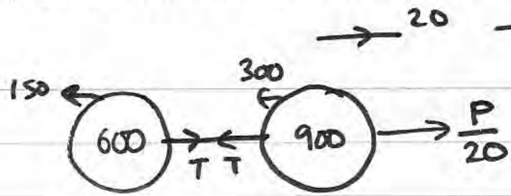
M2 JUNE 13 INT

1. A caravan of mass 600 kg is towed by a car of mass 900 kg along a straight horizontal road. The towbar joining the car to the caravan is modelled as a light rod parallel to the road. The total resistance to motion of the car is modelled as having magnitude 300 N. The total resistance to motion of the caravan is modelled as having magnitude 150 N. At a given instant the car and the caravan are moving with speed 20 m s^{-1} and acceleration 0.2 m s^{-2}.

(a) Find the power being developed by the car's engine at this instant. **(5)**

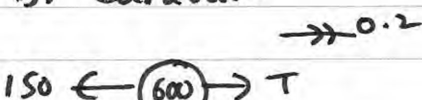
(b) Find the tension in the towbar at this instant. **(2)**



$\overrightarrow{Rf} = ma \quad \Rightarrow \quad \frac{P}{20} - 300 - 150 + T - T = 1500 \times 0.2$

$\therefore \frac{P}{20} = 750 \quad \Rightarrow \quad \underline{P = 15000\,W}$

b) Caravan



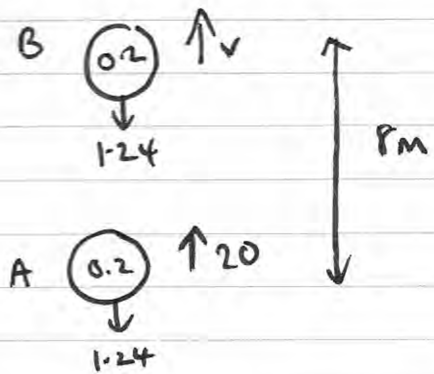
$\overrightarrow{Rf} = ma$

$T - 150 = 600 \times 0.2$

$\therefore T = \underline{270N}$

2. A ball of mass 0.2 kg is projected vertically upwards from a point O with speed 20 m s^{-1}. The non-gravitational resistance acting on the ball is modelled as a force of constant magnitude 1.24 N and the ball is modelled as a particle. Find, using the work-energy principle, the speed of the ball when it first reaches the point which is 8 m vertically above O.

(6)



KE_A − wd against n.g. Res = $KE_B + PE_B$

$$\frac{1}{2}\cancel{m}(20)^2 - \frac{1.24\times 8}{(0.2)} = \frac{1}{2}\cancel{m}v^2 + \cancel{m}g\times 8$$

$$150.4 = \frac{1}{2}v^2 + 78.4 \quad \Rightarrow \frac{1}{2}v^2 = 72 \quad \therefore \underline{v = 12}$$

3. A particle P moves along a straight line in such a way that at time t seconds its velocity v m s^{-1} is given by

$$v = \frac{1}{2}t^2 - 3t + 4$$

Find

(a) the times when P is at rest,

(4)

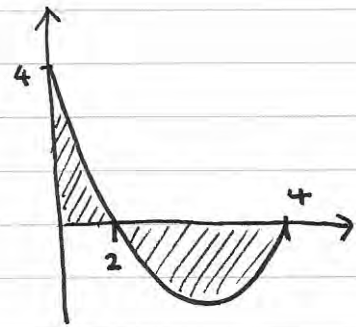
(b) the total distance travelled by P between $t = 0$ and $t = 4$.

(5)

a) $\frac{1}{2}t^2 - 3t + 4 = 0$ $\quad$ $\times 2$ $\quad$ $t^2 - 6t + 8 = 0 \Rightarrow (t-4)(t-2) = 0$

$\underline{t = 4}$ $\quad$ $\underline{t = 2}$

b)



$s = \int v\,dt = \frac{1}{6}t^3 - \frac{3}{2}t^2 + 4t \; (+c)$

$\text{Area} = \left[\frac{1}{6}t^3 - \frac{3}{2}t^2 + 4t\right]_2^4 = \frac{8}{3} - \frac{10}{3} = -\frac{2}{3}$

$\left[\frac{1}{6}t^3 - \frac{3}{2}t^2 + 4t\right]_0^2 = \frac{10}{3} - 0 = \frac{10}{3}$

$\therefore$ area $= \frac{10}{3} + \frac{2}{3} = 4 =$ total distance

4. A rough circular cylinder of radius $4a$ is fixed to a rough horizontal plane with its axis horizontal. A uniform rod AB, of weight W and length $6a\sqrt{3}$, rests with its lower end A on the plane and a point C of the rod against the cylinder. The vertical plane through the rod is perpendicular to the axis of the cylinder. The rod is inclined at 60° to the horizontal, as shown in Figure 1.

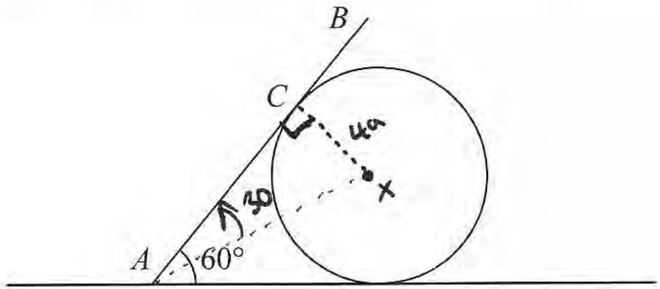


Figure 1

(a) Show that $AC = 4a\sqrt{3}$

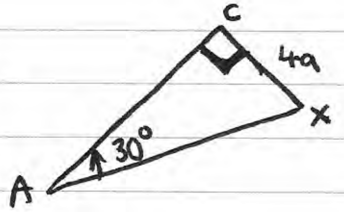
(2)

The coefficient of friction between the rod and the cylinder is $\frac{\sqrt{3}}{3}$ and the coefficient of friction between the rod and the plane is μ. Given that friction is limiting at both A and C,

(b) find the value of μ.

(9)

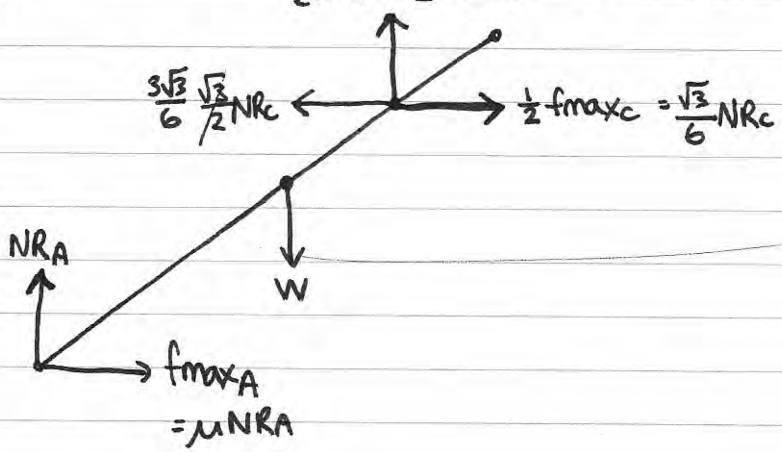
4a)

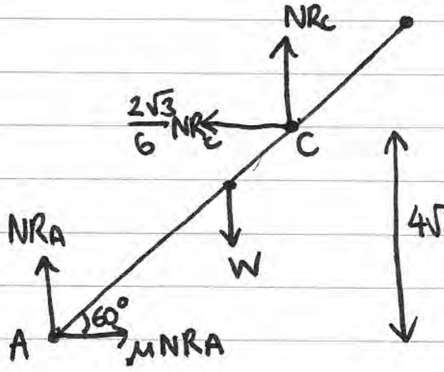


$\tan 30 = \dfrac{4a}{AC} \Rightarrow AC = \dfrac{4a}{\left(\frac{1}{\sqrt{3}}\right)} \Rightarrow \underline{AC = 4\sqrt{3}\,a}$ #

b)

$\frac{1}{2}NR_C + \frac{\sqrt{3}}{2} f_{max_C} = \frac{1}{2}NR_C + \frac{1}{2}NR_C = NR_C$





$\uparrow = \downarrow \Rightarrow NR_A + NR_C = W$ (1)

$\rightarrow = \leftarrow \Rightarrow \mu NR_A = \frac{2\sqrt{3}}{6}NR_C$ (2)

$4\sqrt{3}\,a \sin 60$

$= \frac{4\sqrt{3}}{} \times \frac{\sqrt{3}}{2}a = 6a$

$3\sqrt{3}\,a \cos 60$

$= \frac{3}{2}\sqrt{3}\,a$

$4\sqrt{3}\,a \cos 60$

$= 2\sqrt{3}\,a$

$A\circlearrowright$ $W \times \frac{3\sqrt{3}}{2}\cancel{a} = NR_C \times 2\sqrt{3}\cancel{a} + \frac{2\sqrt{3}}{\cancel{6}}NR_C \times \cancel{6a}$

$W \times \frac{3\cancel{\sqrt{3}}}{2} = 4\cancel{\sqrt{3}} \times NR_C$

$\therefore W = \frac{8}{3}NR_C$ sub in (1)

$NR_A + NR_C = \frac{8}{3}NR_C \Rightarrow NR_A = \frac{5}{3}NR_C$

sub in (2)

$\mu \times \frac{5}{3}\cancel{NR_C} = \frac{2\sqrt{3}}{6}\cancel{NR_C} \Rightarrow \mu = \frac{6\sqrt{3}}{30}$ $\therefore \mu = \frac{\sqrt{3}}{5}$

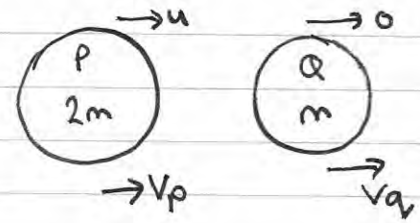
5. Two particles P and Q, of masses $2m$ and m respectively, are on a smooth horizontal table. Particle Q is at rest and particle P collides directly with it when moving with speed u. After the collision the total kinetic energy of the two particles is $\frac{3}{4}mu^2$. Find

(a) the speed of Q immediately after the collision,

(10)

(b) the coefficient of restitution between the particles.

(3)



KE before $= \frac{1}{2}(2m)u^2 = mu^2$

KE after $= \frac{3}{4}mu^2$

$\therefore \frac{1}{4}mu^2 =$ energy lost

$\cancel{\frac{1}{2}}(2\cancel{m})V_p^2 + \frac{1}{2}\cancel{m}V_q^2 = \frac{3}{4}\cancel{m}u^2$

$\Rightarrow V_p^2 + \frac{1}{2}V_q^2 = \frac{3}{4}u^2 \Rightarrow 4V_p^2 + 2V_q^2 = 3u^2 \quad (1)$

CLM $\Rightarrow 2\cancel{m}u = 2\cancel{m}V_p + \cancel{m}V_q \Rightarrow 2u = 2V_p + V_q \quad (2)$

$2V_p = 2u - V_q \Rightarrow 4V_p^2 = 4u^2 - 4uV_q + V_q^2$ sub in (1)

$(4u^2 - 4uV_q + V_q^2) + 2V_q^2 = 3u^2$

$u^2 - 4uV_q + 3V_q^2 = 0 \Rightarrow (u - 3V_q)(u - V_q) = 0$

$\therefore u = 3V_q \quad u = V_q$

$u \neq V_q \quad \therefore \underline{u = V_q}$

b) $e = \dfrac{V_q - V_p}{u}$

$(2) - 2u = 2V_p + u$

$\Rightarrow u = 2V_p \Rightarrow V_p = \frac{1}{2}u$

$\Rightarrow e = \dfrac{u - \frac{1}{2}u}{u} = \dfrac{\frac{1}{2}u}{u} \quad \therefore e = \frac{1}{2}$

6.

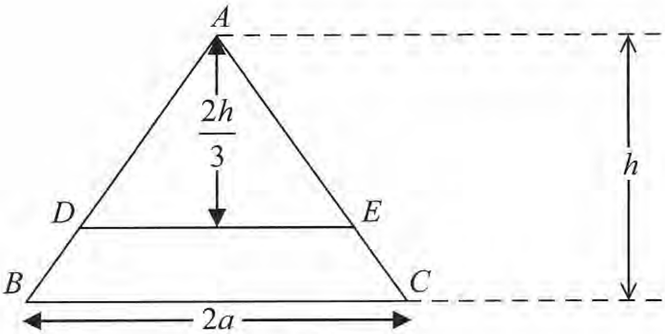


Figure 2

A uniform triangular lamina ABC of mass M is such that $AB = AC$, $BC = 2a$ and the distance of A from BC is h. A line, parallel to BC and at a distance $\frac{2h}{3}$ from A, cuts AB at D and cuts AC at E, as shown in Figure 2.

It is given that the mass of the trapezium $BCED$ is $\frac{5M}{9}$.

(a) Show that the centre of mass of the trapezium $BCED$ is $\frac{7h}{45}$ from BC.

(5)

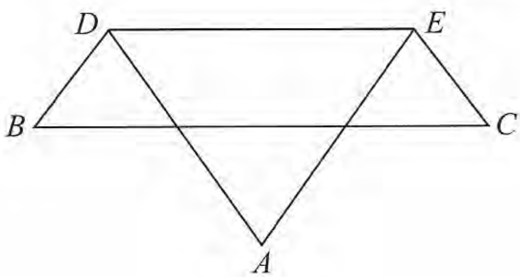


Figure 3

The portion ADE of the lamina is folded through 180° about DE to form the folded lamina shown in Figure 3.

(b) Find the distance of the centre of mass of the folded lamina from BC.

(4)

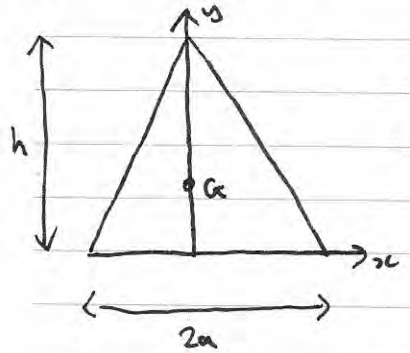
The folded lamina is freely suspended from D and hangs in equilibrium. The angle between DE and the downward vertical is α.

(c) Find $\tan \alpha$ in terms of a and h.

(4)

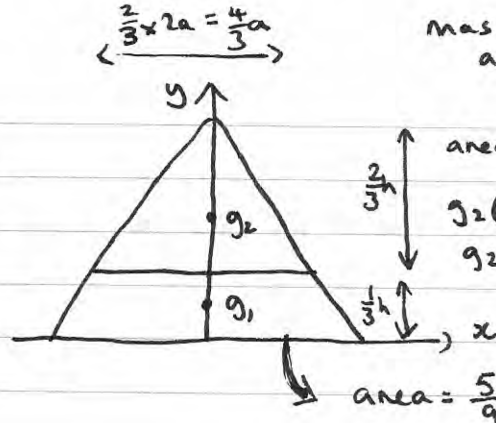
Question 6 continued

mass per unit area = k.



area = ah

$G(0, \frac{1}{3}h)$



area $= \frac{2}{3}a \times \frac{2}{3}h = \frac{4}{9}ah$

$g_2(0, \frac{1}{3}h + \frac{2}{9}h)$

$g_2(0, \frac{5}{9}h)$

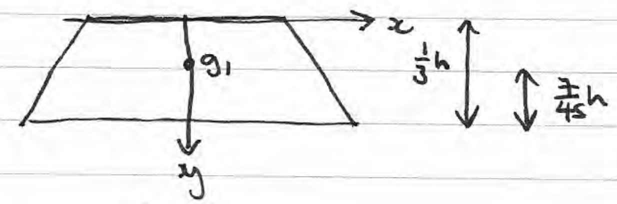
area $= \frac{5}{9}ah$

$g_1(0, \bar{y})$

$\curvearrowleft x \quad \frac{5}{9}\cancel{ak}\cancel{kg} \times \bar{y} + \frac{4}{9}\cancel{ak}\cancel{kg} \times \frac{5}{9}h = \cancel{ak}\cancel{kg} \times \frac{1}{3}h$

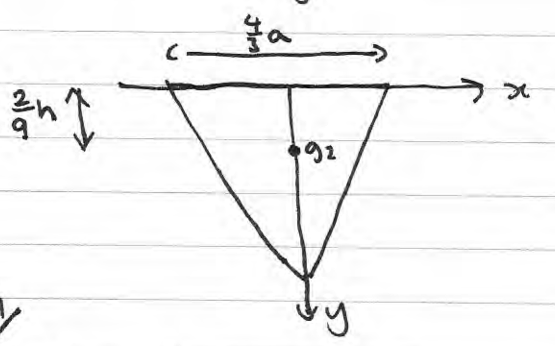
$\Rightarrow \frac{5}{9}\bar{y} + \frac{20}{81}h = \frac{27}{81}h \Rightarrow \frac{45}{\cancel{81}}\bar{y} = \frac{7}{\cancel{81}}h \quad \therefore \bar{y} = \frac{7}{45}h$ #

b)

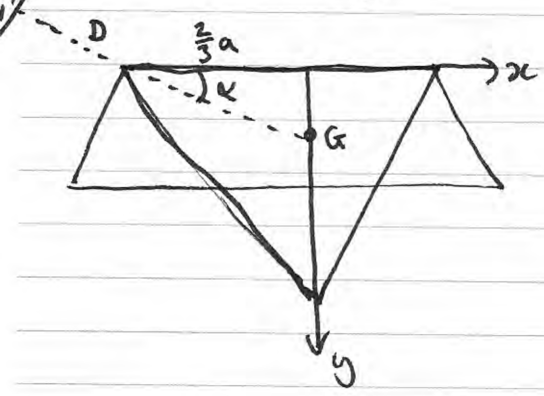


$\updownarrow = \frac{1}{3}h - \frac{7}{45}h = \frac{8}{45}h$

area $= \frac{5}{9}ah \quad g_1(0, \frac{8}{45}h)$



area $= \frac{4}{9}ah \quad g_2(0, \frac{2}{9}h)$



area = ah (combined!) $G(0, \bar{y})$

$\curvearrowleft x \quad \frac{5}{9}\cancel{ak}\cancel{kg} \times \frac{8}{45}h + \frac{4}{9}\cancel{ak}\cancel{kg} \times \frac{2}{9}h = \cancel{akg} \times \bar{y}$

$\Rightarrow \frac{8}{81}h + \frac{8}{81}h = \bar{y} \quad \therefore \bar{y} = \frac{16}{81}h$

from $BC = \frac{1}{3}h - \frac{16}{81}h = \frac{11}{81}h$

c) $\tan\alpha = \dfrac{\frac{16}{81}h}{\frac{2}{3}a} = \dfrac{\frac{16}{\cancel{81}}h}{\frac{54}{\cancel{81}}a} = \frac{16}{54}\frac{h}{a} \quad \therefore \tan\alpha = \frac{8h}{27a}$

Q6

7.

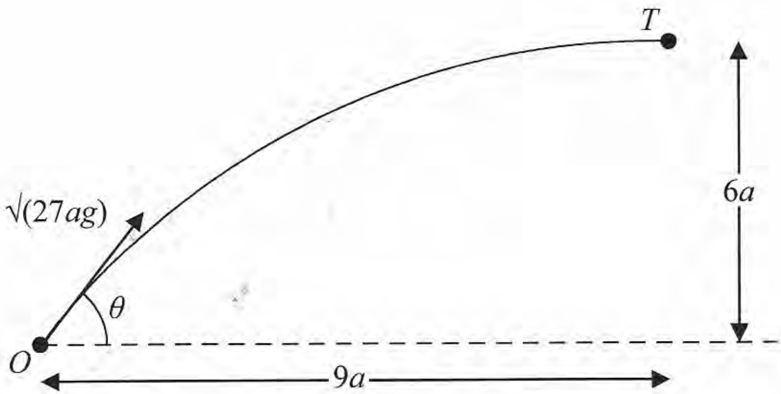


Figure 4

A small ball is projected from a fixed point O so as to hit a target T which is at a horizontal distance $9a$ from O and at a height $6a$ above the level of O. The ball is projected with speed $\sqrt{(27ag)}$ at an angle θ to the horizontal, as shown in Figure 4. The ball is modelled as a particle moving freely under gravity.

(a) Show that $\tan^2 \theta - 6 \tan \theta + 5 = 0$

(7)

The two possible angles of projection are θ_1 and θ_2, where $\theta_1 > \theta_2$.

(b) Find $\tan \theta_1$ and $\tan \theta_2$.

(3)

The particle is projected at the larger angle θ_1.

(c) Show that the time of flight from O to T is $\sqrt{\left(\frac{78a}{g}\right)}$.

(3)

(d) Find the speed of the particle immediately before it hits T.

(3)

$\vec{H}$ $\quad Vel = \sqrt{(27ag)}\cos\theta \quad \Rightarrow \quad t = \dfrac{9a}{\sqrt{27ag}\cos\theta}$

$\quad dist = 9a$

$V\uparrow$ $\quad s = 6a \qquad\qquad S = ut + \frac{1}{2}at^2$

$u = \sqrt{27ag}\sin\theta$

v

$a = -g$

$t = \dfrac{9a}{\sqrt{27ag}\cos\theta}$

$$6a = \frac{9a\sqrt{27ag}\sin\theta}{\sqrt{27ag}\cos\theta} - \frac{1}{2}g\left(\frac{9a}{\sqrt{27ag}\cos\theta}\right)^2$$

$$6\cancel{a} = 9\cancel{a}\tan\theta - \frac{1}{2}\cancel{g}\left(\frac{81a^{\cancel{2}}}{27\cancel{a}\cancel{g}\cos^2\theta}\right)$$

$$\Rightarrow 6 = 9\tan\theta - \frac{3}{2}\sec^2\theta$$

$$\Rightarrow 6 = 9\tan\theta - \frac{3}{2}(\tan^2\theta + 1)$$

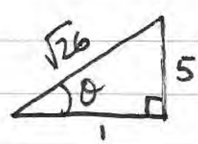
$$\Rightarrow \frac{3}{2}\tan^2\theta - 9\tan\theta + \frac{15}{2} = 0$$

$$\therefore \tan^2\theta - 6\tan\theta + 5 = 0 \quad \#$$

b) $(\tan\theta - 5)(\tan\theta - 1) = 0 \Rightarrow \tan\theta_1 = 5 \quad \tan\theta_2 = 1 \quad \theta_1 = 78.7$

$\theta_2 = 45$

c) $\tan\theta = \frac{5}{1}$



$\cos\theta = \frac{1}{\sqrt{26}}$

$$\Rightarrow t = \frac{9a}{\sqrt{\frac{27ag}{26}}} = \sqrt{\frac{81a^{\cancel{2}} \times 26}{27\cancel{a}g}} = \sqrt{\frac{78a}{g}}$$

d) $V_v\uparrow \qquad v^2 = u^2 + 2as \Rightarrow V\uparrow^2 = 27ag\sin^2\theta - 2g \times 6a$

$$\Rightarrow V^2\uparrow = 27ag\sin^2\theta - 12ga$$

$$\text{Speed}^2 = V_v^2\uparrow + \vec{V_h}^2$$

$$\text{speed}^2 = 27ag\sin^2\theta - 12ga + 27ag\cos^2\theta = 27ag - 12ga$$

$$\therefore \text{Speed}^2 = 15ag \qquad \therefore \text{Speed} = \sqrt{15ag}$$