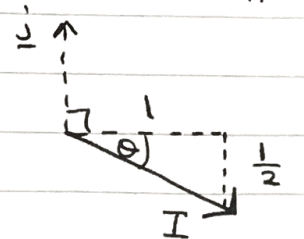


M2 June 2017 (MA)

Q1a)
$$\mathbf{I} = m(\mathbf{v} - \mathbf{u}) = 0.5(2\mathbf{i} + 3\mathbf{j} - 4\mathbf{j}) = 0.5(2\mathbf{i} - \mathbf{j})$$

$$= \mathbf{i} - \frac{1}{2}\mathbf{j} \quad \therefore |\mathbf{I}| = \sqrt{1^2 + (\frac{1}{2})^2} = \boxed{\frac{\sqrt{5}}{2} \text{ N s}}$$

b)



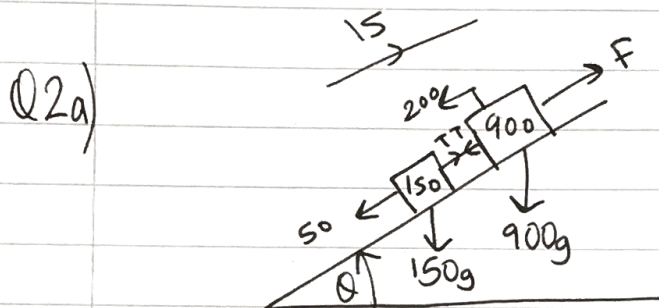
$$\tan \theta = \frac{\frac{1}{2}}{1} = \frac{1}{2}$$

$$\therefore \theta = \tan^{-1}\left(\frac{1}{2}\right) = 26.6^\circ$$

so angle between $\mathbf{I}$ and $\mathbf{j} = 90 + 26.6^\circ$

$$= 116.6^\circ$$

$$= \boxed{117^\circ}$$



$$P = Fv$$

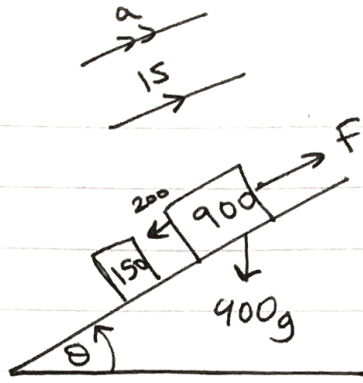
$$P = F \cdot 15$$

N2L (system): $F - 900g \sin \theta - 150g \sin \theta - 250 = 1050(0)$

($\sin \theta = \frac{1}{9}$) $F = \frac{900g + 150g}{9} + 250 = \frac{4180}{3} \text{ N}$

$$\therefore P = 15 \cdot \frac{4180}{3} = 20900 \text{ W} = \boxed{20.9 \text{ kW}}$$

b)



from (a), $F = \frac{4180}{3}$

N2L (Truck): $\frac{4180}{3} - 200 - \frac{900g}{1} = 900a$

$$\therefore a = \frac{\frac{4180}{3} - 200 - 100g}{900} = \boxed{0.24 \text{ ms}^{-2}}$$

c) Trailer: Resistance = 50 N.

Total Initial Energy = $\frac{1}{2} \cdot 150 \cdot (15)^2 = \underline{\underline{16875 \text{ J}}}$

Initially
Energy = 16875 J
(KE)

Finally
KE = 0
GPE = $150gd \sin \theta$

+ W.D by Resistance = $50d$. (F x d)

By C.O.E: $16875 = \frac{150gd}{1} + 50d$

$$d \left(\frac{150g}{1} + 50 \right) = 16875$$

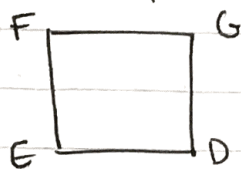
$$d = \frac{16875}{50 + \frac{150g}{1}} = \boxed{79.1 \text{ m}}$$

Q3a)

Shape

Area

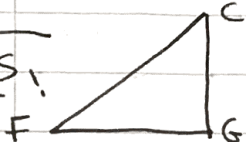
Distance of c.o.m from...

EFED

$$4a^2$$

$$a$$

$$a$$



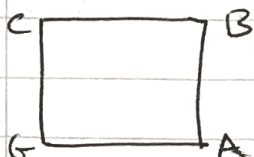
$$2 \times 2a^2$$

$$\frac{2}{3} \times 2a$$

$$2a + \frac{2a}{3}$$

$$= \frac{4a}{3}$$

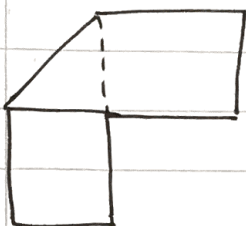
$$= \frac{8a}{3}$$



$$4a^2$$

$$3a$$

$$3a$$



$$12a^2$$

$$\bar{x}$$

$$\bar{y}$$

x2
because
this bit is
folded over!

$$\sum m_i x_i = \bar{x} \sum m_i$$

$$\Rightarrow 4a^2 \begin{pmatrix} a \\ a \end{pmatrix} + 2 \times 2a^2 \begin{pmatrix} \frac{4a}{3} \\ \frac{8a}{3} \end{pmatrix} + 4a^2 \begin{pmatrix} 3a \\ 3a \end{pmatrix} = 12a^2 \begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 4a + \frac{16a}{3} + 12a \\ 4a + \frac{32a}{3} + 12a \end{pmatrix} = 12 \begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix}$$

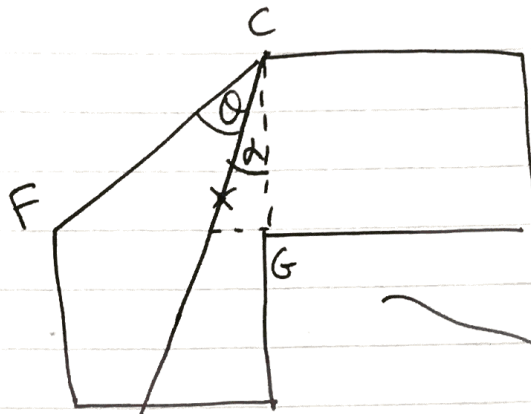
$$\Rightarrow 12 \begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix} = \begin{pmatrix} \frac{64a}{3} \\ \frac{80a}{3} \end{pmatrix} \therefore \begin{pmatrix} \bar{x} \\ \bar{y} \end{pmatrix} = \begin{pmatrix} \frac{16a}{9} \\ \frac{20a}{9} \end{pmatrix}$$

hence $\bar{x}$ = distance from EF = $\frac{16a}{9}$



□

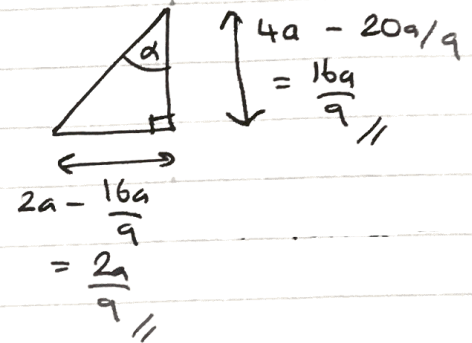
b)



Downward vertical

let θ be angle required,

let α be $\angle GCF - \theta$.



[notice $\theta + \alpha = 45$]

$$\therefore \tan \alpha = \frac{\frac{2a}{9}}{\frac{16a}{9}} = \frac{1}{8}$$

$$\therefore \alpha = \tan^{-1}\left(\frac{1}{8}\right)$$

$$\Rightarrow \theta = 45 - \alpha = 45 - \tan^{-1}\left(\frac{1}{8}\right) = \boxed{37.9^\circ}$$

Q4a) $v = 3t^2 - 16t + 21 = 0$

$$(3t - 7)(t - 3) = 0$$

$$t = \frac{7}{3}, t = 3$$

$$\therefore \boxed{t_1 = \frac{7}{3}} \text{ and } \boxed{t_2 = 3}$$

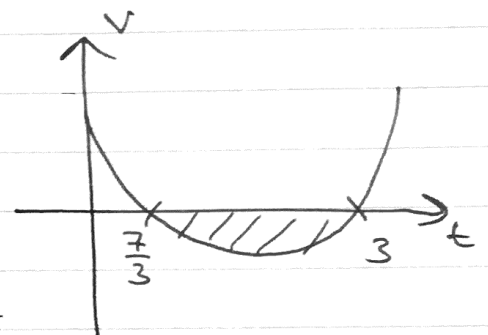
b) $a = \frac{dv}{dt} = 6t - 16$

$$t = \frac{7}{3} : a = 6\left(\frac{7}{3}\right) - 16 = -2 \text{ ms}^{-2}$$

$$\therefore |a| = \boxed{2 \text{ ms}^{-2}}$$

c) distance = $\left| \int_{\frac{7}{3}}^3 [v] dt \right|$

$$\int_{\frac{7}{3}}^3 (v) dt = \int_{\frac{7}{3}}^3 [3t^2 - 16t + 21] dt$$



$$= \left[t^3 - 8t^2 + 21t \right]_{\frac{7}{3}}^3 = \left[27 - 72 + 63 \right] - \left[\frac{490}{27} \right]$$

$$= -\frac{4}{27}$$

$$\therefore \text{distance} = \left| -\frac{4}{27} \right| = \boxed{\frac{4}{27} \text{ m}}$$

$$d) \quad x = \int (v) dt = t^3 - 8t^2 + 21t + c$$

$$\underline{t=0, x=0} : 0 = c //$$

$$\therefore x = t^3 - 8t^2 + 21t$$

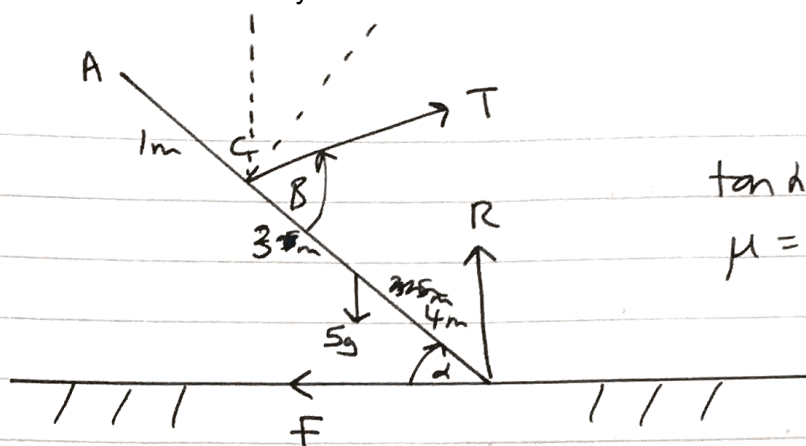
$$\underline{\text{at } t=3} : x = (3)^3 - 8(3^2) + 21(3) \\ = 18 //$$

$$x > 0 \text{ at } t = 3 .$$

and after $t=3$, v is an increasing function indefinitely. (See graph on previous page).

$\therefore P$ does not return to 0.

Q5a)



$$\tan \alpha = \frac{3}{4}$$

$$\mu = \frac{2}{3} \rightarrow F = \frac{2}{3} R //$$

$$M(C) : R \cos \alpha (7) = 5g \cos \alpha (3) + F \sin \alpha (7)$$

$$\div \cos \alpha \rightarrow 7R = 15g + 7F \tan \alpha$$

$$\Rightarrow 7R = 15g + 7F \left(\frac{3}{4} \right)$$

$$F = \frac{2}{3} R : 7R = 15g + \frac{7}{2} R$$

$$R = \frac{15g}{\frac{7}{2}} = \frac{30g}{7} = \boxed{42N}$$

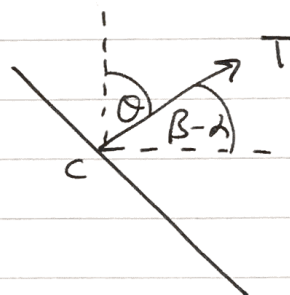
$$b) R(\updownarrow) : T \cos \theta = 5g - R$$

$$\therefore T \cos \theta = 7 // \sim (1)$$

$$R(\leftrightarrow) : T \cos(\beta - \alpha) = F$$

$$F = \frac{2}{3} \cdot 42 = 28N //$$

$$\therefore T \cos(\beta - \alpha) = 28 \sim (2)$$



$$\theta = 90 - (\beta - \alpha)$$

but $\theta = 90 - (\beta - \alpha) : (1) \text{ becomes : } T \cos(90 - (\beta - \alpha)) = 7$
 $\Rightarrow T \sin(\beta - \alpha) = 7$

Our 2 equations are:

$$T \sin(\beta - \alpha) = 7 \quad \text{--- (1)}$$

$$\text{and } T \cos(\beta - \alpha) = 28 \quad \text{--- (2)}$$

$$\frac{\textcircled{1}}{\textcircled{2}} : \frac{T \sin(\beta - \alpha)}{T \cos(\beta - \alpha)} = \frac{7}{28}$$

$$\Rightarrow \tan(\beta - \alpha) = \frac{7}{28}$$

$$\Rightarrow \tan^{-1}\left(\frac{7}{28}\right) = \beta - \alpha$$

$$\Rightarrow \beta = \tan^{-1}\left(\frac{7}{28}\right) + \alpha$$

$$\Rightarrow \beta = \tan^{-1}\left(\frac{7}{28}\right) + \tan^{-1}\left(\frac{3}{4}\right)$$

$$\boxed{\beta = 50.9^\circ}$$

Q6a(i) $s = ut$ for both P and Q

P: $x = (30 \cos 60)(2) = \overrightarrow{\text{distance to collision}}$

$\therefore$ Q travels $\overleftarrow{(40 - 60 \cos 60)} \text{ m to collision}$.

$$\Rightarrow (40 - 60 \cos 60) = q \cos \theta (2)$$

$$\Rightarrow q \cos \theta = 20 - 30 \cos 60 \quad \sim (1)$$

both P and Q travel the same vertical distance.

$$\underbrace{(30 \sin 60)(2) - \frac{g}{2}(4)}_{\left[s = ut + \frac{1}{2}at^2\right] \text{ for P}} = \underbrace{(q \sin \theta)(2) - \frac{g}{2}(4)}_{\text{for Q}}$$

$$\Rightarrow 60 \sin 60 = 2q \sin \theta$$

$$\Rightarrow \therefore q \sin \theta = 30 \sin 60 \quad \sim (2)$$

$$\frac{(2)}{(1)} : \frac{q \sin \theta}{q \cos \theta} = \frac{30 \sin 60}{20 - 30 \cos 60} = \tan \theta = 3\sqrt{3}$$

$$\therefore \theta = \tan^{-1}(3\sqrt{3}) = \boxed{79.10}$$

$$\therefore q = \frac{30 \sin 60}{\sin(79.1)} = \boxed{26.5 \text{ m/s}}$$

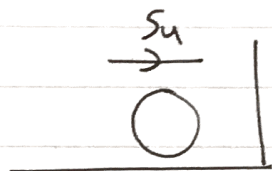
b) For P

$$\left. \begin{array}{l} \uparrow S = \\ u = 30 \sin 60 \\ v = v \\ a = -g \\ t = 2 \end{array} \right\} \begin{array}{l} v = u + at \\ \uparrow v = 30 \sin 60 - 2g // \end{array}$$

$$\therefore |\text{speed}_p| = \sqrt{(30 \sin 60 - 2g)^2 + (30 \cos 60)^2}$$

$$= 16.3 \text{ m/s} //$$

Q7a)



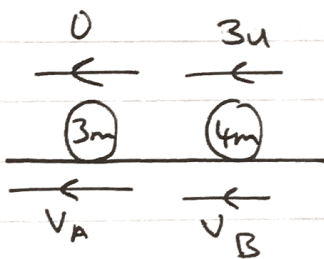
$$I = 4m(v - u)$$

$$\overleftarrow{S_u \times \frac{3}{5}} = 3u$$

$$= 4m(3u - -5u)$$

$$= \boxed{32mu}$$

b)



$$\underline{\text{C.L.M}} : 4m(3u) = 3m(v_A) + 4m(v_B)$$

$$\Rightarrow 12u = 3v_A + 4v_B$$

$$\underline{\text{N.I.L}} : e = \frac{v_A - v_B}{3u}$$

$$\therefore 3ue = v_A - v_B$$

$$\Rightarrow 9ue = 3v_A - 3v_B \sim \textcircled{2}$$

①

$$- \begin{bmatrix} 12u = 3V_A + 4V_B \\ 9ue = 3V_A - 3V_B \end{bmatrix}$$

$$12u - 9ue = 7V_B$$

$$7V_B = 3u(4 - 3e)$$

$$V_B = \frac{3u}{7}(4 - 3e) //$$

$$V_A = 3ue + V_B = 3ue + \frac{12u}{7} - \frac{9ue}{7}$$

$$V_A = \frac{12ue}{7} + \frac{12u}{7} = \frac{12u}{7}(1 + e) //$$

both V_A and V_B are greater than 0.

(as $0 \leq e \leq 1$).

$\therefore$ They are travelling in the same dir.

$$c) \text{ KE after} = \frac{1}{2} (4m) \left[\frac{3u}{7}(4 - 3e) \right]^2$$

$$\text{KE before} = \frac{1}{2} (4m) [3u]^2$$

$$\Rightarrow 2m \left[\frac{9u^2}{49} \right] [4 - 3e]^2 = \frac{1}{4} [2m \cdot 9u^2]$$

$$\Rightarrow \frac{18mu^2}{49} (16 - 24e + 9e^2) = \frac{9}{2} mu^2$$

$$\Rightarrow 16 - 24e + 9e^2 = 49/4$$

$$\Rightarrow 9e^2 - 24e + \frac{15}{4} = 0$$

By Quadratic formula, $e = \frac{5}{2}$, $e = \frac{1}{6}$
x ↑

($e \leq 1$
 $\therefore$ reject)

so

$$e = \frac{1}{6}$$