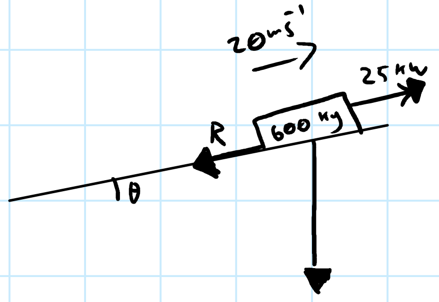
June 2014 (R) - M2

1)



In 1s, 20 m moved up the plane

Work done against R :

$R \times 20 = 20R$ J

Work done against W :

$600 \times 9.8 \times 20\sin(\theta)$

$= 7350$ J

$25{,}000\ \text{W} \times 1\text{s} = 25{,}000$ J

$25{,}000 = 7350 + 20R$

$R = 882.5$ N

Because we use g to (2sf), we cannot be more accurate than (2sf) in our answer.

$R = 880$ N (2sf)

1)

b)

Resultant force requires a power of 5kw (30-25)

$P = F \times V$, $F = ma$

$5{,}000 = 600a \times 20$

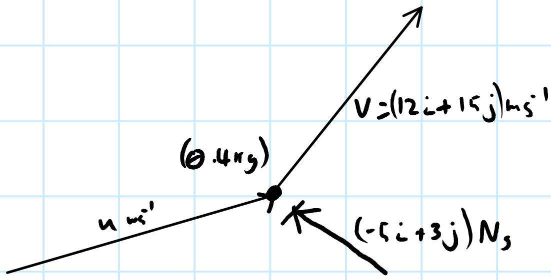
$\frac{5{,}000}{12{,}000} = a$

$a = 0.417\,ms^{-2}$

Be no more accurate than 3sf in an M2 question, unless specified otherwise

2)

a)



Initial momentum $0.4u$

$0.4u + (-5i + 3j) =$ Final momentum

$$V = \frac{0.4u + (-5i + 3j)}{0.4}$$

$$12i + 15j = u - 12.5i + 7.5j$$

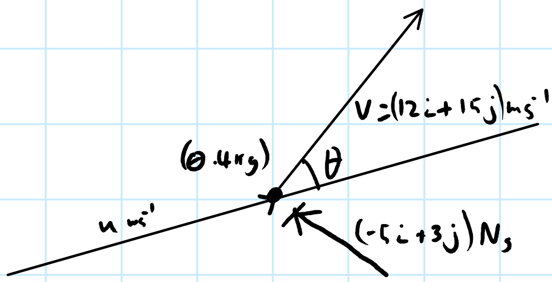
$$24.5i + 7.5j = u$$

$$|u| = \sqrt{(24.5)^2 + (7.5)^2}$$

$$= 25.6 \text{ ms}^{-1} \quad (3sf)$$

2)

b)



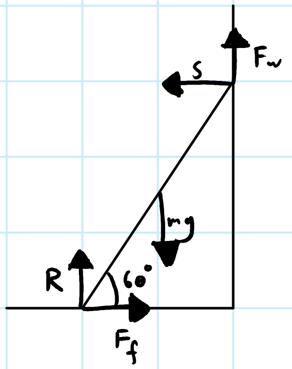
$$\arg(v) = \tan^{-1}\left(\frac{15}{12}\right)$$
$$= 51.3^\circ \ (3sf)$$

$$\arg(u) = \tan^{-1}\left(\frac{7.5}{24.5}\right)$$
$$= 17.0^\circ \ (3sf)$$

$$\theta = 51.3 - 17.0$$
$$= 34.3^\circ \ (3sf)$$

3)

a)



+ve

$\mu_f = \frac{1}{4}$, $\mu_w = \frac{2}{3}$

Forces (↑)

$F_w + R - mg = 0$

$\frac{2}{3}S + R = mg$

Forces (→)

$F_f - S = 0$

$\frac{1}{4}R = S$

$\frac{2}{3}\left(\frac{1}{4}R\right) + R = mg$

$\frac{7}{6}R = mg \quad \therefore R = \frac{6}{7}mg$

b) moments around (A)

Let x = distance AG

moments (↻) = moments (↺)

$x\cos(60)mg = 2L\sin(60)S + 2L\cos(60)F_w$

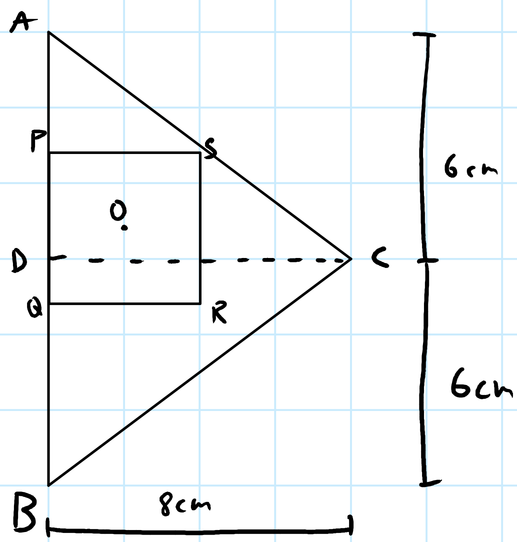
$\frac{1}{2}xmg = \frac{2\sqrt{3}}{2}L\frac{1}{4}R + \frac{2}{2}L\left(\frac{2}{3} \times \frac{1}{4}R\right)$

$xmg = \frac{2\sqrt{3}}{4}L\frac{6}{7}mg + \frac{2 \times 2}{3 \times 4}\frac{6}{7}mg$

$x = \frac{3\sqrt{3}}{7}L + \frac{2}{7}L$

$x = \frac{2 + 3\sqrt{3}}{7}L$

4)



a)

In the horizontal direction only, letting AB pass through the origin

Since PQRS is a square of side length 4cm and PQ lies on AB, O is 2cm from AB

Since ABC is symmetrical around CD, this is a median line of the triangle. COM of ABC lies on this line, 2/3 of the way from vertex C (or 1/3 from AB)
Distance is (8/3)cm from AB

Area of PQRS is 4*4 = 16cm^2 (let this be a negative since it is removed from L)
Area of ABC is (1/2)*(12*8) = 48cm^2
Total area of L is 48 + -16 = 32cm^2

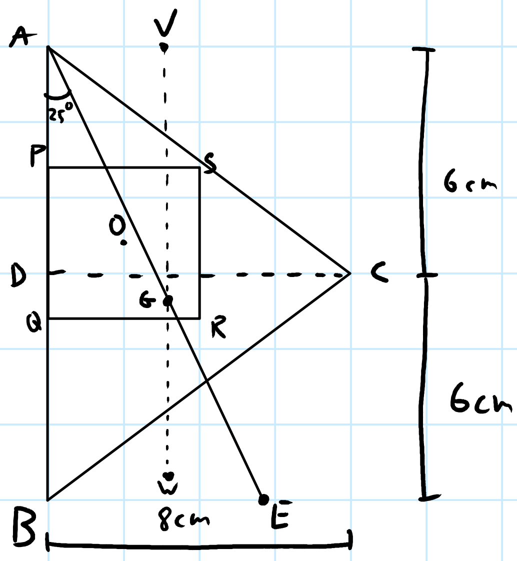
$$-16(2) + 48\left(\frac{8}{3}\right) = 32(\bar{x})$$

$$-32 + 128 = 32(\bar{x})$$

$$\bar{x} = 3\text{cm}$$

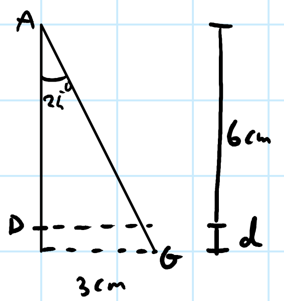
4)

b)



Line VW is parallel to AB at distance of 3cm, so that G lies on VW
AGE is a straight line angled at 25° from AB. When L is freely suspended from A, AE becomes the vertical.

G is the intersect of AE and VW



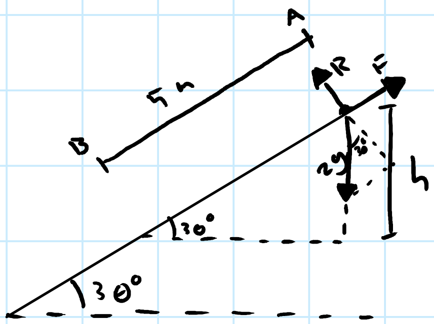
$$\frac{3cm}{\tan(25°)} = 6.434cm$$

$$d = 6.43 - 6$$
$$= 0.434 \text{ cm}$$

G is 0.434cm away from CD.
We know the COM of ABC lies on CD as it is symmetrical around it.
We know the mass ratios of L, ABC and PQRS from part (a)
Let |y| be the distance from O to CD

$$48(0) + -16\,(y) = 32\,(-0.434)$$
$$y = -2(-0.434)$$
$$|y| = 0.867 \text{ (3sf)}$$

5)



a) $h = 5\sin(30°)$

$= 2.5\text{m}$

$mgh = 2 \times 9.8 \times 2.5$

$= 49\text{J}$

b) i) Final $KE = \frac{1}{2} \times 2 \times 4^2$

$= 16\text{J}$

$\therefore$ Work done by friction $49 - 16$

$= 33$

$F \times 5\text{m} = 33\text{J} \therefore F = 6.6\text{ N}$

ii) $6.6 = \mu R$, $R = 2g\cos(30°)$

$6.6 \div \frac{49\sqrt{3}}{5} = \mu$

$= 0.389$ (3sf)

5)

c) Initial energy:

KE + GPE

$\frac{1}{2} \times 2 \times 3^2 + 2 \times 9.8 \times 5 \sin(30^\circ) = 58\text{ J}$

Final Energy:

workdone by F + Final KE

$33 + \frac{1}{2} \times 2 \times v^2 = 58\text{ J}$

$v^2 = 25$

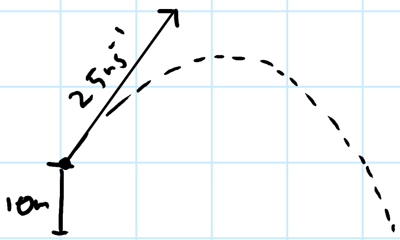
$|v| = 5\text{ ms}^{-1}$

Question asks for speed so use positive v.

Constant force in both (a) and (b) so same amount of work done in each since same distance is travelled.

6)

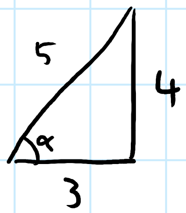
a)



$\sin(\alpha)=\frac{4}{5}$

$\therefore \cos(\alpha)=\frac{3}{5}$

$\& \tan(\alpha)=\frac{4}{3}$



Horizontal velocity is $25\cos(\alpha)$

$15 ms^{-1}$

Initial GPE + Initial KE = final GPE + final KE

$4.8\cancel{m}(10)+\frac{1}{2}\cancel{m}(25)^2 = 4.8\cancel{m}h+\frac{1}{2}\cancel{m}(15)^2$

$48+312.5 = 4.8h+112.5$

$\frac{298}{4.8} = h$

$h = 30.4m$

b)

S	$-10m$
U	$25\sin(\alpha) ms^{-1}$
V	X
A	$-9.8 ms^{-2}$
T	—

$s=ut+\frac{1}{2}at^2$

$-10=20t-4.9t^2$

$4.9t^2-20t-10=0$

$\frac{20\pm\sqrt{400--196}}{4.8} = 4.53, -0.45$

$t>0 \therefore t=4.53s$

Time taken to hit the floor is 4.53s

6)

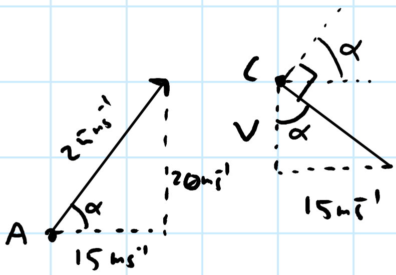
b)

In part (a) we found the horizontal speed was 15ms^{-1}
The ball was moving at this speed for 4.53s

$15 \times 4.53 = 68.0\text{m}$ (3sf)

c)

Horizontal speed is constant at A and C at 15ms^{-1}
Let the vertical speed at C be v



$\tan(\alpha) = \frac{4}{3}$, $\tan(\alpha) = \frac{15}{v}$

$\frac{4}{3} = \frac{15}{v}$

$v = 11.25\text{ms}^{-1}$

V is negative here because it is travelling down towards the ground.

S x

U 20ms^{-1}

V -11.25ms^{-1}

A -9.8ms^{-2}

T ___

$v = u + at$

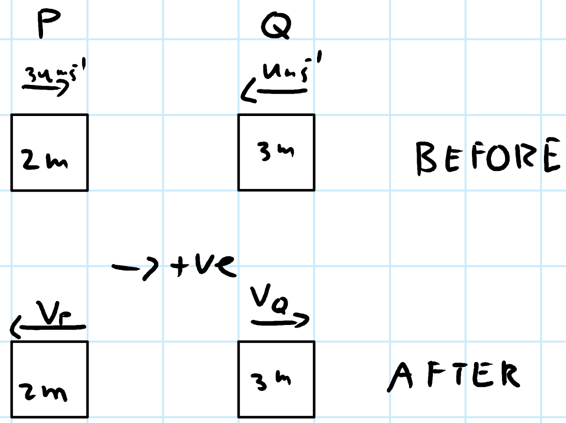
$-11.25 = 20 - 9.8t$

$\frac{-31.25}{-9.8} = t$

$t = 3.19\text{s}$ (3sf)

7)

a)



Initial momentum = Final Momentum

$$2\cancel{m} \times 3u - 3\cancel{m} \times u = -2\cancel{m}V_P + 3\cancel{m}V_Q$$

$$3u = 3V_Q - 2V_P$$

$$e = \frac{\text{speed of seperation}}{\text{speed of approach}}$$

$$e = \frac{V_Q - -V_P}{3u - -u}$$

$$e = \frac{V_Q + V_P}{4u}$$

$$8eu - 2V_Q = 2V_P$$

$$3u = 3V_Q - 8eu + 2V_Q$$

$$3u + 8eu = 5V_Q$$

$$V_Q = \frac{u}{5}(3 + 8e)$$

7) $3u = 3V_Q - 2V_p \qquad e = \frac{V_Q + V_p}{4u}$

b)

$12eu - 3V_p = 3V_Q$

$3u = 12eu - 3V_p - 2V_p$

$V_p = \frac{u}{5}(12e - 3)$

$V_p > 0 \quad , \quad u > 0$

$(12e - 3) > \frac{5}{u} 0$

$12e > 3 \quad \therefore \quad e > \frac{1}{4}$

$\frac{1}{4} < e \leq 1$

We always assume no extra energy is added in a collision, therefore e is always assumed to be at most 1.

7)

c) Initial KE:

$P: \frac{1}{2} 2m(3u)^2 = 9mu^2$

$Q: \frac{1}{2} 3mu^2 = \frac{3}{2}mu^2$

$\therefore T = \frac{21}{2}mu^2$

Final KE:

$P: \frac{1}{2} 2m\left(\frac{u}{5}\left(\frac{12}{2}-3\right)\right)^2 = \frac{9}{25}mu^2$

$Q: \frac{1}{2} 3m\left(\frac{u}{5}\left(\frac{3}{2}+3\right)\right)^2 = \frac{147}{50}mu^2$

$KT = \frac{33}{10}mu^2$

$$K = \frac{\frac{33}{10}mu^2}{\frac{21}{2}mu^2}$$

$$= \frac{11}{35}$$

$= 0.314$ (3sf)