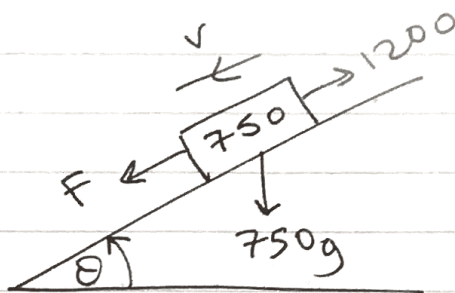


M2 June 2018 (MA)

Q1a)



$$P = Fv$$

$$9000 = Fv$$

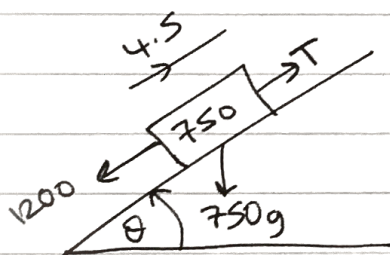
N2L (truck): $F + 750g \sin \theta - 1200 = 750(a)$

$$F = 1200 - 750g \left(\frac{3}{49} \right)$$

$$F = 750 \text{ N} //$$

$$\therefore v = \frac{9000}{750} = \boxed{12 \text{ ms}^{-1}}$$

b)



$$P = Tv$$

$$9000 = T(4.5)$$

$$T = \frac{9000}{4.5} = 2000 \text{ N} //$$

N2L (truck): $T - 1200 - 750g \sin \theta = 750(a)$

$$\therefore a = \frac{2000 - 1200 - 750g \left(\frac{3}{49} \right)}{750}$$

$$= \boxed{0.47 \text{ ms}^{-2}}$$

(Q2a) $u = 4\hat{i}$

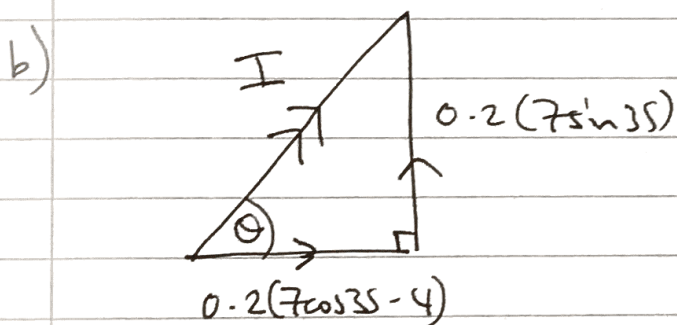
$$v = (7\cos 35)\hat{i} + (7\sin 35)\hat{j}$$

$$I = m(v \cdot u) = 0.2((7\cos 35)\hat{i} + 7\sin 35\hat{j}) - 4\hat{i})$$

$$I = 0.2((7\cos 35 - 4)\hat{i} + (7\sin 35)\hat{j})$$

$$\text{so } |I| = 0.2\sqrt{(7\cos 35 - 4)^2 + (7\sin 35)^2}$$

$$= 0.8747 \dots = \boxed{0.875 \text{ N s}}$$



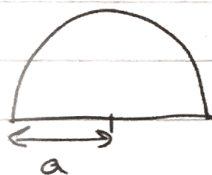
angle required = θ .

$$\tan \theta = \frac{0.2 \times 7\sin 35}{0.2 \times (7\cos 35 - 4)} = \frac{7\sin 35}{7\cos 35 - 4}$$

$$\therefore \theta = \tan^{-1} \left(\frac{7\sin 35}{7\cos 35 - 4} \right) = \boxed{66.6^\circ}$$

Q3a) Shape Mass (area) Distance of c.o.m from BD

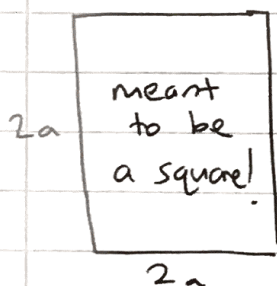
(+)



$$\boxed{\frac{\pi a^2}{2}}$$

$$\boxed{-\frac{4a}{3\pi}}$$

(+)



$$\boxed{4a^2}$$

$$\boxed{a}$$

(=)



$$\boxed{a^2 \left(4 + \frac{\pi}{2} \right)}$$

$$\boxed{\bar{y}}$$

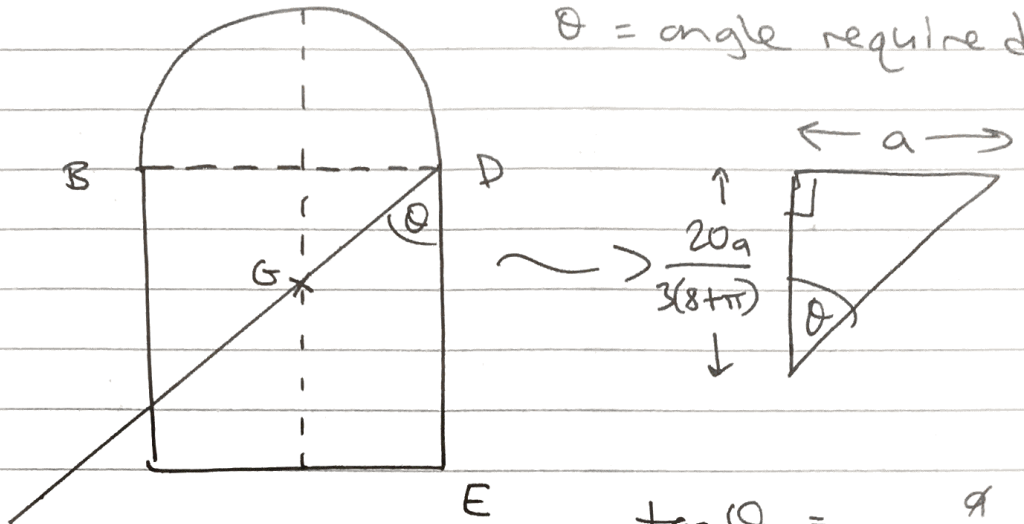
taking moments about BD...

$$\frac{\pi a^2}{2} \left[-\frac{4a}{3\pi} \right] + 4a^2 [a] = a^2 \left(4 + \frac{\pi}{2} \right) \bar{y}$$

$$\frac{-\frac{2a}{3} + 4a}{4 + \frac{\pi}{2}} = \bar{y} = \frac{6 \left(-\frac{2a}{3} + 4a \right)}{6 \left(4 + \frac{\pi}{2} \right)}$$

$$= \frac{20a}{3(8 + \pi)} \quad \neq$$

b)



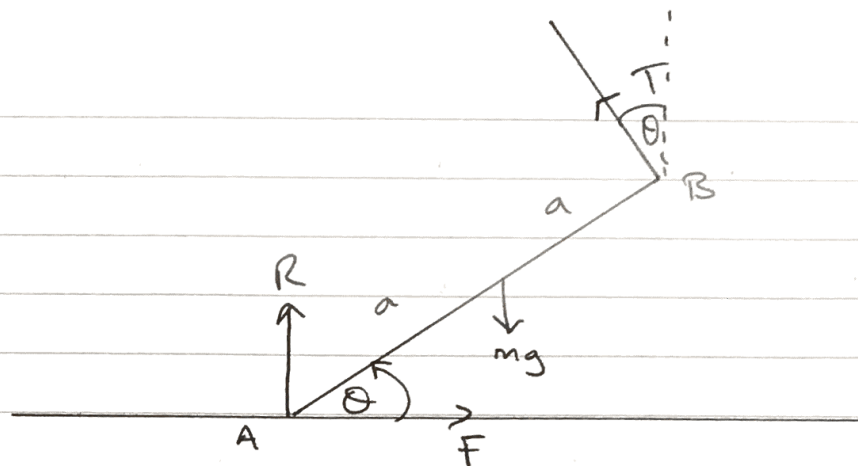
$$\tan \theta = \frac{a}{\frac{20a}{3(8+\pi)}}$$

$$\tan \theta = \frac{3(8+\pi)}{20}$$

$$\therefore \theta = \tan^{-1} \left(\frac{3(8+\pi)}{20} \right)$$

$$= \boxed{59^\circ} \text{ to nearest degree}$$

Q4)



$$R(\uparrow): T \cos \theta + R = mg \quad \sim (1)$$

$$R(\leftrightarrow): F = T \sin \theta \quad \sim (2)$$

$$M(A): mg \cos \theta (a) = T(2a)$$

$$\therefore T = \frac{1}{2} mg \cos \theta //$$

$$(2): F = \frac{1}{2} mg \cos \theta \sin \theta //$$

$$(1): R = mg - \frac{1}{2} mg \cos^2 \theta = mg(1 - \frac{1}{2} \cos^2 \theta) //$$

$$\text{limiting equilibrium} \therefore F = \mu R$$

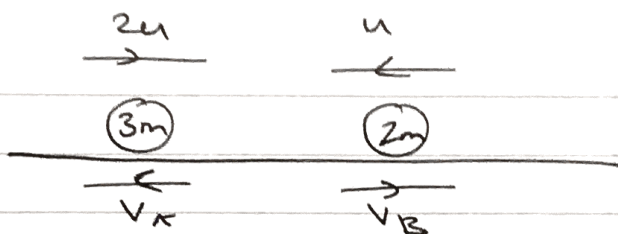
$$\Rightarrow \frac{1}{2} mg \sin \theta \cos \theta = \mu \cdot mg (1 - \frac{1}{2} \cos^2 \theta)$$

$$\Rightarrow \mu = \frac{\frac{1}{2} mg \sin \theta \cos \theta}{mg(1 - \frac{1}{2} \cos^2 \theta)} = \frac{2(\frac{1}{2} \sin \theta \cos \theta)}{2(1 - \frac{1}{2} \cos^2 \theta)}$$

$$\therefore \boxed{\mu = \frac{\sin \theta \cos \theta}{2 - \cos^2 \theta}} \quad \text{QED}$$

$$e = \frac{1}{3}$$

Q5 a i)



C.L.M : $3m(2u) - 2m(u) = -3m(V_A) + 2m(V_B)$

$$6mu - 2mu = -3mV_A + 2mV_B$$

$$4u = -3V_A + 2V_B \quad \text{--- (1)}$$

N.I.L : $\frac{1}{3} = \frac{V_A + V_B}{3u}$

$$\therefore u = V_A + V_B$$

$$\begin{aligned} \times 3 : 3u &= 3V_A + 3V_B \\ + (2) : + [4u &= -3V_A + 2V_B] \end{aligned}$$

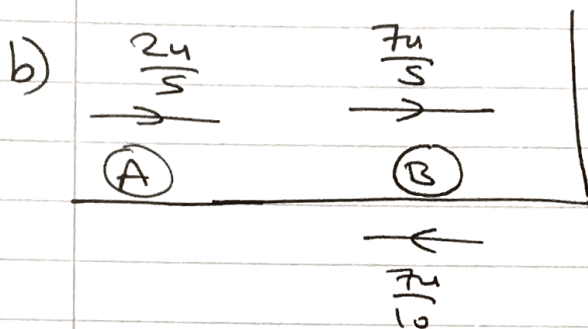
$$7u = 0 + 5V_B$$

$$\therefore V_B = \boxed{\frac{7u}{5}} = \text{speed of B}$$

ii) $V_A = u - V_B = u - \frac{7u}{5} = -\frac{2u}{5} //$

so speed of A = $\left| -\frac{2u}{5} \right|$

$$= \boxed{\frac{2u}{5}}$$



until B hits the wall

For B

$$\begin{cases} S = ut_1 \\ x = \frac{7u}{5} t_1 \\ t_1 = \frac{Sx}{7u} \end{cases}$$

t_1 = time from first collision to wall

from when B hits the wall to 2nd collision

For B

$$\begin{cases} S = ut_2 \\ y = \frac{7u}{10} t_2 \\ t_2 = \frac{10y}{7u} \end{cases}$$

t_2 = time from when B hits the wall to 2nd collision

from first collision to second collision

For A

$$\begin{cases} S = ut \\ x - y = \frac{2u}{5} t \\ t = \frac{S(x - y)}{2u} \end{cases}$$

t = total time

$$\text{but } t = t_1 + t_2 \quad | \quad 0$$

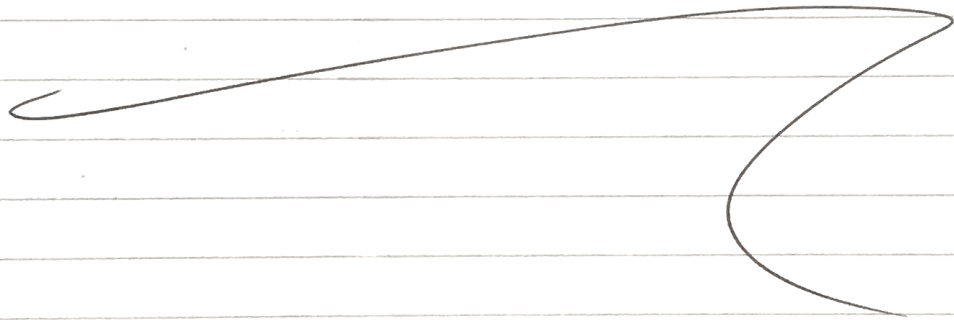
$$\frac{5x}{7u} + \frac{10y}{7u} = \frac{5(x-y)}{2u}$$

$$\times 14u : 10x + 20y = 35(x-y)$$

$$10x + 20y = 35x - 35y$$

$$55y = 25x$$

$$\therefore y = \frac{5}{11}x$$



(Q6ai) $v = (4t - 3t^2)\underline{i} + (t^2 - 8t - 40)\underline{j}$

$$a = \frac{dv}{dt} = (4 - 6t)\underline{i} + (2t - 8)\underline{j}$$

$t=3$: $a = (4 - 18)\underline{i} + (6 - 8)\underline{j}$

$$a = -14\underline{i} - 2\underline{j}$$

$$F = ma = -7\underline{i} - \underline{j} \quad (= 0.5(-14\underline{i} - 2\underline{j}))$$

$$\therefore |F| = \sqrt{(-7)^2 + (-1)^2} = \boxed{5\sqrt{2}} \text{ N}$$

ii) parallel to $-\underline{i} - \underline{j}$ means $\underline{i}$ and $\underline{j}$ components of v will be equal!

$$\therefore 4t - 3t^2 = t^2 - 8t - 40$$

$$4t^2 - 12t - 40 = 0$$

$$t^2 - 3t - 10 = 0$$

$$(t - 5)(t + 2) = 0$$

$$t = 5, \quad t = -2 \quad \begin{array}{l} \nearrow \text{not valid, } t \geq 0 \\ \parallel \end{array}$$

at $t=5$, $a = (4 - 30)\underline{i} + (10 - 8)\underline{j}$

$$a = \boxed{-26\underline{i} + 2\underline{j}}$$

$$b) \quad x = \int (v) dt = (2t^2 - t^3) \underline{i} + \left(\frac{t^3}{3} - 4t^2 - 40t \right) \underline{j} + c //$$

$$\text{let } x = 0 \text{ at } t = 1,$$

$$\Rightarrow 0 = (2 - 1) \underline{i} + \left(\frac{1}{3} - 4 - 40 \right) \underline{j} + c$$

$$\therefore c = -\underline{i} + \frac{131}{3} \underline{j} //$$

$$\therefore x = (2t^2 - t^3 - 1) \underline{i} + \left(\frac{t^3}{3} - 4t^2 - 40t + \frac{131}{3} \right) \underline{j}$$

$$t = 2, \quad x = (2(4) - (8) - 1) \underline{i} + \left(\frac{8}{3} - 4(4) - 40(2) + \frac{131}{3} \right) \underline{j}$$

$$x = -\underline{i} - \frac{149}{3} \underline{j} = \overrightarrow{AB} //$$

$$(Q7a) \quad \vec{u} = u \cos \alpha \rightarrow \vec{s} = \vec{u} t.$$

$$x = ut \cos \alpha // \therefore t = \frac{x}{u \cos \alpha} //$$

$$y = u \sin \alpha$$

$$\left. \begin{array}{l} s = y \\ u = u \sin \alpha \\ v = \\ a = -g \\ t = t \end{array} \right\}$$

$$s = ut + \frac{1}{2} at^2$$

$$y = ut \sin \alpha - \frac{g}{2} t^2 //$$

$$\text{but } t = \frac{x}{u \cos \alpha},$$

$$\therefore y = u \sin \alpha \left(\frac{x}{u \cos \alpha} \right) - \frac{g}{2} \left(\frac{x}{u \cos \alpha} \right)^2$$

$$y = \frac{x \sin \alpha}{\cos \alpha} - \frac{g x^2}{2 u^2 \cos^2 \alpha}$$

$$y = x \tan \alpha - \frac{g x^2 (\sec^2 \alpha)}{2 u^2}$$

$$\tan \alpha = 2$$

$$1 + \tan^2 \alpha = \sec^2 \alpha$$

$$\therefore \sec^2 \alpha = 1 + (2)^2 = 5 //$$

$$\text{so } y = x(2) - \frac{5g x^2}{2 u^2} (5)$$

$$\therefore y = 2x - \frac{5g x^2}{2 u^2}$$



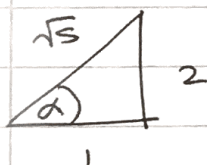
b) $x=36, y=0$: $0 = 2(36) - \frac{5g}{2u^2} (36)^2$

$$\underline{\div 36} : 2 = \frac{5g(36)}{2u^2}$$

$$\frac{5g(36)}{2} = 2u^2$$

$$\therefore u = \sqrt{\frac{5 \times 9.8 \times 36}{4}} = \boxed{21} \text{ ms}^{-1}$$

- c) minimum KE occurs at highest point ...
ie when speed = $u \cos \alpha = 21 \times \frac{1}{\sqrt{5}} = \frac{21}{\sqrt{5}}$



$$\begin{aligned} KE &= \frac{1}{2} (0.3) \left(\frac{21}{\sqrt{5}} \right)^2 \\ &= \boxed{13.2 \text{ J}} \quad (3 \text{ sf}) \end{aligned}$$

- d) expressing position of particle (relative to O) as a vector,

$$\mathbf{r} = \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ 2x - \frac{5g}{2u^2} x^2 \end{pmatrix} = \begin{pmatrix} u \cos \alpha \\ u \sin \alpha - \frac{gt^2}{2} \end{pmatrix}$$

$$\left(\frac{d\mathbf{r}}{dt} \right) = \dot{\mathbf{r}} = \mathbf{v} = \begin{pmatrix} u \cos \alpha \\ u \sin \alpha - gt \end{pmatrix}$$

from a

$$\sin \alpha = \frac{2}{\sqrt{5}}, \cos \alpha = \frac{1}{\sqrt{5}} \therefore \mathbf{v} = \begin{pmatrix} \frac{u}{\sqrt{5}} \\ \frac{2u}{\sqrt{5}} - gt \end{pmatrix}$$

$$\text{So } v = \begin{pmatrix} \frac{u}{\sqrt{5}} \\ \frac{2u}{\sqrt{5}} - gt \end{pmatrix}$$

$$\text{initially } v = \begin{pmatrix} \frac{u}{\sqrt{5}} \\ \frac{2u}{\sqrt{5}} \end{pmatrix}$$

we want when the direction of motion is perpendicular to the initial direction...

$$\text{ie when } \begin{pmatrix} \frac{u}{\sqrt{5}} \\ \frac{2u}{\sqrt{5}} - gt \end{pmatrix} \cdot \begin{pmatrix} \frac{u}{\sqrt{5}} \\ \frac{2u}{\sqrt{5}} \end{pmatrix} = 0$$

$$\Rightarrow \frac{u^2}{5} + \frac{2u}{\sqrt{5}} \left(\frac{2u}{\sqrt{5}} - gt \right) = 0$$

$$\Rightarrow \frac{u^2}{5} + \frac{4u^2}{5} - \frac{2ugt}{\sqrt{5}} = 0$$

$$\Rightarrow u^2 = \frac{2ugt}{\sqrt{5}}$$

$$\Rightarrow t = \frac{\sqrt{5}u}{2g} \quad \leftarrow \text{this is when the particle passes through B.}$$

$$x = ut \cos \alpha = \frac{u}{\sqrt{5}} \times \frac{\sqrt{5}u}{2g} = \frac{u^2}{2g} = \frac{21^2}{2g} = \boxed{22.5 \text{ m}}$$