1. Two uniform rods AB and BC are rigidly joined at B so that $\angle ABC = 90°$. Rod AB has length 0.5 m and mass 2 kg. Rod BC has length 2 m and mass 3 kg. The centre of mass of the framework of the two rods is at G.

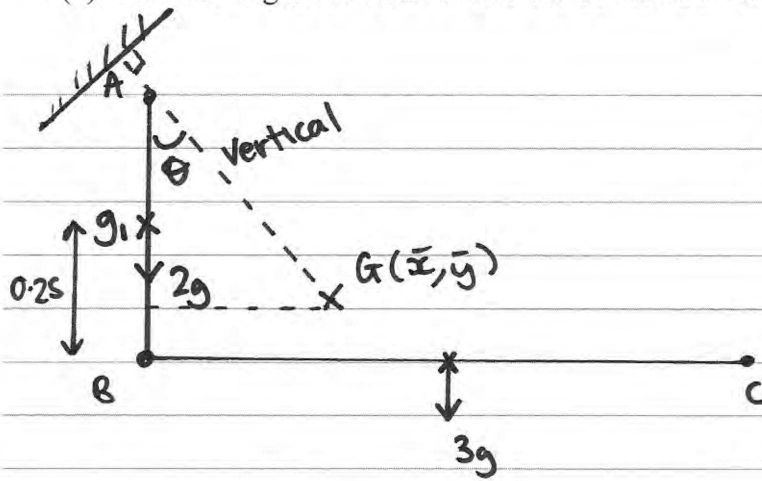
(a) Find the distance of G from BC.

(2)

The distance of G from AB is 0.6 m.
The framework is suspended from A and hangs freely in equilibrium.

(b) Find the angle between AB and the downward vertical at A.

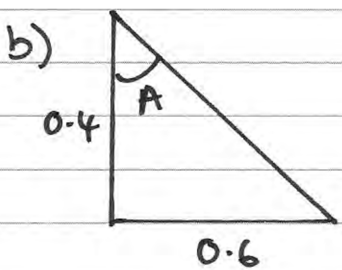
(3)



BC ↺ $2\cancel{g}\times 0.25 = 5\cancel{g}\times\bar{y}$

$\therefore\ \bar{y} = \underline{0.1\text{ m}}$

b)



$A = \tan^{-1}\left(\frac{0.6}{0.4}\right) \Rightarrow A = 56.3°$

2. A lorry of mass 1800 kg travels along a straight horizontal road. The lorry's engine is working at a constant rate of 30 kW. When the lorry's speed is 20 m s^{-1}, its acceleration is 0.4 m s^{-2}. The magnitude of the resistance to the motion of the lorry is R newtons.

(a) Find the value of R.

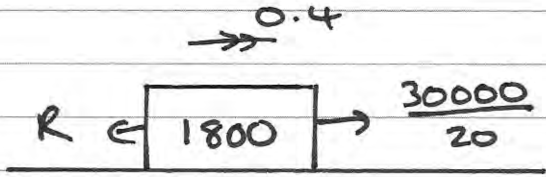
(4)

The lorry now travels up a straight road which is inclined at an angle α to the horizontal, where $\sin \alpha = \frac{1}{12}$. The magnitude of the non-gravitational resistance to motion is R newtons. The lorry travels at a constant speed of 20 m s^{-1}.

(b) Find the new rate of working of the lorry's engine.

(5)

a)

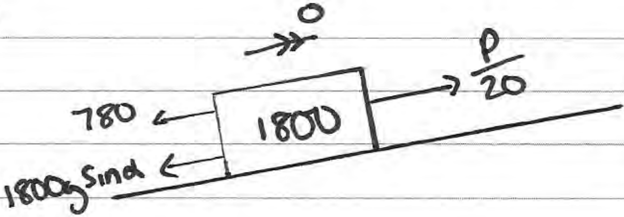


$Rf = ma$

$1500 - R = 1800 \times 0.4$

$\therefore R = \underline{780N}$

b)



$\frac{P}{20} = 780 + 1800g\left(\frac{1}{12}\right)$

$\frac{P}{20} = 2250$

$\therefore P = \underline{45000W} \, (45kW)$

3.

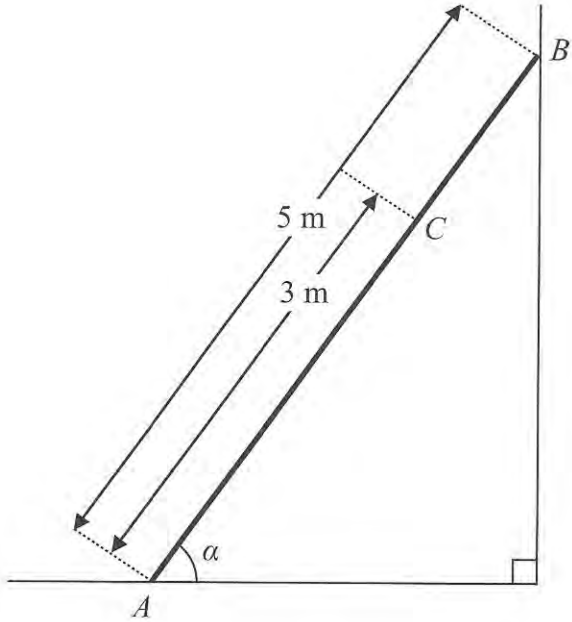
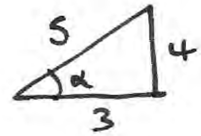


Figure 1

A ladder, of length 5 m and mass 18 kg, has one end A resting on rough horizontal ground and its other end B resting against a smooth vertical wall. The ladder lies in a vertical plane perpendicular to the wall and makes an angle α with the horizontal ground, where $\tan\alpha = \frac{4}{3}$, as shown in Figure 1. The coefficient of friction between the ladder and the ground is μ. A woman of mass 60 kg stands on the ladder at the point C, where $AC = 3$ m. The ladder is on the point of slipping. The ladder is modelled as a uniform rod and the woman as a particle.

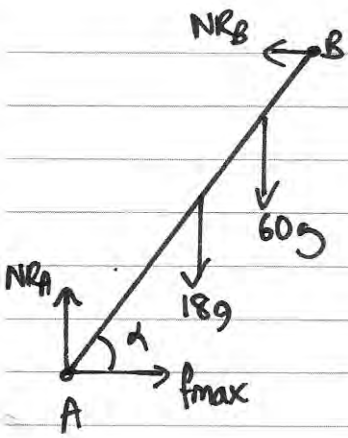
Find the value of μ. **(9)**

$\tan\alpha = \frac{4}{3}$



$\therefore$ $\sin\alpha = \frac{4}{5}$

$\cos\alpha = \frac{3}{5}$



$R(\uparrow) = 0 \Rightarrow NR_A = 78g$

$f_{max} = \mu \times NR_A = 78g\mu$

$R(\rightarrow) = 0 \Rightarrow NR_B = 78g\mu.$

$A \circlearrowleft$ $18g \times 2.5\cos\alpha + 60g \times 3\cos\alpha = 78g\mu \times \sin\alpha$

$\Rightarrow 27\cancel{g} + 108\cancel{g} = 312g\mu$

$\therefore \mu = \frac{45}{104}$

4. At time t seconds the velocity of a particle P is $[(4t-5)\mathbf{i}+3\mathbf{j}]\ \mathrm{m\,s^{-1}}$. When $t=0$, the position vector of P is $(2\mathbf{i}+5\mathbf{j})$ m, relative to a fixed origin O.

(a) Find the value of t when the velocity of P is parallel to the vector $\mathbf{j}$.

(1)

(b) Find an expression for the position vector of P at time t seconds.

(4)

A second particle Q moves with constant velocity $(-2\mathbf{i}+c\mathbf{j})\ \mathrm{m\,s^{-1}}$. When $t=0$, the position vector of Q is $(11\mathbf{i}+2\mathbf{j})$ m. The particles P and Q collide at the point with position vector $(d\mathbf{i}+14\mathbf{j})$ m.

(c) Find

(i) the value of c,

(ii) the value of d.

(5)

a) Parallel to $j \Rightarrow i=0 \quad \therefore 4t-5=0 \Rightarrow t=1.25$

b) $s=\int v\,dt = \int \begin{pmatrix} 4t-5 \\ 3 \end{pmatrix} dt = \begin{pmatrix} 2t^2-5t+C_1 \\ 3t+C_2 \end{pmatrix}$

when $t=0$ $\quad s=\begin{pmatrix} 2 \\ 5 \end{pmatrix} \therefore \begin{matrix} C_1=2 \\ C_2=5 \end{matrix} \Rightarrow s=\begin{pmatrix} 2t^2-5t+2 \\ 3t+5 \end{pmatrix}$

c) $q=\begin{pmatrix} 11 \\ 2 \end{pmatrix}+t\begin{pmatrix} -2 \\ c \end{pmatrix}=\begin{pmatrix} 11-2t \\ 2+ct \end{pmatrix}$

$\therefore \begin{pmatrix} 2t^2-5t+2 \\ 3t+5 \end{pmatrix}=\begin{pmatrix} d \\ 14 \end{pmatrix}=\begin{pmatrix} 11-2t \\ 2+ct \end{pmatrix}$

$\Rightarrow 3t+5=14 \Rightarrow 3t=9 \Rightarrow t=3$

$\Rightarrow \begin{pmatrix} d \\ 14 \end{pmatrix}=\begin{pmatrix} 5 \\ 2+3c \end{pmatrix} \quad \therefore d=5$

$14=2+3c \Rightarrow 3c=12 \Rightarrow c=4$

$\underline{c=4}, \underline{d=5}$

5. The point A lies on a rough plane inclined at an angle θ to the horizontal, where $\sin\theta = \frac{24}{25}$.

A particle P is projected from A, up a line of greatest slope of the plane, with speed U m s^{-1}. The mass of P is 2 kg and the coefficient of friction between P and the plane is $\frac{5}{12}$. The particle comes to instantaneous rest at the point B on the plane, where $AB = 1.5$ m. It then moves back down the plane to A.

(a) Find the work done against friction as P moves from A to B.

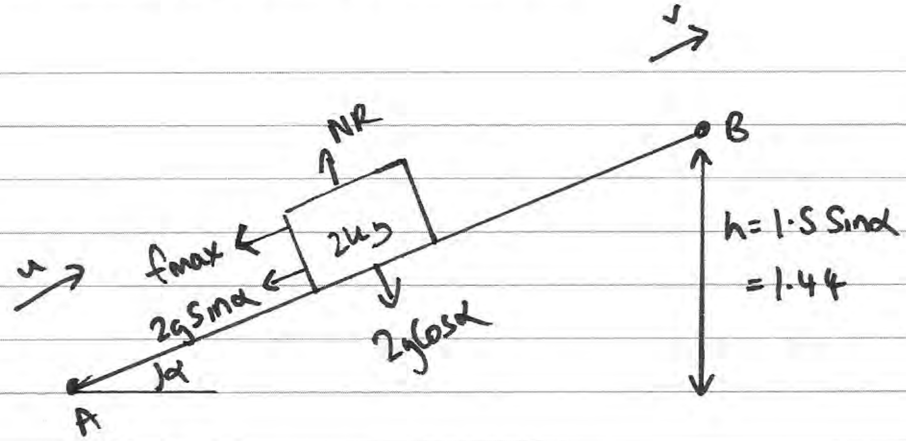
(4)

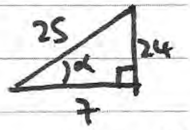
(b) Use the work-energy principle to find the value of U.

(4)

(c) Find the speed of P when it returns to A.

(3)





$\cos\alpha = \frac{7}{25}$

$\sin\alpha = \frac{24}{25}$

$\uparrow RF = 0 \Rightarrow NR = 2g\cos\alpha = 5.488$

$\mu = \frac{5}{12} \Rightarrow f_{max} = \frac{343}{150}$

$\therefore$ wd against friction $= \frac{343}{150} \times 1.5 = 3.43$J

b) KE_A − Wd against friction $= PE_B$

$\cancel{\tfrac{1}{2}}(\cancel{2})u^2 - 3.43 = 2g(1.44)$ $\therefore u^2 = 31.654\ldots$

$u = \underline{5.63}$ (3sf)

c) PE_B − Wd against friction $= KE_A$

$2g(1.44) - 3.43 = \cancel{\tfrac{1}{2}}(\cancel{2})v^2$ $\Rightarrow v^2 = 24.794\ldots$

$v = \underline{4.98\ (3sf)}$

6.

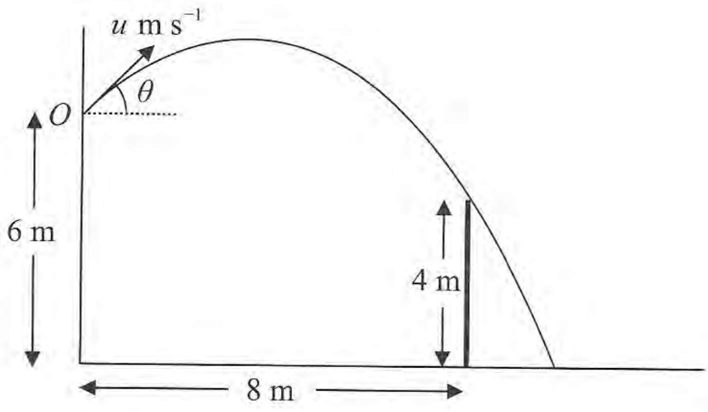


Figure 2

A ball is thrown from a point O, which is 6 m above horizontal ground. The ball is projected with speed u m s^{-1} at an angle θ above the horizontal. There is a thin vertical post which is 4 m high and 8 m horizontally away from the vertical through O, as shown in Figure 2. The ball passes just above the top of the post 2 s after projection. The ball is modelled as a particle.

(a) Show that $\tan\theta = 2.2$

(5)

(b) Find the value of u.

(2)

The ball hits the ground T seconds after projection.

(c) Find the value of T.

(3)

Immediately before the ball hits the ground the direction of motion of the ball makes an angle α with the horizontal.

(d) Find α.

(5)

$\overrightarrow{H}$ $\quad vel = \frac{dist}{time} \Rightarrow u\cos\theta = \frac{8}{2} \Rightarrow u\cos\theta = 4$

$V\uparrow$ $\quad s = -2 \qquad s = ut + \frac{1}{2}at^2$

$u = u\sin\theta$

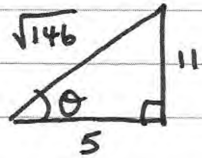
$v = \qquad -2 = 2u\sin\theta - 19.6 \Rightarrow u\sin\theta = 8.8$

$a = -9.8$

$t = 2$

$\frac{u\sin\theta}{u\cos\theta} = \frac{8.8}{4} \Rightarrow \tan\theta = 2.2$ #

b) $\tan\theta = 2.2 = \frac{11}{5}$



$\Rightarrow \cos\theta = \frac{5}{\sqrt{146}}$

$\sin\theta = \frac{11}{\sqrt{146}}$

$u\cos\theta = 4 \Rightarrow u \times \frac{5}{\sqrt{146}} = 4 \quad \therefore u = \frac{4\sqrt{146}}{5}$

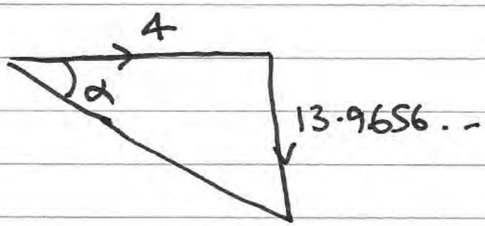
c) $s = -6$
$u = u\sin\theta = \frac{4\cancel{\sqrt{146}}}{5} \times \frac{11}{\cancel{\sqrt{146}}} = \frac{44}{5} = 8.8$
$v =$
$a = -9.8$
t

$s = ut + \frac{1}{2}at^2 \Rightarrow -6 = 8.8t - 4.9t^2 \Rightarrow 4.9t^2 - 8.8t - 6 = 0$

$\Rightarrow t = \frac{8.8 \pm \sqrt{8.8^2 - 4(4.9)(-6)}}{9.8} \qquad t = 2.323\ldots$
$\qquad\qquad -0.527\ldots$

$\therefore T = \underline{2.32}$ (3sf)

d) $v = u + at \Rightarrow v = 8.8 - 9.8 \times 2.32\ldots = -13.9656\ldots$



$\Rightarrow \alpha = \tan^{-1}\left(\frac{13.96\ldots}{4}\right)$

$\therefore \alpha = 74°$ (2sf)
below horizontal.

7. A particle A of mass m is moving with speed u on a smooth horizontal floor when it collides directly with another particle B, of mass $3m$, which is at rest on the floor. The coefficient of restitution between the particles is e. The direction of motion of A is reversed by the collision.

(a) Find, in terms of e and u,

(i) the speed of A immediately after the collision,

(ii) the speed of B immediately after the collision.

(7)

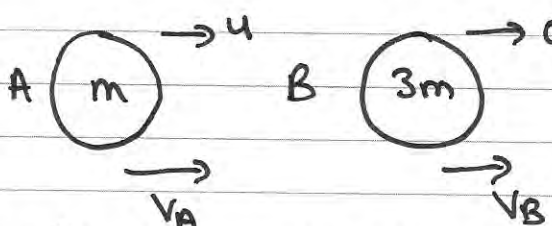
After being struck by A the particle B collides directly with another particle C, of mass $4m$, which is at rest on the floor. The coefficient of restitution between B and C is $2e$. Given that the direction of motion of B is reversed by this collision,

(b) find the range of possible values of e,

(6)

(c) determine whether there will be a second collision between A and B.

(3)



$e = \frac{V_B - V_A}{u}$

$V_A < 0$ $\quad V_A < V_B$

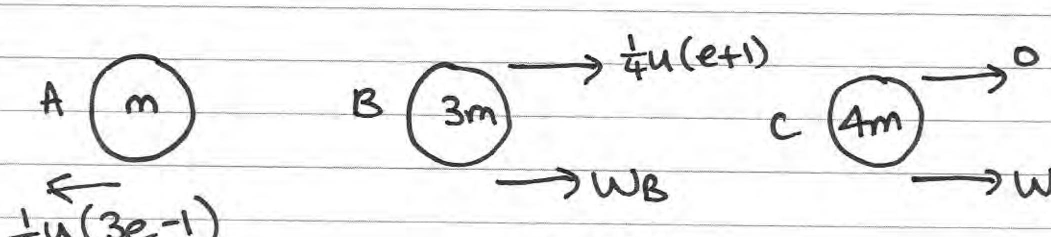
CLM $\Rightarrow$ $\cancel{m}u = \cancel{m}V_A + \cancel{m}3V_B \Rightarrow V_A = u - 3V_B$

$\Rightarrow e = \frac{V_B - u + 3V_B}{u} \Rightarrow eu = 4V_B - u \Rightarrow 4V_B = eu + u$

$\therefore V_B = \frac{1}{4}u(e+1) \qquad V_A = u - \frac{3}{4}u(e+1) = \frac{1}{4}u - \frac{3}{4}ue$

$\therefore V_A = \frac{1}{4}u(1-3e)$

b)



$W_B < 0$

$\therefore e > \frac{1}{3}$

$$2e = \frac{W_C - W_B}{\frac{1}{4}u(e+1)} \Rightarrow \frac{1}{2}eu(e+1) = W_C - W_B$$

$$\text{CLM} \Rightarrow \frac{3}{4}\cancel{m}u(e+1) = 3\cancel{m}W_B + 4\cancel{m}W_C$$

$$\therefore \quad 3u(e+1) = 12W_B + 16W_C \quad -$$

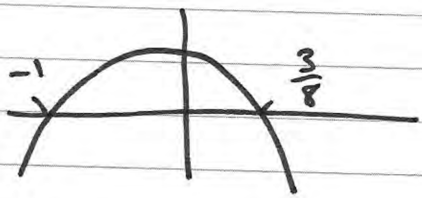
$$\text{i} \times 16 \quad 8eu(e+1) = -16W_B + 16W_C$$

$$u(3-8e)(e+1) = 28W_B$$

$$\Rightarrow \overrightarrow{W_B} = \frac{1}{28}u(3-8e)(e+1)$$

$$\text{Since } W_B < 0 \Rightarrow (3-8e)(e+1) < 0$$

$$\frac{3}{8} \qquad -1$$



$$\therefore \cancel{e < -1} \qquad e > \frac{3}{8}$$

$$e > \frac{3}{8} \quad e > \frac{1}{3} \quad \therefore e > \frac{3}{8}$$

c) $\overleftarrow{\text{speed } A} = \frac{1}{4}u(3e-1)$ $\qquad$ $\overleftarrow{\text{Speed } B} = \frac{1}{28}u(8e-3)(e+1)$

Collide if $\overleftarrow{\text{Speed } B} > \overleftarrow{\text{Speed } A}$

$$\frac{1}{28}\cancel{u}(8e-3)(e+1) > \frac{1}{4}\cancel{u}(3e-1)$$

$$\Rightarrow (8e-3)(e+1) > 7(3e-1)$$

$$\Rightarrow 8e^2 + 5e - 3 > 21e - 7$$

$$\Rightarrow 8e^2 - 16e + 4 > 0$$

$\Rightarrow \quad e^2 - 2e + \frac{1}{2} > 0$

$\Rightarrow \quad (e-1)^2 - \frac{1}{2} > 0 \qquad \Rightarrow (e-1)^2 > \frac{1}{2}$

$\Rightarrow \qquad -\frac{\sqrt{2}}{2} > e - 1 > +\frac{\sqrt{2}}{2}$

$\Rightarrow \qquad 1 - \frac{\sqrt{2}}{2} > e > 1 + \frac{\sqrt{2}}{2}$ (not possible!)

$\therefore \quad e < 1 - \frac{\sqrt{2}}{2}$ if they collide

$e < 0.29$ if they collide

but $e > \frac{3}{8}$, $e > 0.375$

$\therefore$ they do <u>not</u> collide.