M2 June 2016 (MA)

Q1a) $t=0, v=11$: $\boxed{11} = r$

$t=2, v=3$: $3 = 4p + 2q + 11$

$$4p + 2q = -8 \quad (1)$$

$t=2$, v is min $\therefore a = 0$:

$$a = \frac{dv}{dt} = 2pt + q = 0$$

$$\Rightarrow \quad 4p + q = 0$$

$$\Rightarrow \quad 4p = -q \quad (2)$$

sub (2) into (1) : $-q + 2q = -8$

$$q = \boxed{-8}$$

$$\text{so} \quad p = -\frac{-8}{4} = \boxed{2}$$

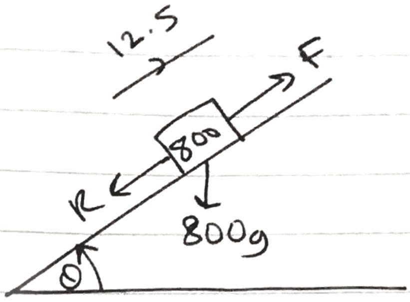
at $t=3$, $a = 2(2)(3) - 8 = \boxed{4\text{ms}^{-2}}$

b) 3^{rd} second means from $t=2$ to $t=3$.

$$\text{distance} = \int_2^3 \left[2t^2 - 8t + 11\right] dt = \left[\frac{2t^3}{3} - 4t^2 + 11t\right]_2^3$$

$$= \left[15\right] - \left[\frac{34}{3}\right] = \boxed{\frac{11}{3}}$$

Q2ai)



$P = Fv$

$3P = 12.5(F)$

$F = \frac{3P}{12.5}$

N2L(car) : $F - R - 800g\sin\theta = 0$

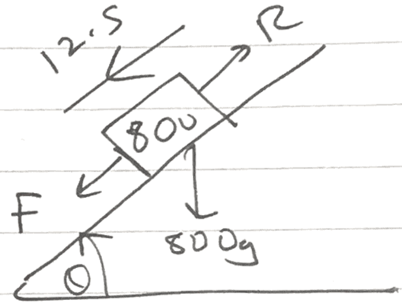
$$\frac{3P}{12.5} = \frac{800g}{20} + R$$

$$P = \frac{12.5}{3}\left(\frac{800g}{20} + R\right) \sim (1)$$

$P = Fv$

$P = 12.5F$

$\frac{P}{12.5} = F$



N2L(car) : $F + 800g\sin\theta - R = 0$

$$\frac{P}{12.5} = R - \frac{800g}{20}$$

$$P = 12.5\left(R - \frac{800g}{20}\right) \sim (2)$$

solve (1) and (2) simultaneously :

$$\cancel{12.5}\left(R - \frac{800g}{20}\right) = \frac{\cancel{12.5}}{3}\left(\frac{800g}{20} + R\right)$$

$$R - 40g = \frac{40}{3}g + \frac{R}{3}$$

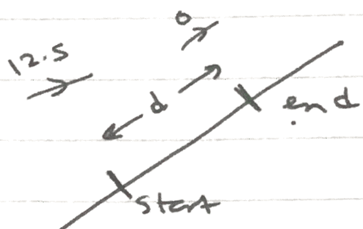
$$\therefore \quad \frac{2R}{3} = \left(40 + \frac{40}{3}\right) g$$

$$R = \boxed{784N}$$

and from ②, $P = 12.5(784 - 40g)$

$$= \boxed{4900W}$$

b)



$\underline{KE} = 0$

$\underline{GPE} = 800g(d\sin\theta)$.

$\underline{KE} = \frac{1}{2}(800)(12.5^2)$

$\underline{GPE} = 0$

+ W.D due to R = $R \times d$. $= 784d$.

<u>Conservation of energy</u> . . .

$$400(12.5^2) = \frac{800g}{20} d + 784d$$

$$d\left(784 + 40g\right) = 400(12.5^2)$$

$$\therefore d = \frac{400 \times 12.5^2}{784 + 40g} = \boxed{53.1m}$$

Q3) Impulse $= 0.6(v-u) = 0.6(2c\underline{i} - c\underline{j} - c\underline{i} - 2c\underline{j})$

$\Rightarrow 0.6(c\underline{i} - 3c\underline{j}) = \text{Impulse}$

$\Rightarrow |0.6c\underline{i} - 1.8c\underline{j}| = 2\sqrt{10}$

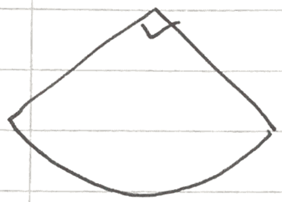
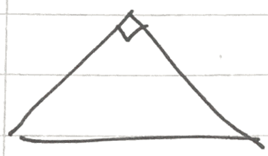
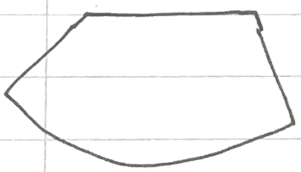
$\Rightarrow \sqrt{(0.6c)^2 + (1.8c)^2} = 2\sqrt{10}$

$\Rightarrow 0.36c^2 + 3.24c^2 = (2\sqrt{10})^2$

$\Rightarrow 3.6c^2 = 40 \quad \therefore c^2 = \frac{40}{3.6}$

$\Rightarrow c = \sqrt{\frac{40}{3.6}} = \sqrt{\frac{100}{9}} = \boxed{\frac{10}{3}} \quad (c > 0)$

Q4a)

Shape	Area	$\downarrow$ Displacement from O al c.o
	$\frac{\pi(4)^2}{4} = \boxed{4\pi}$	$\boxed{\frac{16\sqrt{2}}{3\pi}}$ from formula
	$\frac{1}{2}\cdot 3 \cdot 3 = \boxed{\frac{9}{2}}$	$3\cos 45 \times \frac{1}{3} = \boxed{\frac{\sqrt{2}}{2}}$
	$\boxed{4\pi - \frac{9}{2}}$	$\boxed{\bar{y}}$

$$\bar{x}\sum m_i = \sum m_i x_i$$

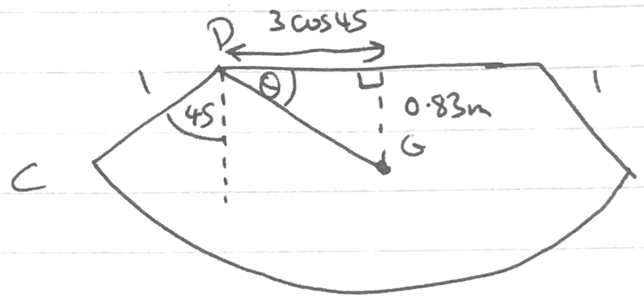
$$4\pi\left(\frac{16\sqrt{2}}{3\pi}\right) - \frac{9}{2}\left(\frac{\sqrt{2}}{2}\right) = \left(4\pi - \frac{9}{2}\right)\left(\bar{y}\right)$$

$$\bar{y} = \frac{\frac{4(16\sqrt{2})}{3} - \frac{9\sqrt{2}}{4}}{4\pi - \frac{9}{2}} = 2.951\ldots$$

$\therefore$ distance from AD $= 2.951\ldots - 3\cos 45$

$= \boxed{0.83\text{m}}$

b)



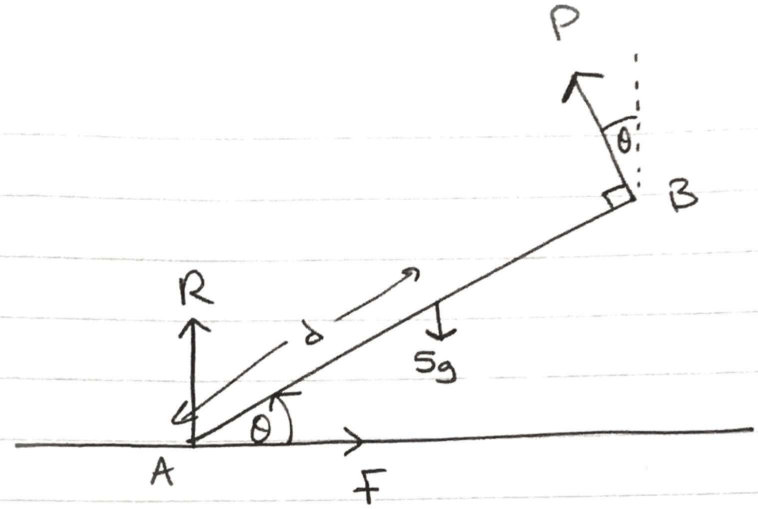
$$\tan\theta = \frac{0.83}{3\cos 45}$$

$$\therefore \theta = \tan^{-1}(0.3918\ldots) = 21.4°$$

angle required $= 90 - 21.4° + 45°$

$= \boxed{114°}$

Q5a i)



$M(A): \quad 5g\cos\theta\,(d) = P(4)$

$$P = \frac{5gd\cos\theta}{4}$$

$R(\updownarrow): \quad P\cos\theta + R = 5g$ —— ①

$R(\leftrightarrow): \quad F = P\sin\theta$ —— ②

①: $R = 5g - P\cos\theta = 5g - \dfrac{5gd\cos^2\theta}{4}$

$$\boxed{R = 5g\left(1 - \frac{d}{4}\cos^2\theta\right)}$$

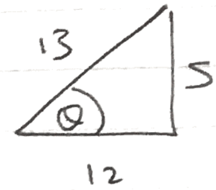
ii) limiting equilibrium $\therefore F = \mu R$

②: but $F = P\sin\theta = \boxed{\dfrac{5gd\sin\theta\cos\theta}{4}}$

b) $F = \frac{1}{2}R: \quad \dfrac{\cancel{5}\cancel{g}d\sin\theta\cos\theta}{4} = \dfrac{\cancel{5}\cancel{g}}{2}\left(1 - \dfrac{d}{4}\cos^2\theta\right)$

$$d\sin\theta\cos\theta = 2 - \frac{d}{2}\cos^2\theta$$

$\tan\theta = \frac{5}{12}$



$\therefore \sin\theta = \frac{5}{13}$

$\cos\theta = \frac{12}{13}$

$\Rightarrow d\left(\sin\theta\cos\theta + \frac{1}{2}\cos^2\theta\right) = 2$

$\Rightarrow d = \dfrac{2}{\sin\theta\cos\theta + \frac{1}{2}\cos^2\theta} = \dfrac{2}{\left(\frac{5}{13}\right)\left(\frac{12}{13}\right) + \frac{1}{2}\left(\frac{12}{13}\right)^2}$

$= \boxed{\dfrac{169}{66}}$

Q6a) $r = \begin{pmatrix} 3t \\ 4t - \frac{9}{2}t^2 \end{pmatrix} = \begin{pmatrix} \lambda \\ -\lambda \end{pmatrix}$ $\quad (\uparrow) \left.\begin{array}{l} s = y \\ u = 4 \\ v = \\ a = -9 \\ t = t \end{array}\right\} y = 4t - \frac{9}{2}t^2$

expressing position as a vector.

so $3t = \lambda$ and $4t - 4.9t^2 = -\lambda$

$t = \frac{\lambda}{3}$

$\Rightarrow \frac{4}{3}\lambda - 4.9\left(\frac{\lambda^2}{9}\right) + \lambda = 0$

$\Rightarrow \frac{4}{3}\cancel{\lambda} + \cancel{\lambda} = \frac{49}{90}\lambda^{\cancel{2}}$

$$\Rightarrow \frac{49}{90}\lambda = \frac{7}{3} \quad \therefore \lambda = \frac{\frac{7}{3}}{\frac{49}{90}} = \boxed{\frac{30}{7}}$$

bi) from (a), $t = \frac{\lambda}{3} = \frac{10}{7} =$ time at A.

horizontal speed = constant = $3\,ms^{-1}$

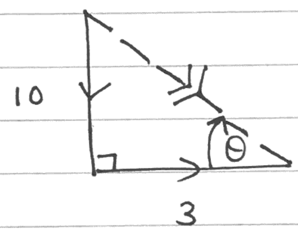
<u>finding vertical component:</u>

$+\uparrow$

$$\left.\begin{aligned} s &= \\ u &= 4 \\ v &= v \\ a &= -g \\ t &= \frac{10}{7} \end{aligned}\right\}$$

$v = u + at$

$v = 4 - \frac{10g}{7} = -10\,ms^{-1}$



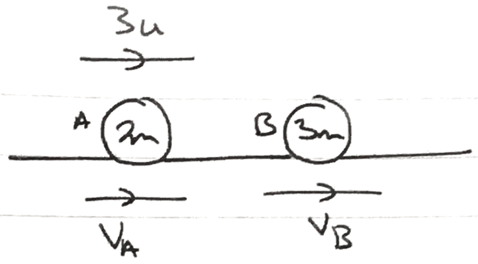
$$\text{speed} = \sqrt{(10)^2 + (3)^2} = \sqrt{109}\ ms^{-1} = \boxed{10.4}\ m/s.$$

direction: $\tan\theta = \frac{10}{3} \quad \therefore \theta = \arctan\left(\frac{10}{3}\right)$

$= 73.3°$

So P is moving at $\boxed{73.3° \text{ below the horizontal}}$

Q7a)



$e = \frac{3}{4}$

$\overrightarrow{C \cdot L \cdot M}$: $2m(3u) = 2m(V_A) + 3m(V_B)$

$$6u = 2V_A + 3V_B \quad — ①$$

$\overrightarrow{N \cdot I \cdot L}$: $\frac{3}{4} = \frac{V_B - V_A}{3u}$

$$\frac{9u}{4} = V_B - V_A$$

$$\overset{\times 2}{\Rightarrow} \quad \frac{9u}{2} = 2V_B - 2V_A \quad — ②$$

① + ② : $\left(\frac{9}{2} + 6\right)u = 5V_B$

$$\frac{\frac{21}{2}u}{5} = V_B = \boxed{\frac{21u}{10}}$$

and $V_A = V_B - \frac{9u}{4} = -\frac{3u}{20} \rightarrow$ speed $= \boxed{\frac{3u}{20}}$

b) $I = m(v - u)$

$$\frac{27}{4}\cancel{m}\cancel{u} = 3\cancel{m}\left(\frac{21\cancel{u}e}{10} + \frac{21\cancel{u}}{10}\right)$$

$$\frac{27}{12} = \frac{21}{10}e + \frac{21}{10}$$

$$\therefore e = \frac{\frac{27}{12} - \frac{21}{10}}{21/10} = \boxed{\frac{1}{14}}$$

c) final speed of B $= \frac{21u}{10} \times \frac{1}{14} = \frac{3u}{20}$

speed of A = speed of B

so no further collision.