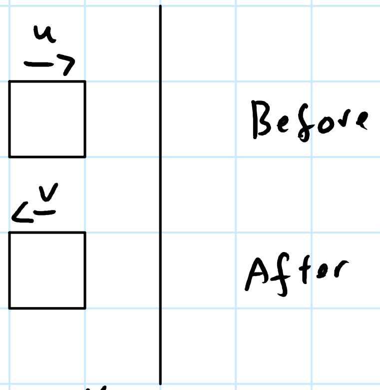
# Mock MA - M2

1)



$$e = \frac{0 - -v}{u - 0}$$

$$= \frac{v}{u}$$

$$= \frac{1}{2}$$

Before KE $= \frac{1}{2}mu^2$

after KE $= \frac{1}{2}mv^2$

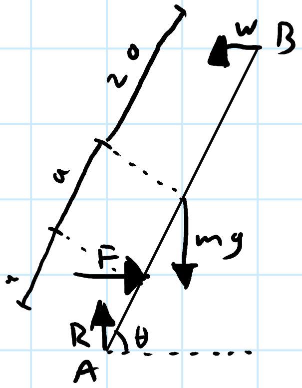
Fraction of KE lost $= \dfrac{\frac{1}{2}mu^2 - \frac{1}{2}mv^2}{\frac{1}{2}mu^2}$

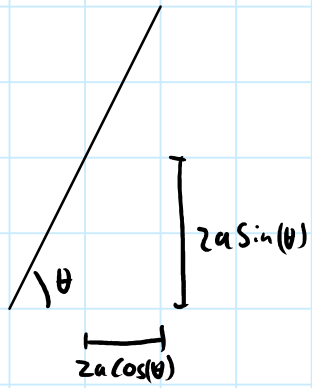
$$= 1 - \frac{v^2}{u^2}$$

$$= 1 - \left(\frac{1}{2}\right)^2$$

$$= \frac{3}{4}$$

2)




$R = mg$ , $F = W$

moments ↻ = moments ↺

moments around A

$$4a\sin(\theta)W = a\sin(\theta)F + 2a\cos(\theta)mg$$

$W = F$ ∴

$$3a\sin(\theta)F = 2a\cos(\theta)mg$$

$$\frac{3a\sin\theta}{2a\cos(\theta)}F = mg$$

$$\frac{3}{2}\tan(\theta)F = mg$$

$$F = \frac{1}{3}mg \quad \because \tan(\theta) = 2$$

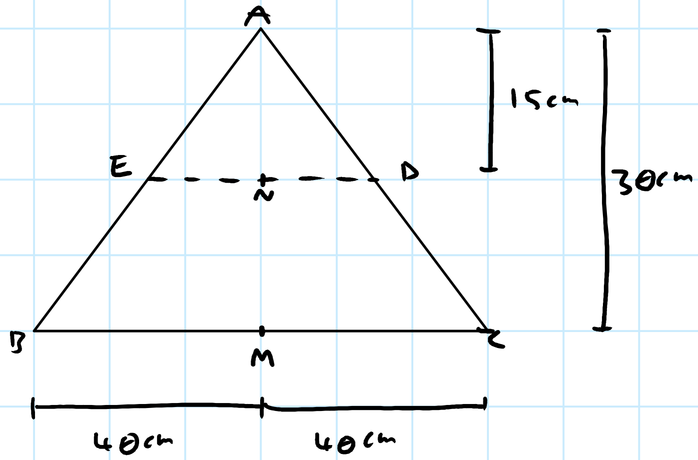
3)

a)

Lamina symmetrical around AM therefore COM lies on AM

COM of triangle ABC lies 2/3 along a median line from a vertex. Since AM is a line of symmetry and A is a vertex, COM of ABC lies 2/3 away from A along AM

Let N be the mid-point of DE so that ANM is a straight line.
COM of triangle ADE lies 2/3 along a median line from a vertex. Since AN is a line of symmetry and A is a vertex, COM of ADE lies 2/3 away from A along AN




$\frac{2}{3} 15 = 10cm$ COM of ADE from A

$\frac{2}{3} 30 = 20cm$ COM of ABC from A

Therefor from BC:
COM of ADE is 30 - 10 = 20cm

COM of ABC is 30 - 20 = 10cm

Since ABC is isosceles. Angles ABC, ACB, AED, ADE are all equal
Therefore ABC and ADE are similar
Since height of ADE is half that of ADE, then DE must also be half that of BC
DE = 40cm

3)

a) mass of : ABC: $\frac{1}{2}(40+40)\times 30$

$= 1200\text{cm}^2$

(negative as it is being taken away from mass of ABC) ADE : $-\frac{1}{2}(40)\times 15$

$= -300\text{cm}^2$

$\therefore$ mass of BCDE: $1200 - 300$

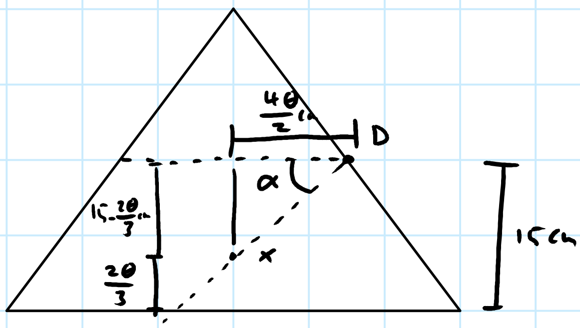
$= 900\text{cm}^2$

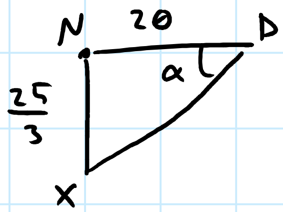
$$900\bar{y} = 1200\times 10 + -300\times 20$$

$$\bar{y} = \frac{12000-6000}{900}$$

$$\bar{y} = 6\frac{2}{3}$$

b)



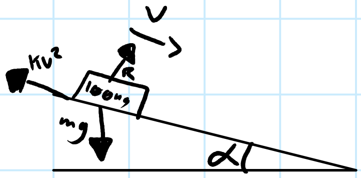

$$\tan(\alpha) = \frac{\frac{25}{3}}{20}$$

$$= \frac{5}{12}$$

$$\alpha = 22.6^\circ \text{ (1dp)}$$

4)

a)




Power from loss of GPE:

$$P = mgV\sin(\alpha)$$

Power used by resistive force

$$P = kv^2 v$$

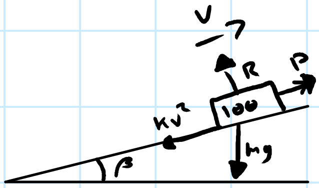
Since no gain in KE, all power from loss of GPE must be being used by resistive force

$$mgV\sin(\alpha) = k\,v^3$$

$$\frac{980}{20} = k\left(\frac{7}{2}\right)^2$$

$$k = 4$$

b)



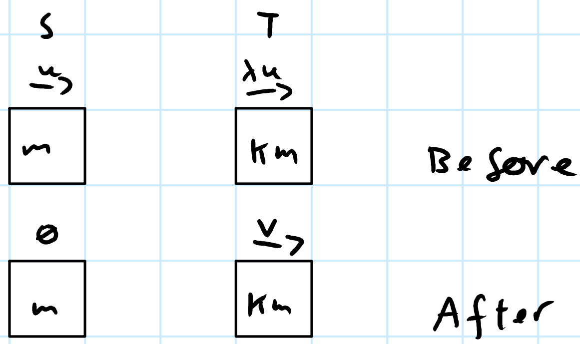

Power from cyclist - Power used by resistive force - power gained in GPE = 0
Because there is no gain in KE and no other forces act on the cyclist

$$P - mgV\sin(\beta) - kv^2 v = 0$$

$$P = \frac{1960}{40} + 4(2)^3$$

$$P = 81\,W$$

a)




$$e = \frac{\text{speed of separation}}{\text{speed of approach}}$$

$$e = \frac{v - 0}{u - \lambda u}$$

momentum is conserved

$$\therefore mu + km\lambda u = 0m + vkm$$

$$mu(1 + k\lambda) = kmv$$

$$\frac{\cancel{m}u}{k\cancel{m}}(1 + k\lambda) = v$$

$$e = \frac{\frac{\cancel{u}}{k}(1 + k\lambda)}{\cancel{u}(1 - \lambda)}$$

$$e = \frac{1 + k\lambda}{k(1 - \lambda)}$$

5)

b) $e \leq 1 \quad \& \quad 0 < \lambda < \frac{1}{2}$

$$\therefore \quad \frac{1 + k\lambda}{k(1-\lambda)} \leq 1$$

$$1 + k\lambda \leq k - k\lambda$$

K is positive because km & m must be positive
Mass cannot be a negative value

$$1 \leq k - 2k\lambda$$

$$\frac{1}{(1-2\lambda)} \leq k$$

Because 0 < λ < 0.5 , 1-2λ > 0 so no sign change

Check the extremes of λ:

$\lambda = 0 \quad \frac{1}{1-0} = 1 \quad \cup \quad 1 < k$

$\lambda = 0.5 \quad \frac{1}{1-2\left(\frac{1}{2}\right)} = \frac{1}{0} \quad$ undefined

Any value of λ above 0 will create an inequality that includes 1 < k
Therefore, for all values of λ, 1 < k is the valid inequality.

Note: 1 < k uses a hard inequality (not 1 ≤ k) because λ ≠ 0, therefore k ≠ 1

6)

a)

$$V = \int 2i + 6tj \; dt$$

$$= 2ti + 6\frac{t^2}{2}j + C$$

at $t=0$, $V = 2i - 4j$ $\therefore$ $C = 2i - 4j$

$$V_t = (2t+2)i + (3t^2 - 4)j$$

b)

$$V_2 = (2(2)+2)i + (3(2)^2 - 4)j$$

$$= 6i + 8j$$

$$P_2 = (6i + 8j)ms^{-1} \times 0.5kg$$

$$= (3i + 4j)\, kgms^{-1}$$

New momentum P is

$$(3i+4j)kgms^{-1} + (3i - 1.5j)Ns$$

$$P = 6i + 2.5j \; kgms^{-1}$$

$$\therefore V = 12i + 5j \; ms^{-1} \qquad \because P = mV$$

$$|V| = \sqrt{12^2 + 5^2}$$

$$= 13ms^{-1}$$

7) a) ($\downarrow$)

S 52.5

U $-28\sin(30°)$

V x

A $9.8\,ms^{-2}$

T —

$S = ut + \frac{1}{2}at^2$ $\downarrow \overset{\rightarrow}{+ve}$

$52.5 = -14t + 4.9t^2$

$4.9t^2 - 14t - 52.5 = 0$

$$\frac{14 \pm \sqrt{14^2 - -1029}}{9.8} = \frac{14 \pm 35}{9.8}$$

$t = 5, -\frac{15}{7}$

$t > 0 \therefore t = 5$

($\rightarrow$)

S —

U $28\cos(30°)$

V x

A $0\,ms^{-2}$

T 5s

$s = ut + \frac{1}{2}at^2$

$S = 14\sqrt{3} \times 5 + \frac{0}{2}5^2$

$= 70\sqrt{3}$

b) initial

$GPE = mg(52.5)$ $KE = \frac{1}{2}m(28)^2$

final KE: $\cancel{m}g(52.5) + \frac{1}{2}\cancel{m}(28)^2 = \frac{1}{2}\cancel{m}v^2$

$514.5 + 392 = \frac{1}{2}v^2$

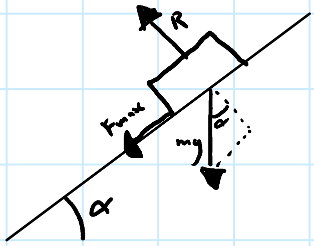
$v^2 = 906.5 \times 2$

$v^2 = 1813 (ms^{-1})^2$

$v = 7\sqrt{37}$

$v \approx 42.6\, ms^{-1}$ (3 sf)

8)




a)

$R = mg\cos(\alpha)$

$F_{max} = \mu R$

$= \frac{1}{2}\frac{4}{5}mg$

$= \frac{2}{5}mg$

Initial KE: $\frac{1}{2}m\,10^2$

$= 50m$ J

Final GPE: $mg\,OA\,\sin(\alpha)$

$= \frac{3}{5}g(OA)m$ J

workdone by $F_{max} = \frac{2}{5}g(OA)m$ J

$50\cancel{m} = \frac{3}{5}g(OA)\cancel{m} + \frac{2}{5}g(OA)\cancel{m}$

$50 = g(OA)$

$OA = 5.10$ m (3sf)

8)

b) $F_{max} = \frac{2}{5}mg$

Weight acting against $F_{max} = mg\sin(\alpha)$

$= \frac{3}{5}mg$

$\frac{3}{5}mg > \frac{2}{5}mg \quad \therefore$ slides down

c) Final KE = Initial KE - work done by friction

$\frac{1}{2}\cancel{m}v^2 = \frac{1}{2}\cancel{m}(10)^2 - 2(0A)\frac{2}{5}\cancel{m}g$

$v^2 = 100 - \frac{8}{5} \times 5.1 \times g$

$v^2 = 100 - 80$

$v = \sqrt{20}$

$v = 2\sqrt{5}$

$v = 4.47\ ms^{-1}$ (3sf)