\rm P}$	A1	Any form
			NEL		
			$\frac{v_{\rm Q} - v_{\rm P}}{0 - 13} = -e$	M1	Use of NEL. Allow sign errors but not inversion
			$v_{\rm Q} - v_{\rm P} = 13e$	A1	Any form
				M1	Eliminating one of $v_Q$ or $v_P$ OR allow substitution of given result in one equation and check both answers in other equation
			$v_{\rm Q} = 3(1+e)$	B1	cao; aef
			$v_{\rm P}=3-10e$	E1 [ <b>7</b> ]	Properly shown

#### Mark Scheme

PMT

Question		n Answer	Marks	Guidance
4	(iii)	Need $v_{\rm P} < 0$ so $3 - 10e < 0$	M1	Accept ≤
		Hence $\frac{3}{10} < e \le 1$	A1	cao (Allow $e \le 1$ omitted) Correct answer www gets $2/2$
			[2]	
4	(iv)	When $e > \frac{3}{10}$ , its speed is $10e - 3$	M1	
		We require $(10e - 3) > 3(1 + e)$	M1	FT their $v_Q$ SC1 for $(3 - 10e) > \pm 3(1 + e)$
			A1	FT their $v_Q$
		so $7e > 6$ and so $\frac{6}{7} < e \le 1$	A1	cao. Allow $e > \frac{6}{7}$ (0.857) Correct answer www gets 4/4
			[4]	
4	(v)	Either $y_0 = 4.5$ and $y_0 = -2$	M1	Substitute $e = 0.5$ ; FT their $v_Q$
		When they collide the speed of Q is $-4.5$ and of P is 2 PCL M	M1	Change signs of their velocities
		$10 \times -4.5 + 3 \times 2 = 13V$	M1	Use of PCLM Allow sign errors
		so $V = -3$ and velocity is $-3$ m s <sup>-1</sup>	A1	cao; OR 3 m s <sup>-1</sup> to the right
			E 43	or use argument about final LM is -ve of original LM
			[4]	
		Or		
		10(-3(1+e)) + 3(10e-3) = 13V	M1	Use of PCLM; Allow sign errors ; FT their $v_Q$
			M1	Change signs of their velocities
		-39 = 13V	M1	Simplify
		so $V = -3$ and velocity is $-3 \text{ m s}^{-1}$	A1 [4]	cao; OR 3 m s <sup><math>-1</math></sup> to the right
4	(vi)	3(-3-2) = -15 N s	B1	FT 3(their( $v$ ) – 2) Using 10(–3 +4.5) = 15 gets B0 until it leads to correct
			[1]	

January 2013

Question		on	Answer	Marks	Guidance
1	(a)		Take <b>j</b> north and <b>i</b> east velocity: before 5 <b>i</b> - $5\sqrt{3}$ <b>j</b> (after 3 <b>i</b> ) <b>I</b> = m( <b>v</b> - <b>u</b> ) so <b>I</b> = 120 000 000(-2 <b>i</b> + $5\sqrt{3}$ <b>j</b> ) Modulus is 120 000 000 × 8.888194 = 1.0665 × 10 <sup>9</sup> N s so 1.07 × 10 <sup>9</sup> N s (to 3 s. f.)	B1 M1 A1 A1 [4]	Resolving initial velocity (may be implied). Allow $5\mathbf{i} + 5\sqrt{3}\mathbf{j}$ or $5\mathbf{i} - 5\sqrt{3}\mathbf{j}$ oe May be implied Allow if only one direction considered or both combined without vectors. Must include an attempt to resolve 10 Accept mass of 120 000 cao Alternative method using a diagram, cos and sine rules
1	(b)	(i)	PCLM $0.4 \times 6 = 0.5 V$ $V = 4.8 \text{ ms}^{-1}$ direction is opposite to that of P	M1 A1 [ <b>2</b> ]	Implied by 4.8 or -4.8 Allow -4.8 as the speed
1	(b)	(ii)	P travels $6 \times \frac{2}{3} = 4$ m before the collision so Q travels $4 - 2 \times 0.75 = 2.5$ m in $\frac{2}{3}$ s $2.5 = \frac{(4.8 + v_Q)}{2} \times \frac{2}{3}$ Hence $v_Q = 2.7$ ms <sup>-1</sup>	B1 B1 M1 E1 [4]	Or find $t = \frac{13}{24}$ for time from edge to collision AND $d = 3.25$ 3.25 - 0.75 = 2.5 Using appropriate <i>suvat</i> FT their 2.5 Answer given

Question		on	Answer	Marks	Guidance
1	<b>(b)</b>	(iii)	Suppose friction on Q is F		
			$-F \times \frac{2}{3} = 0.5(2.7 - 4.8)$ so $F = 1.575$	B1	Using $Ft = m(v - u)$ or find $a = -3.15$ and use $F = ma$ . FT their 2.7
			$1.575 = \mu \times 0.5 \times 9.8$	M1	$F = \mu R$
				A1	R correct (4.9)
			$\mu = 0.32142$ so 0.321 (3 s. f.)	A1	cao
				[4]	Note: <i>F</i> and <i>R</i> need not be explicit:
					$F=ma$ and $R=mg$ give $\mu = \frac{a}{g}$ (M1A1). Find $a = -3.15$ (B1) gives 0.321
					(A1)
1	<b>(b)</b>	(iv)	Let the speeds after be $V_{\rm P}$ and $V_{\rm O}$ .		
			× ×		
			PCLM		
			$0.4 \times 6 + 0.5 \times 2.7 = 0.4 V_{\rm P} + 0.5 V_{\rm Q}$	M1	PCLM. FT <b>their</b> 2.7 from (ii). Award M1A0 for use of their 4.8 from (i) instead of 2.7
			so $4V_{\rm P} + 5V_{\rm Q} = 37.5$	A1	FT their 2.7 from (ii). Accept any form
			NEL		
			$V_{\rm Q} - V_{\rm P} = 1$	M1	NEL. FT their 2.7 from (ii). Award M1A0 for use of their 4.8 from (i)
			$\frac{1}{2.7-6} = -\frac{1}{8}$		instead of 2.7
			so $V_{\rm Q} - V_{\rm P} = 0.4125$	A1	FT their 2.7 from (ii). Accept any form
			$V_{\rm Q} = 4.35 \text{ so } 4.35 \text{ m s}^{-1}$	A1	cao
				[5]	
(	Juestion	Answer	Marks	Guidance	
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2	(i)	$455 = 0.01 \times 80 \times 9.8 \times \cos 4 \times 12 + WD$	M1	Use of Fx	
			B1	rolling friction force correct (7.82) 12 not needed	
			A1	All correct terms in an equation (allow sign errors)	
		WD = 361.149 so 361 J (3 s. f.)	A1	cao	
			[4]	SC B1B1 for final answer 30.1 seen	
2	(ii)	$0.5  imes 80  imes v^2 - 0.5  imes 80  imes 2^2$	M1	Use of W-E equation. Must include GPE, at least one KE and the WD	
			B1	Either KE term	
		$= 80 \times 9.8 \times 12 \times \sin 4 - 455$	B1	GPE term (656.27)	
			A1	All correct terms in an equation (allow sign errors)	
		v = 3.0052. so 3.01 m s <sup>-1</sup> (3 s. f.)	A1	cao	
			[5]		
2	(iii)	Using N2L with driving force S	M1	N2L with at most one force term missing	
		$S - (15 + 0.01 \times 80 \times 9.8 \times \cos 5)$	B1	Both resistance terms seen (15 and 7.81)	
		$-80 \times 9.8 \times \sin 5$	B1	Condone wrong sign (68.33)	
		$= 80 \times 1.5$	A1	All correct terms present; allow sign errors	
		S = 211.1402	A1	May be implicit	
		405 = Sv	M1	Use of Power = $Sv$ with any $S$ calculated using N2L	
		so $v = 1.918$ so $1.92 \text{ m s}^{-1}$ (3 s. f.)	A1	FT their S	
			[7]	Note: missing out one term in N2L can earn 4/7 (M1B1B0A0A0M1A1)	

Questi	ion	Answer	Marks	Guidance
3	(i)	$15\left(\frac{\overline{x}}{\overline{y}}\right) = 2\left(\frac{-1}{2.5}\right) + 9\left(\frac{1.5}{1.5}\right) + 2\left(\frac{4}{0.5}\right) + 2\left(\frac{4.5}{-1}\right)$	M1	A systematic method for at least 1 cpt
			A1 A1	<i>Either</i> all <i>x</i> or all <i>y</i> values correct <i>or</i> 2 vector terms correct on RHS Completely correct expressions seen for all components
		$= \begin{pmatrix} 28.5\\17.5 \end{pmatrix}$		Need not be explicit
		so $\overline{x} = 1.9$	A1	Accept any form
		$\overline{y} = \frac{7}{6}$	A1	Accept any form (1.17, 1.2) but not 1.16
			[5]	
3	(ii)	Referred to Fig 3.1 with c.m. G, G is 2 + 1.9 to the right of K and $3 - \frac{7}{6} = \frac{11}{6}$	B1	FT from (i) May be implied
		When hanging, G is vertically below K	B1	May be implied
		Angle is $\arctan\left(\frac{\frac{11}{6}}{3.9}\right)$	M1	o.e. FT their values but must be attempting to find the appropriate angle
		$= 25.1775$ so $25.2^{\circ}$ (3 s. f.)	A1 [4]	cao
3	(iii)	New c.m. is at (1.5, 1.5) & mass of object is 0.3 kg	B1	Do not penalise below if mass of lamina is taken to be 15
		For $\overline{x}$ :	M1	Recognising need first to produce an equation in terms of <i>m</i> for the <i>x</i> -component
		$(0.3 + m) \times 1.5 = 0.3 \times 1.9 + m \times 0$	M1	Must be 0 not x
		m = 0.08 For $\overline{n} : (0.3 \pm 0.08) \times 1.5 = 0.3 \times 7.4$	A1	FT their 1.9 from (i). If 15 used, accept $m = 4$
		$\begin{array}{c} 1.01  y \\ 0.08y \end{array}$	M1	
		so particle should be at $(0, 2.75)$	A1	cao. Condone no reference to x component. Allow obtained using 15. Allow 2.74, 2.7375 (from 1.17), 2.775 (from 1.16), 2.625 (from 1.2)
			[6]	

## Mark Scheme

January	201	3
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Question		n	Answer	Marks	Guidance
3		(iv)	The c.m. must lie inside KFDL as seen in	E1	Some indication of this is what is required. Accept a closed region with
			the plan in Fig. 3.2		KF correct and sides parallel to KL and FD.
				E1	Correct. Accept freehand.
			The c.m. shown to be in this region	M1	Recognition that com is at (1.7, their $\overline{y}$ ) and is related to their critical
					region even if region is incorrect
					or calculation with at least 1 correct equation $(3y+2x=9 \text{ and } 3y+4x=6)$
					Do NOT award simply for a recalculation of com as (1.7, 7/6)
				E1	Properly established including a statement. (i.e. correct region, correct
					com marked and statement of stability)
				[4]	

## Mark Scheme

C	Questi	on	Answer	Marks	Guidance
4		(i)	Let vertical force from support be R N and tension in string T N. moments about A $30 \times 0.5 \times 2.4 - R \times (2.4 - 0.6) = 0$ R = 20 so force from block is 20 N $\uparrow R + T - 30 = 0$ T = 10 so tension is 10 N	M1 A1 M1 F1 [4]	Use of moments with all relevant moments attempted (FT from <i>T</i> if <i>T</i> found first) FT from <i>R</i>
4	(ii)	(A)	$ \begin{array}{c} \underset{F}{\overset{B}{\rightarrow}} & \underset{R}{\overset{R}{\rightarrow}} & \underset{P}{\overset{P}{\rightarrow}} & \underset{R}{\overset{1.2 \text{ m}}{\rightarrow}} & \underset{A}{\overset{S}{\rightarrow}} & \underset{A}{\overset{N}{\rightarrow}} \\ \xrightarrow{0.9 \text{ m}} & \xrightarrow{1.2 \text{ m}} & \underset{A}{\overset{S}{\rightarrow}} & \underset{A}{\overset{N}{\rightarrow}} \\ \xrightarrow{0.9 \text{ m}} & \xrightarrow{0.9 \text{ m}} & \underset{A}{\overset{N}{\rightarrow}} & \underset{A}{\overset{N}{\rightarrow}} \\ \xrightarrow{0.9 \text{ m}} & \xrightarrow{0.9 \text{ m}} & \xrightarrow{0.6 \text{ m}} \\ \xrightarrow{0.8 \text{ m}} & \xrightarrow{R} & \underset{R}{\overset{N}{\rightarrow}} & \underset{R}{\overset{N}{\rightarrow}} & \underset{R}{\overset{N}{\rightarrow}} & \underset{R}{\overset{N}{\rightarrow}} \\ \xrightarrow{0.9 \text{ m}} & \xrightarrow{R} & \underset{R}{\overset{N}{\rightarrow}} & \underset{R}{\overset{N}{\rightarrow}} & \underset{R}{\overset{N}{\rightarrow}} & \underset{R}{\overset{N}{\rightarrow}} \\ \xrightarrow{0.9 \text{ m}} & \xrightarrow{0.9 \text{ m}} & \underset{R}{\overset{N}{\rightarrow}} & \underset{R}{\overset{N}{\rightarrow}} & \underset{R}{\overset{N}{\rightarrow}} \\ \xrightarrow{0.9 \text{ m}} & \xrightarrow{0.9 \text{ m}} & \underset{R}{\overset{N}{\rightarrow}} & \underset{R}{\overset{N}{\rightarrow}} & \underset{R}{\overset{N}{\rightarrow}} \\ \xrightarrow{0.9 \text{ m}} & \xrightarrow{0.9 \text{ m}} & \underset{R}{\overset{N}{\rightarrow}} & \underset{R}{\overset{N}{\rightarrow}} & \underset{R}{\overset{N}{\rightarrow}} \\ \xrightarrow{0.9 \text{ m}} & \xrightarrow{0.9 \text{ m}} & \underset{R}{\overset{N}{\rightarrow}} & \underset{R}{\overset{N}{\rightarrow}} & \underset{R}{\overset{N}{\rightarrow}} \\ \xrightarrow{0.9 \text{ m}} & \underset{R}{\overset{N}{\rightarrow}} & \underset{R}{\overset{N}{\rightarrow}} & \underset{R}{\overset{N}{\rightarrow}} & \underset{R}{\overset{N}{\rightarrow}} & \underset{R}{\overset{N}{\rightarrow}} \\ \xrightarrow{0.9 \text{ m}} & \underset{R}{\overset{N}{\rightarrow}} & \underset{R}{\overset{N}{\rightarrow}} & \underset{R}{\overset{N}{\rightarrow}} & \underset{R}{\overset{N}{\rightarrow}} \\ \xrightarrow{0.9 \text{ m}} & \underset{R}{\overset{N}{\rightarrow}} & \underset{R}{\overset{N}{\rightarrow}} & \underset{R}{\overset{N}{\rightarrow}} & \underset{R}{\overset{N}{\rightarrow}} & \underset{R}{\overset{N}{\rightarrow}} \\ \xrightarrow{0.9 \text{ m}} & \underset{R}{\overset{N}{\rightarrow}} & \underset{R}{\overset{N}{\rightarrow} & \underset{R}{\overset{N}{\rightarrow}} & \underset{R}{\overset{N}{\rightarrow}} & \underset{R}{\overset{N}{\rightarrow}} & \underset{R}{\overset{N}{\rightarrow}} & \underset{R}{\overset$	M1 A1 M1 E1 [5] M1 A1 M1 M1 M1	Must be consideration of a force at A <i>F</i> and <i>R</i> must be identified, e.g. on a diagram Complete argument Resolve parallel and perpendicular to rod Both correct <i>F</i> and <i>R</i> must be identified, e.g. on a diagram Divide factored expressions with <i>S</i> included
			$\tan\theta = 0.6$	E1 [ <b>5</b> ]	

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(	)uesti	on	Answer	Marks	Guidance
4	(ii)	(B)	$\begin{array}{c} B \\ \hline 0.9 \text{ m} \\ \hline 0.3 \text{ m} \\ \hline \theta \\ \hline 0.3 \text{ m} \\ \hline \theta \\ \hline 0.3 \text{ N} \\ \hline \end{array} \begin{array}{c} 1.2 \text{ m} \\ F' \text{ N} \\ \hline F' \text{ N} \\ \hline 30 \text{ N} \end{array}$		
			AP is 1.5 gives $\sin \theta = 0.6$ or $\cos \theta = 0.8$ c. w. moments about A	B1	oe. or $\theta = 36.9^{\circ}$
			$1.5R' - 30 \times 1.2 \times \cos \theta = 0$ R' = 19.2 so $19.2$ N	M1 A1	Moments and all terms present. Accept $\cos\theta$ or 0.8 cao
			$\Upsilon  S' + R' \cos \theta - 30 = 0$	M1	An equilibrium equation with all relevant forces, resolved appropriately, e.g. $R' + S' \cos \theta = 30 \cos \theta + F' \sin \theta$ . Allow $\sin \leftrightarrow \cos$
			( <i>S</i> '=14.64)	A1	Correct equation involving only S'. Numerical answer not required
			$ \rightarrow R' \sin \theta - F' = 0  (F' = 11.52)  \mu = \frac{11.52}{14.64}  = 0.78688 so 0.787 (3 s. f.) $	M1 A1 M1 A1	Second equilibrium equation with all relevant forces, resolved appropriately. e.g. $F' \cos \theta + S' \sin \theta = 30 \sin \theta$ . Allow $\sin \leftrightarrow \cos$ Correct equation involving only $F'$ . Numerical answer not required Use of $F' = \mu S'$ for a calculated $F'$ and $S'$ cao
				[א]	

June 2013

	Questi	on	Answer	Marks	Guidance
1	(a)	(i)	$3 \times 4 + 21 \times 2 = 4U$	M1	Use of PCLM and $I = Ft$
			4U = 54 so $U = 13.5$ and speed is 13.5 m s <sup>-1</sup>	A1	
				[2]	
			<b>OR</b> $21 = 4a : a = 5.25$ and $v = 3 + 2 \times 5.25$	M1	Use of $F = ma$ and suvat
			speed is 13.5 m s <sup><math>-1</math></sup>	A1	
				[2]	
1	(a)	( <b>ii</b> )	Let $V$ be the speed of S in direction PQ		
			$54 - 2 \times 3 = (4 + 2)V$	M1	PCLM for coalescence
			6V = 48 so $V = 8$ and velocity is 8 m s <sup>-1</sup> in direction PQ	E1	Answer given. Accept no reference to direction.
				[2]	
1	(a)	(iii)	Let velocities of R be <i>u</i> before and <i>v</i> after, both in the		
			direction SR		
			$6 \times 8 + 4u = 6 \times 5 + 4v$	M1	Use of PCLM. Allow any sign convention. All masses and speeds must
			y = 4.5	Δ 1	Any form
			v - u - 4.5	AI	Any Ionn.
			$\frac{v-3}{2} = -\frac{1}{4}$	M1	Use of NEL correct way up; allow sign errors
			u-8 4	. 1	
			4v + u = 28	AI	Any form signs consistent with PCLM eqn
			Solving $2 \approx 2 \approx e^{-1}$ in the direction SP	A 1	and NOTE that a sign amon in NEL loads to such this sate A0
			u = 2  so  2  in s in the direction SR $u = 65 \text{ so } 65 \text{ m s}^{-1}$ in the direction SP		cao NOTE that a sign error in NEL leads to $u = -2$ ; this gets AU
			v = 6.5  so  6.5  m s in the direction SR	Al	cao. withhold only 1 of the final A marks if the directions not clear.
				[6]	Directions can be interred from a CLEAR diagram

June 2013

## Mark Scheme

(	Questi	on	Answer	Marks	Guidance
1	(b)	(i)	Find v, the speed at which particle hits the plane $\frac{1}{2} \times 0.2 \times v^2 - \frac{1}{2} \times 0.2 \times 5^2 = 0.2 \times 10 \times 10$ so $v^2 = 225$ and $v = 15$	M1 A1	Use of WE or <i>suvat</i> must use distance of 10 allow $g = 9.8$ Answer not required ( $v = 14.9$ if $g = 9.8$ )
			$\cos\alpha = \frac{4}{5}, \sin\alpha = \frac{3}{5}$	B1	Use of either expression or use of 36.9°
			Let velocity after be at $\beta$ to the plane Parallel to the plane		
			$15\cos\alpha = 13\cos\beta$	M1	Attempt to conserve velocity component parallel to plane. Allow use of 5 instead of 15
			So $\cos\beta = \frac{12}{13}$ and $\beta = 22.61$ so $22.6^{\circ}$ (3 s. f.)	A1	$(\beta = 23.8^{\circ} \text{ if } g = 9.8)$
			Perpendicular to the plane: $13\sin\beta = e \times 15\sin\alpha$	M1	Attempt to use NEL perpendicular to plane: Allow use of 5 instead of 15 or use tan $\beta$ = etan $\alpha$
			$\sin\beta = \frac{5}{13}$	A1	o.e. find $\tan \beta = \frac{5}{12}$
			so $13 \times \frac{5}{13} = 15 \times \frac{3}{5} \times e$ and $e = \frac{5}{9}$	A1	cao Accept 0.56 ( $e = 0.589$ if $g = 9.8$ )
				[8]	
			OR: First three marks as above	M1A1B1	
			Parallel to plane, $u_x = 15 \cos \alpha (= 12)$ and $v_x = u_y (= 12)$	M1	Attempt to conserve velocity component parallel to plane. Allow use of 5 instead of 15
			$\cos \beta = \frac{v_x}{v} = \frac{12}{12} \qquad \beta = 22.6^{\circ}$	A1	
			Perpendicular to plane, $u_y = 15 \sin \alpha (=9)$ and $v_z = e u_z (=9e)$	M1	Attempt to use NEL perpendicular to plane. Allow use of 5 instead of 15
			$v_y = cu_y (-50)^2$ $v_x^2 + v_y^2 = 13^2$	A1	Use Pythagoras' theorem for velocities after collision in attempt to find $e$
			$12^{2} + (9e)^{2} = 13^{2}$ so $e^{2} = \frac{25}{81}$ $e = \frac{5}{9}$	A1	
				[8]	

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•	Questi	on	Answer	Marks	Guidance
1	<b>(b</b> )	( <b>ii</b> )	Impulse is perp to plane with mod		
			$0.2(13\sin\beta - (-15\sin\alpha)) = 0.2(5 - (-9))$	M1	For use of $I = m (v - u)$ perp to the plane 0.2(5-9) gets M1A0
			= 2.8  N s	A1	cao
				[2]	
2	(i)		WD is $800 \times 9.8 \times 6 + 400 \times 6$ J	M1	WD as <i>Fd</i> Used in TWO terms
			40,440	<b>F</b> 1	
			= 49440	EI M1	
			Power 1s $49440 \div 12$		Power is $wD / \Delta t$
			=4120 W		cao
2	(ii)		Power is $(800 \times 9.8 \pm 400) \times 0.55$	<u>[</u> 4] M1	Power as <i>Ev</i> in one term
-	(11)			A1	All correct
			= 4532 W	A1	
				[3]	
2	(iii)		Let speed be <i>v</i>		
			$\frac{1}{2} \times 800v^2 = 800 \times 9.8 \times 3 - 400 \times 3$	M1	Use of W-E equation Must include KE and at least one WD term
			2	A1	Allow only sign errors
				A1	All correct
			$v^2 = 55.8$ so $v = 7.4699$		
					SC: Use of N2L and <i>suvat</i> :
			and speed is $7.47 \text{ m s}^{-1}$ (3 s.f.)	A1	M1 Complete method
					A1 7.47 cao
				[4]	
2	(iv)		$\frac{1}{2} \times 800 \times \frac{v^2}{4} - \frac{1}{2} \times 800 \times v^2$	M1	Use of W-E equation Must include 2 KE terms and a WD term
				B1	Final KE term correct. FT their v.
			$=(800 \times 9.8 - 400) \times 0.8$	B1	One correct WD term
			– WD	A1	All terms present. Allow sign errors and FT their v.
					cao
			WD is 22 692 so 22 700 $I(3 \text{ s} \text{ f})$	Δ1	SC Use of N2L and <i>suvat</i> :
				411	Award maximum of B1 for 'Average force (28365) x 0.8'
				۲ <i>۳</i> ٦	
				[5]	

Mark Scheme

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Question		ion	Answer	Marks	Guidance
3	(i)		c.w. moments about A		Condone using cm not m in moments in any part if consistent
			$60\cos 40 \times 0.3 - 60\sin 40 \times 0.1$	M1	oe e.g. $60(0.3 - 0.1 \tan 40)\sin 50$ or $60 \times \frac{1}{\sqrt{10}}\cos(90^\circ - \arctan 3 + 40^\circ)$ Method of dealing with moment of weight. Allow $\cos \leftrightarrow \sin$ Both weight terms correct. Allow wrong overall sign but not both terms
			= 9.93207 so 9.93 N m (3 s. f.)	E1 [3]	with the same sign
3	(ii)		$P\cos 40 \times 0.2 - 9.93207 = 0$	M1	Moments of all relevant forces attempted. No extra terms. Allow $\cos \leftrightarrow \sin (14.012)$
			P = 64.827 so $64.8(3  s. t.)$	A1 [2]	cao (64.813 11 9.93 used)
3	(iii)		a.c. moments about A to find NR, <i>R</i> , at B $R \times 0.8 = 9.93$ or $R \times 0.8 + 60 \sin 40 \times 0.1 - 60 \cos 40 \times 0.3 = 0$ R = 12.4150 Resolve vertically $Y - 60 + R \cos 40 = 0$	M1 A1 depM1	Attempt to use moments to find <i>R</i> . Moments of all relevant forces attempted. No extra terms. Allow $\cos \leftrightarrow \sin$ Note that mmts about B can score M1 only if mmt of horiz compt of force at A is included. If <i>R</i> is taken as vertical, M0 FT their moment of weight from (i) Not a required answer Note that the second M mark awarded in this part must be for a complete method to find <i>Y</i> :
			so <i>Y</i> = 50.489 so 50.5 N (3 s. f.)	A1 [ <b>4</b> ]	FT their calculated R

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Question		n Answer	Marks	Guidance
3	(iv)	resolve perp to plane		
		$R - 60\cos 40 - 200\sin 40 = 0$	M1	All terms present and no extra terms. Components of 60 and 200; allow $\cos \leftrightarrow \sin$
			A1	
		R = 174.52		Not a required answer
		N2L up the plane		
		$200\cos 40 - F - 60\sin 40 = \frac{60}{9.8} \times 1.75$	M1	Use of N2L with all terms present and no extras. Components of 60 and 200; allow $\cos \leftrightarrow \sin$ Allow use of 60 for mass
			B1	Use of mass not weight
			A1	FT use of weight and/or sign errors
		F = 103.927	A1	All correct. Not a required answer
		As friction limiting $F = \mu R$ so		
		$\mu = \frac{103.927}{174.520}$	M1	FT their $F$ and their $R$
		= 0.59550 so 0.596 (3 s. f.)	A1	cao
			[8]	

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Question		Answer	Marks	Guidance
(a)	(i)	Write $d = 0.8$		
		$(2.5+1.2+1.3+2.4) \times d \times \left(\frac{\overline{x}}{\overline{y}}\right)$	M1	Method for c.m (length is 7.4 m, mass is 5.92 kg)
		$= 2.5d \begin{pmatrix} 1.2 \\ -0.35 \end{pmatrix} + 1.2d \begin{pmatrix} 2.4 \\ -0.1 \end{pmatrix} + 1.3d \begin{pmatrix} 1.8 \\ 0.25 \end{pmatrix} + 2.4d \begin{pmatrix} 1.2 \\ 0 \end{pmatrix}$	B1	One rod mass and cpts correct or if done by separate $x$ and $y$ equations 2 rod components and masses correct. (Allow length used instead of mass)
		OR: $(2+0.96+1.04+1.92) \times \left(\frac{\overline{x}}{\overline{y}}\right)$		
		$= 2 \binom{1.2}{-0.35} + 0.96 \binom{2.4}{-0.1} + 1.04 \binom{1.8}{0.25} + 1.92 \binom{1.2}{0}$		
			B1	Another rod dealt with correctly or if done by separate <i>x</i> and <i>y</i> equations, the other equation attempted with 2 rod components and masses correct. (Allow length used instead of mass)
		$7.4\left(\frac{\overline{x}}{\overline{y}}\right) = \left(\begin{array}{c} 3+2.88+2.34+2.88\\-0.875-0.12+0.325+0\end{array}\right) = \left(\begin{array}{c} 11.1\\-0.67\end{array}\right)$		
		OR:		
		$5.92 \left(\frac{\overline{x}}{\overline{y}}\right) = \left(\begin{array}{c} 2.4 + 2.304 + 2.304 + 1.872\\ -0.7 - 0.096 + 0.26 + 0 \end{array}\right) = \left(\begin{array}{c} 8.88\\ -0.536 \end{array}\right)$		
		$\overline{x} = 1.5$	E1	Clearly shown, with at least one intermediate step
		$\overline{y} = -0.090540 = -0.0905$ (3 s. f.)	A1	Condone - 0.09
			[5]	

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Question		ion	Answer	Marks	Guidance
4	(a)	(ii)	EITHER: New c.m. has $\overline{x} = 1.2$	M1	Identifying and using a suitable condition.
			$(5.92 + m) \times 1.2 = 5.92 \times 1.5 + m \times 0$	M1	Complete method
			m = 1.48	A1	cao
				[3]	
			OR: Moment about any point is zero	M1	Identifying a suitable condition.
			e.g. about S: $1.2mg = 0.3 \times 5.92g$	M1	Allow g omitted. Correct number of terms must be included
			m = 1.48	A1	cao
				[3]	
4	<b>(b)</b>	(i)	Consider the equilibrium at R		
			Resolving horizontally gives $T_{QR} = 0$	E1	
			Then resolving vertically gives $T_{OR} = 0$	E1	
		(••)		[2]	
4	(D)	(11)	c.w. moments about $O$	M1	Max also he around he first considering internal foress
			$120 \times 1 + 60 \times 2 = 31$		May also be argued by first considering internal forces
			So $T = 60$ Resolve to give $Y = 80$ and $Y = 180$		FT Y - T Only $V = 180$ scores 0
			Resolve to give $X = 60$ and $T = 100$	[3]	11X - 1. Only $1 - 100$ scores 0
4	(b)	(iii)			All correct Accept T, X and Y labelled but not substituted Accept
	(~)	()		B1	mixes of T and C. Require pairs of arrows with label on OO. OP
					and PQ.
				[1]	
					Forces internal to the rods have been taken to be tensions.
4	<b>(b)</b>	(iv)	Take angle OPQ as $\alpha$		
			At P $\downarrow 60 + T_{\text{OP}} \sin \alpha = 0$	M1	Equilibrium at ANY pin-joint (not R)
				A1	Correct equation(s) that leads directly to finding $T_{OP}$ or $T_{OP}$
			3		
			$\sin \alpha = \frac{1}{\sqrt{13}}$ : $\alpha = 56.3^{\circ}$		
			= 60 $= 50$ $= 50$ $= 50$		
			$T_{\rm OP} = -\frac{1}{\sin \alpha} = -20\sqrt{13} \text{ so } 20\sqrt{13} \text{ N}$ (C)	A1	o.e. Accept 72.1 N
			At P $\leftarrow$ $T_{\rm QP} + T_{\rm OP} \cos \alpha = 0$	M1	A second equilibrium equation leading to a second internal force
			so $T_{\rm OP} = 40$ so $40$ N (T)	A1	cao T/C correct for both rods
				[5]	