M2 JUNE 2009

1) $Im = m(v - u)$

$5i - 3j = 0.25(v - 3i - 7j)$

$20i - 12j = v - 3i - 7j$

$23i - 5\underline{j} = v$

speed $= \sqrt{(23)^2 + (-5)^2} = \underline{23.5 \text{ ms}^{-1}}$

$x \underset{i}{\overset{d}{\rightleftarrows}} v \underset{i}{\overset{d}{\rightleftarrows}} a$

2a) $v = 8t - t^2$

v_{max} when $a = 0$

$\frac{dv}{dt} = a = 8 - 2t$

$8 - 2t = 0$

$\underline{t = 4}$

$v = 8(4) - (4)^2 = \underline{16 \text{ ms}^{-1}}$

b) $\int v \, dt = x$

$x = \int 8t - t^2 \, dt = 4t^2 - \frac{1}{3}t^3 + c$

when $t = 0$ $x = 0$ $\therefore c = 0$

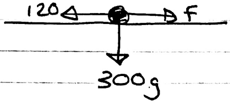
$x = 4t^2 - \frac{1}{3}t^3$

when $x = 0$

$0 = t(4t - \frac{1}{3}t^2)$

$t = 0$ or $4t = \frac{1}{3}t^2$ $\underline{\underline{t = 12}}$

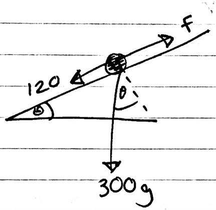
3a)



$\rightarrow^{+}$ $F = 120$

$P = F \times V = 120 \times 10 = \underline{1200\,W}$

b)



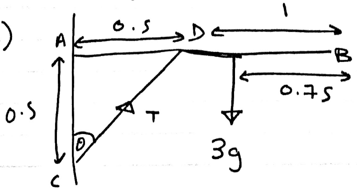
$\sin\theta = \frac{1}{14}$

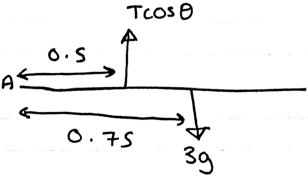
$\nearrow^{+}$ $F - 120 - 300g\sin\theta = 0$

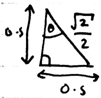
$F = 120 + 300g \times \frac{1}{14} = 210 + 120 = 330$

$\frac{P}{F} = V = \frac{1200}{330} = \underline{\underline{3.6\,ms^{-1}}}$

4a)







$\cos\theta = \frac{\sqrt{2}}{2}$

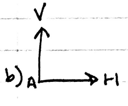
$\circlearrowright A: \ -(0.5 \times T\cos\theta) + (0.75 \times 3g) = 0$

$= -0.5 \times \frac{\sqrt{2}}{2} T + \frac{9}{4}g = 0$

$\frac{\sqrt{2}}{4} T = \frac{9}{4} g$

$T = \frac{9g}{\sqrt{2}} = \underline{62.4 \text{ N}}$

b)

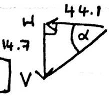


$\uparrow^{+} \ V + T\cos\theta = 3g$

$V = 3g - \frac{9g}{\sqrt{2}} \times \frac{\sqrt{2}}{2} = \underline{-14.7 \text{ N}}$

$\rightarrow^{+} \ H = -T\sin\theta = -\frac{9g}{\sqrt{2}} \times \frac{\sqrt{2}}{2} = \underline{-44.1 \text{ N}}$

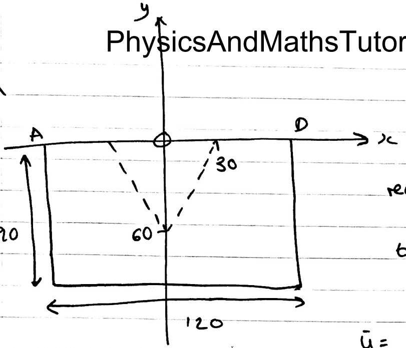
$\therefore$ Force $\sqrt{44.1^2 + 14.7^2} = \boxed{46.5 \text{ N}}$



$\alpha = \tan^{-1}\left(\frac{14.7}{44.1}\right) = \boxed{18.4^\circ \text{ to downwards horizontal}}$

AD $\bar{y}$

5a

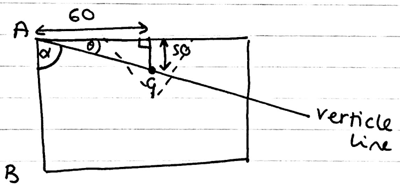


	m	y
rectangle	10800	−45
triangle	−1800	−20

$$\bar{y} = \frac{\Sigma my}{\Sigma m} = \frac{-450000}{9000} = -50$$

distance $\bar{y}$ from AD : $|-50| = \underline{\underline{50cm}}$

b)

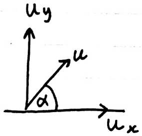


~~ ~~ $90 - \theta = \alpha$

$\theta = \tan^{-1}\left(\frac{50}{60}\right) = 39.8$

$90 - 39.8 = \underline{\underline{50.2^\circ}}$

6a)



$\overset{+}{\uparrow}$ $s = 2$

$u_y = u\sin\alpha$

$v =$

$a = -9.8$

$t = t$

$\xrightarrow{+}$ $s = 10$

$u_x = u\cos\alpha$

$v =$

$a = 0$

$t = t$

$s = ut + \frac{1}{2}at^2$

$s = u\sin\alpha t - \frac{1}{2}gt^2$

$s = ut + \frac{1}{2}at^2$

$s = u\cos\alpha t$

$u\sin\alpha t - \frac{1}{2}gt^2 = 2 \ldots (2)$

$10 = u\cos\alpha t$

$\frac{10}{u\cos\alpha} = t \ldots (1)$

sub (1) into (2)

$$u\sin\alpha\left(\frac{10}{u\cos\alpha}\right) - \frac{1}{2}g\left(\frac{10}{u\cos\alpha}\right)^2 = 2$$

$$\frac{10u\sin\alpha}{u\cos\alpha} - \frac{1}{2}g\frac{100}{u^2\cos^2\alpha} = 2$$

$$10\tan\alpha - \frac{50g}{u^2\cos^2\alpha} = 2$$

b) $$\frac{50g}{u^2\cos^2\alpha} = 10\tan\alpha - 2$$

$$\frac{50g}{u^2 \times \frac{1}{2}} = 10 - 2$$

$$50g = +\frac{8}{2}u^2$$

$$u = \sqrt{\frac{50g}{\ldots}} = \underline{\underline{11.1}}$$

6b) conservation of Energy

$$\frac{1}{2}mu^2 = mg\Delta h + \frac{1}{2}mv^2$$

$$\frac{1}{2}u^2 = g\Delta h + \frac{1}{2}v^2$$

$$\frac{1}{2}(11.1)^2 - 9.8 \times 2 = \frac{1}{2}v^2$$

$$41.65 = \frac{1}{2}v^2$$

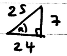
$$v = \sqrt{83.3} = \underline{9.1\ ms^{-1}}$$

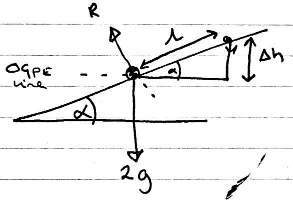
7a)

Initial energy − workdone = final

workdone = friction × distance

$\tan\alpha = \frac{7}{24}$





$\uparrow$ $R = 2g\cos\alpha$

$= \frac{48}{25}g$

friction $= \mu R$

$= \frac{1}{8} \times \frac{48}{25}g$

$= \frac{6}{25}g$

Initial $= \frac{1}{2} \times 2 \times 14^2 = 196$

Final $=$ GPE $= 2 \times 9.8 \times l\sin\alpha = l\frac{14}{25}g$

$$196 - \frac{6}{25}gl = \frac{14}{25}gl$$

$$196 = \frac{4}{5}gl$$

$$l = \frac{196 \times 5}{4 \times 9} = \underline{\underline{25}} \qquad XY = l \therefore XY = 25$$

b) I - wd = final

$X \rightarrow Y \rightarrow X$

$d = 2 \times 25 = 50$

$\Delta GPE = 0$

$\frac{1}{2} m 14^2 - 50 \times \mu R = \frac{1}{2} m v^2$

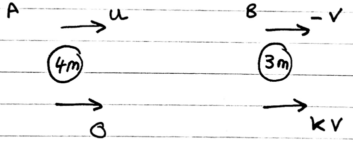
$\frac{1}{2} \times 2 \times 14^2 - 50 \times \frac{1}{8} \times \frac{48}{25} g = \frac{1}{2} \times 2 v^2$

$196 - 12g = v^2$

$v = \sqrt{78.4}$

$v = 8.85 \text{ ms}^{-1}$

8a)



$\frac{kv - 0}{u - -v} = \frac{3}{4} \Rightarrow kv = \frac{3}{4}u + \frac{3}{4}v \ldots (1)$

$4\cancel{m}u - 3\cancel{m}v = 3\cancel{m}kv \ldots (2)$

sub (1) into (2)

$4u - 3v = 3\left(\frac{3}{4}u + \frac{3}{4}v\right)$

$4u - \frac{9}{4}u = \frac{9}{4}v + 3v$

$\frac{7}{4}u = \frac{21}{4}v \qquad \underline{\underline{u = 3v}}$

b) $u = 3v$

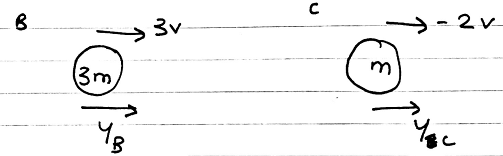
$kv = \frac{3}{4}(u+v)$

$kv = \frac{3}{4}(3v+v)$

$kv = \frac{12v}{4}$

$\underline{\underline{k = 3}}$

c)



$\frac{y_C - y_B}{3v+2v} = e \Rightarrow y_C - y_B = 5ve$

$y_C = 5ve + y_B \cdots (1)$

$3v \times 3m - 2v \times m = y_C m + 3y_B m$

$9v - 2v = y_C + 3y_B$

$7v = y_C + 3y_B \quad \cdots (2)$

sub (1) into (2) $\quad 7v = 5ve + y_B + 3y_B$

$7v - 5ve = 4y_B$

$y_B = \frac{v}{4}(7 - 5e)$

as $e \leq 1$

$\frac{v}{4}(7 - 5e) > 0$

$\therefore y_B \rightarrow$ +ve direction

$\therefore$ no further collisions with A