M2 JAN11

1. A cyclist starts from rest and moves along a straight horizontal road. The combined mass of the cyclist and his cycle is 120 kg. The resistance to motion is modelled as a constant force of magnitude 32 N. The rate at which the cyclist works is 384 W. The cyclist accelerates until he reaches a constant speed of v m s^{-1}.

Find

(a) the value of v, **(3)**

(b) the acceleration of the cyclist at the instant when the speed is 9 m s^{-1}. **(3)**

$32 \leftarrow \boxed{120} \rightarrow \frac{384}{V}$

a) $\frac{384}{V} = 32 \Rightarrow V = \underline{12 \text{ ms}^{-1}}$

b) $\frac{384}{9} - 32 = 120a \Rightarrow a = \underline{\frac{4}{45} \text{ ms}^{-2}}$

2. A particle of mass 2 kg is moving with velocity $(5\mathbf{i}+\mathbf{j})$ m s^{-1} when it receives an impulse of $(-6\mathbf{i}+8\mathbf{j})$ N s. Find the kinetic energy of the particle immediately after receiving the impulse. **(5)**

Initial Mom $= 10i + 2j$

Impulse $= -6i + 8j$

final Mom $= 4i + 10j \Rightarrow V = 2i + 5j$

$KE = \frac{1}{2}mv^2 = \frac{1}{2}(2)(2^2 + 5^2) = \underline{29 \text{ J}}$

3. A particle moves along the x-axis. At time $t = 0$ the particle passes through the origin with speed 8 m s^{-1} in the positive x-direction. The acceleration of the particle at time t seconds, $t \geqslant 0$, is $(4t^3 - 12t)$ m s^{-2} in the positive x-direction.

Find

(a) the velocity of the particle at time t seconds, **(3)**

(b) the displacement of the particle from the origin at time t seconds, **(2)**

(c) the values of t at which the particle is instantaneously at rest. **(3)**

a) $a = 4t^3 - 12t$

$V = t^4 - 6t^2 + C \quad , t = 0, V = 8 \Rightarrow C = 8$

$V = t^4 - 6t^2 + 8$

b) $S = \frac{1}{5}t^5 - 2t^3 + 8t + C \qquad S = 0, t = 0 \Rightarrow C = 0$

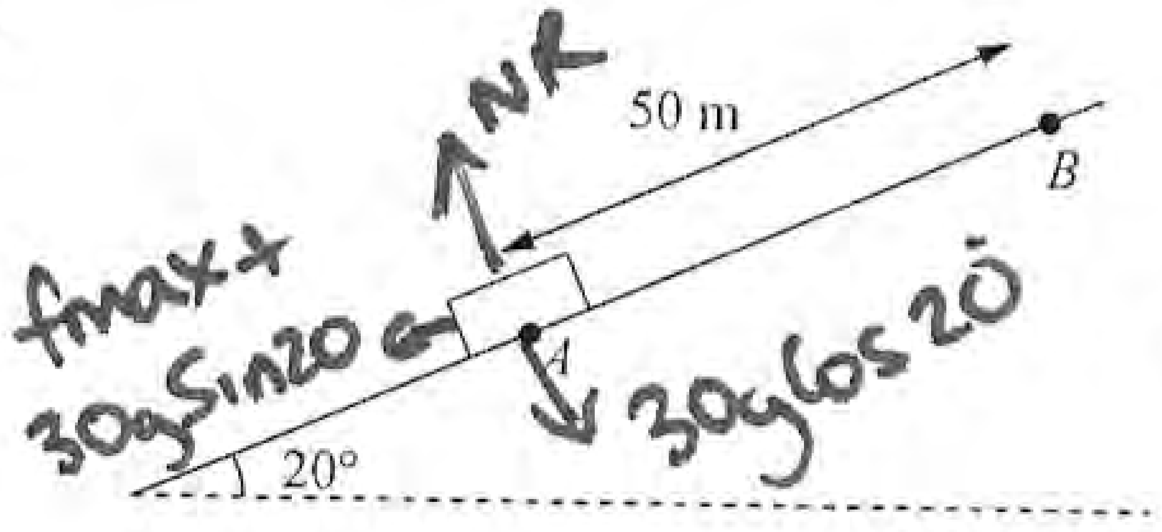
$S = \frac{1}{5}t^5 - 2t^3 + 8t$

c) $V = 0 \qquad t^4 - 6t^2 + 8 = 0$

$(t^2 - 4)(t^2 - 2) = 0$

$t = \sqrt{4} = \underline{2} \qquad t = \sqrt{2} = \underline{1.41 \text{ sec}}$

4. $\mu = \frac{1}{4}$



$h = 50 \sin 20$

Figure 1

A box of mass 30 kg is held at rest at point A on a rough inclined plane. The plane is inclined at 20° to the horizontal. Point B is 50 m from A up a line of greatest slope of the plane, as shown in Figure 1. The box is dragged from A to B by a force acting parallel to AB and then held at rest at B. The coefficient of friction between the box and the plane is $\frac{1}{4}$. Friction is the only non-gravitational resistive force acting on the box. Modelling the box as a particle,

(a) find the work done in dragging the box from A to B.
(6)

The box is released from rest at the point B and slides down the slope. Using the work-energy principle, or otherwise,

(b) find the speed of the box as it reaches A.
(5)

a) $f_{max} = \mu NR = \frac{1}{4}(30g \cos 20)$

$f_{max} = 69.067...$

wd against friction $= 69.067 \times 50 = 3453.4 J$

wd $= 169.62 \times 50 = 8481.1 J$

b) $KE_A = PE_B - WD_{\vec{AB}}$

$\frac{1}{2}mv^2 = 30g \times 50 \sin 20 - 3453.4$

$15v^2 = 1574.3...$ $\quad v = 10.24$

$v = 10.2 ms^{-1}$

The uniform L-shaped lamina $ABCDEF$, shown in Figure 2, has sides AB and FE parallel, and sides BC and ED parallel. The pairs of parallel sides are 9 cm apart. The points A, F, D and C lie on a straight line.

$AB = BC = 36$ cm, $FE = ED = 18$ cm. $\angle ABC = \angle FED = 90°$, and $\angle BCD = \angle EDF = \angle EFD = \angle BAC = 45°$.

(a) Find the distance of the centre of mass of the lamina from

(i) side AB,

(ii) side BC.
(7)

The lamina is freely suspended from A and hangs in equilibrium.

(b) Find, to the nearest degree, the size of the angle between AB and the vertical.
(3)

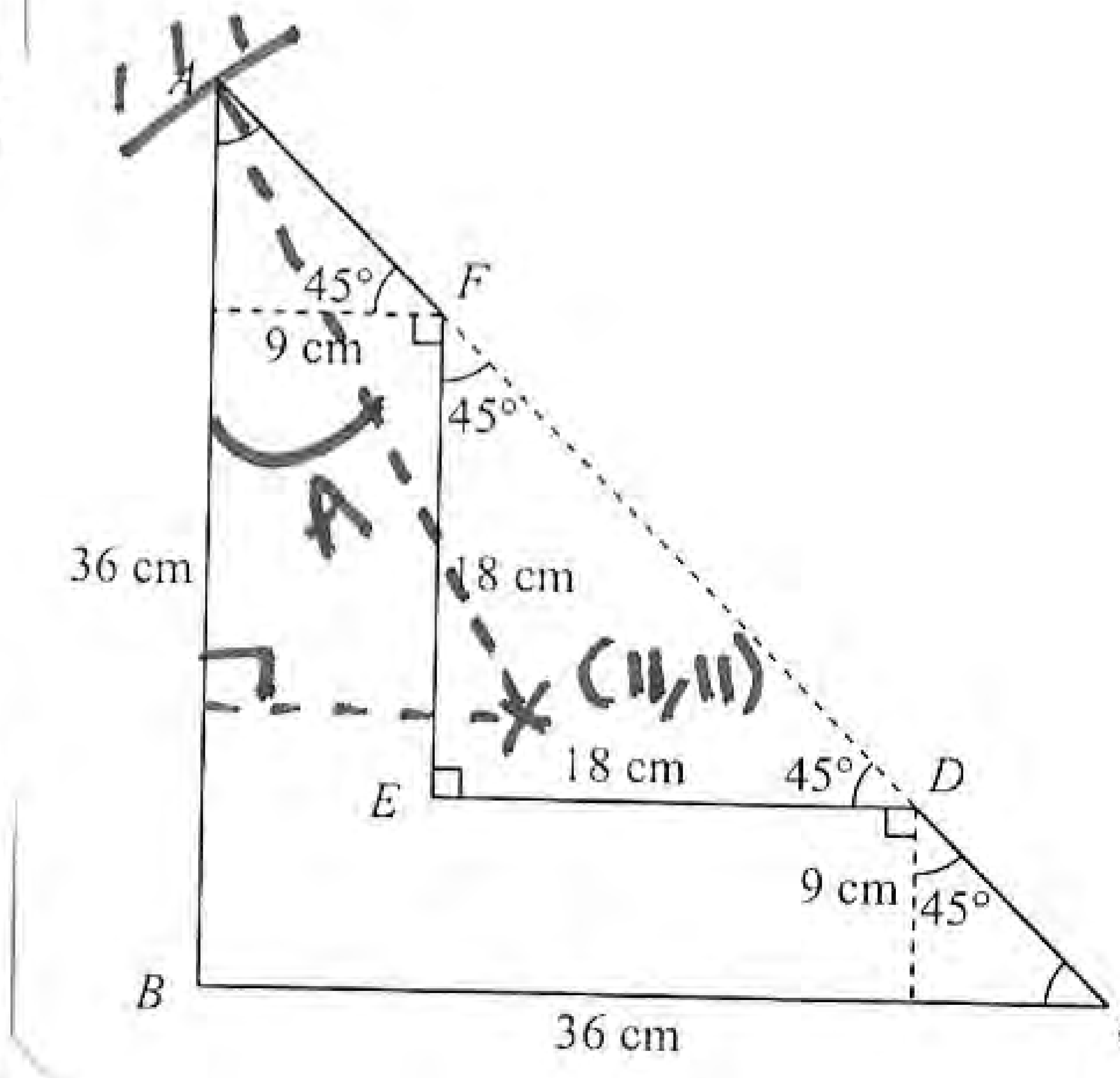
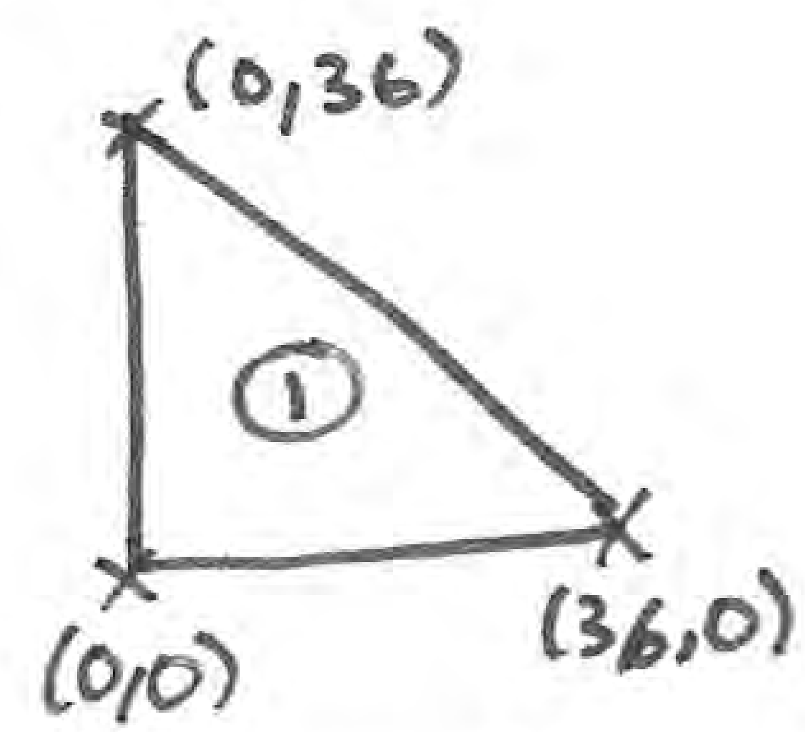


Figure 2



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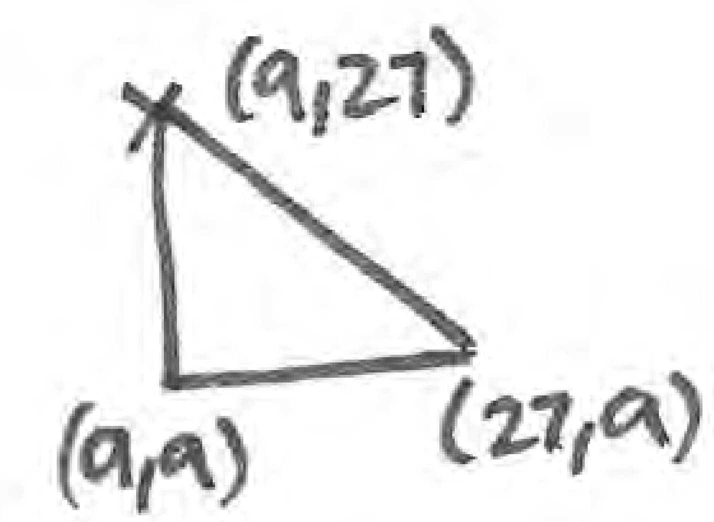


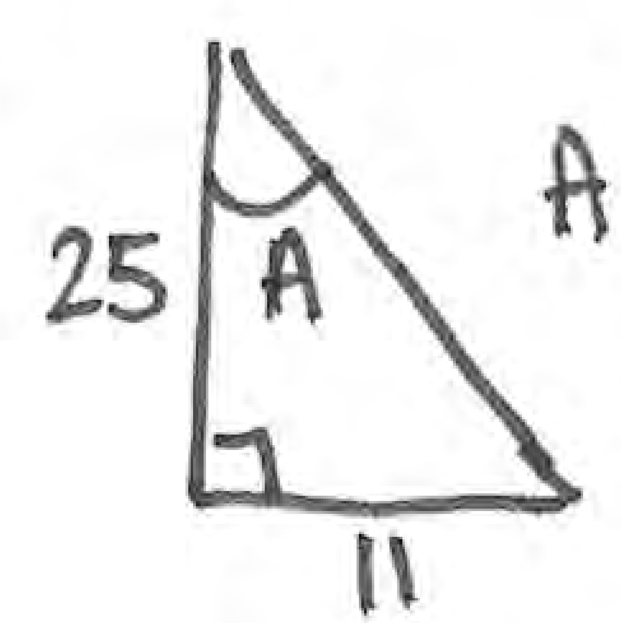
= Total

$\bar{x}_1 = \frac{0+0+36}{3} = 12$ $\quad \bar{x}_2 = \frac{9+9+27}{3} = 15$ $\quad$ Total $= 486\ (\bar{x}, \bar{y})$

$A_1 = \frac{36^2}{2} = 648$ $\quad A_2 = \frac{18^2}{2} = 162$

$648kg \times 12 + 162kg \times 15 = 486kg \times \bar{x}$ $\Rightarrow \bar{x} = 11cm$ (ii) $11cm$

b)



$A = \tan^{-1}\left(\frac{11}{25}\right) = 23.7°$

$A = 24°$

6. [*In this question, the unit vectors* **i** *and* **j** *are in a vertical plane,* **i** *being horizontal and* **j** *being vertically upwards.*]

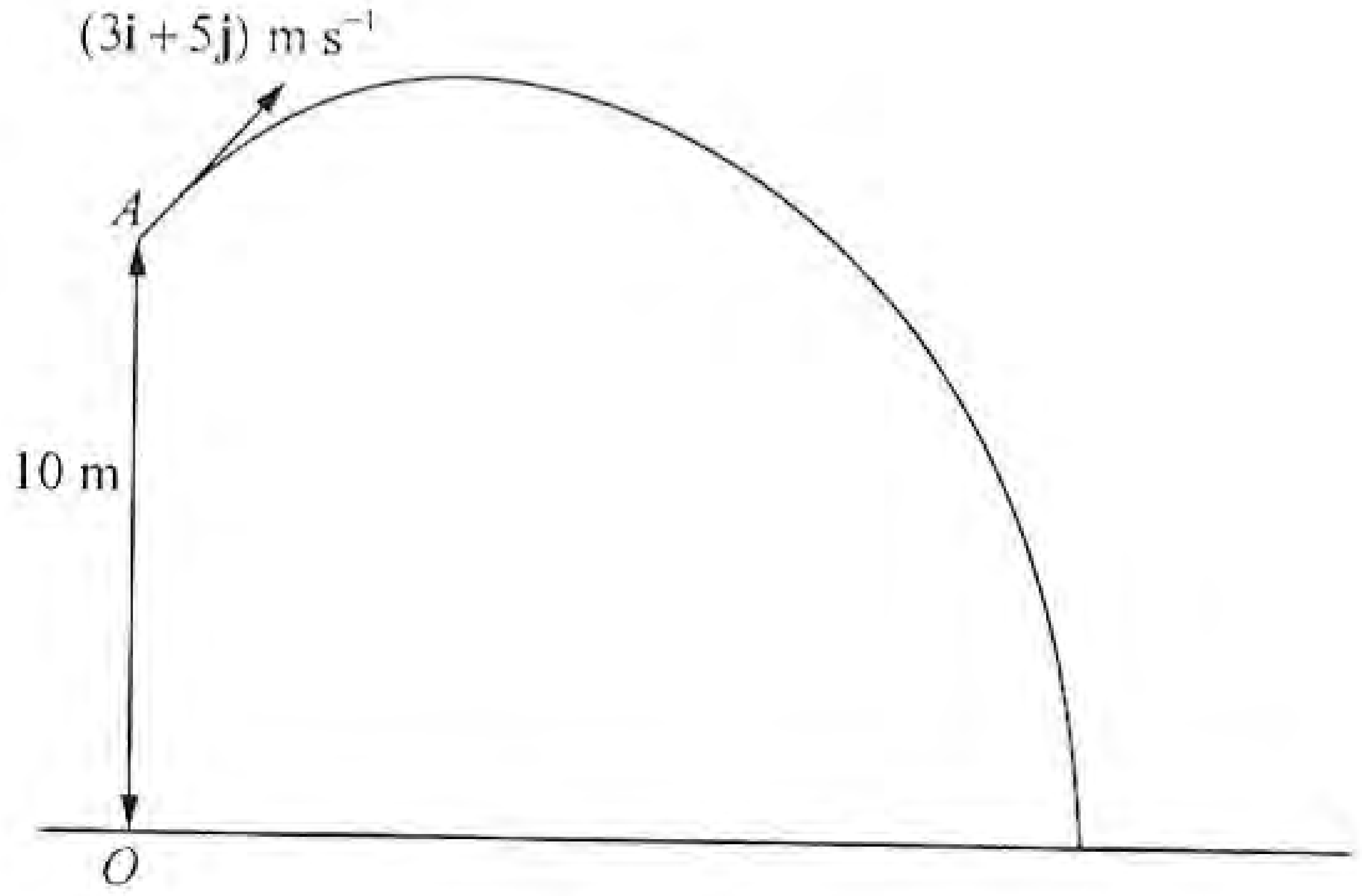


Figure 3

At time $t = 0$, a particle P is projected from the point A which has position vector $10\mathbf{j}$ metres with respect to a fixed origin O at ground level. The ground is horizontal. The velocity of projection of P is $(3\mathbf{i} + 5\mathbf{j})$ m s^{-1}, as shown in Figure 3. The particle moves freely under gravity and reaches the ground after T seconds.

(a) For $0 \leqslant t \leqslant T$, show that, with respect to O, the position vector, $\mathbf{r}$ metres, of P at time t seconds is given by

$$\mathbf{r} = 3t\mathbf{i} + (10 + 5t - 4.9t^2)\mathbf{j}$$

(3)

(b) Find the value of T.

(3)

(c) Find the velocity of P at time t seconds $(0 \leqslant t \leqslant T)$.

(2)

When P is at the point B, the direction of motion of P is 45° below the horizontal.

(d) Find the time taken for P to move from A to B.

(2)

(e) Find the speed of P as it passes through B.

(2)

a) $\overrightarrow{H}$ $V = 3$ $S = 3t$

$V\uparrow$ $u\uparrow = 5$ $y = 5t - \frac{1}{2}(9.8)t^2$

$a\uparrow = -9.8$ $y = 5t - 4.9t^2$

$S = y$ from ground $= 10 + 5t - 4.9t^2$

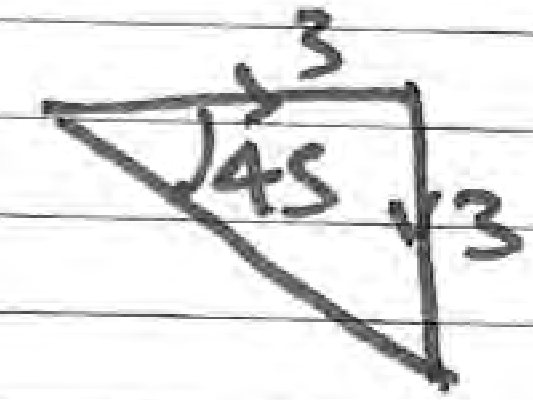
$r = 3t\mathbf{i} + (10 + 5t - 4.9t^2)\mathbf{j}$

b) $t = T$ $\mathbf{j} = 0$ $4.9t^2 - 5t - 10 = 0$

$t = \dfrac{5 + \sqrt{5^2 - 4(4.9)(-10)}}{9.8}$ $t = 2.03\text{ sec}$

c) $V = \dfrac{dr}{dt} = 3\mathbf{i} + (5 - 9.8t)\mathbf{j}$

d)



$5 - 9.8t = -3$ $t = \dfrac{8}{9.8} = 0.82$

e) $\sqrt{3^2 + 3^2} = 3\sqrt{2} = 4.24\text{ ms}^{-1}$

7.

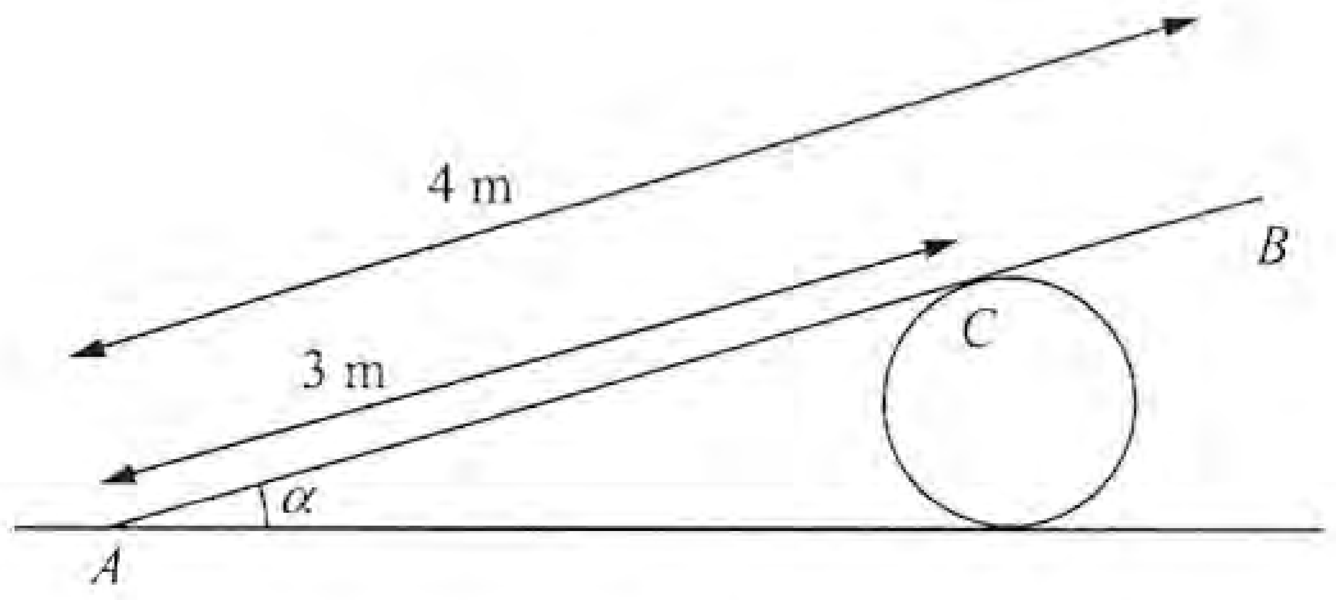
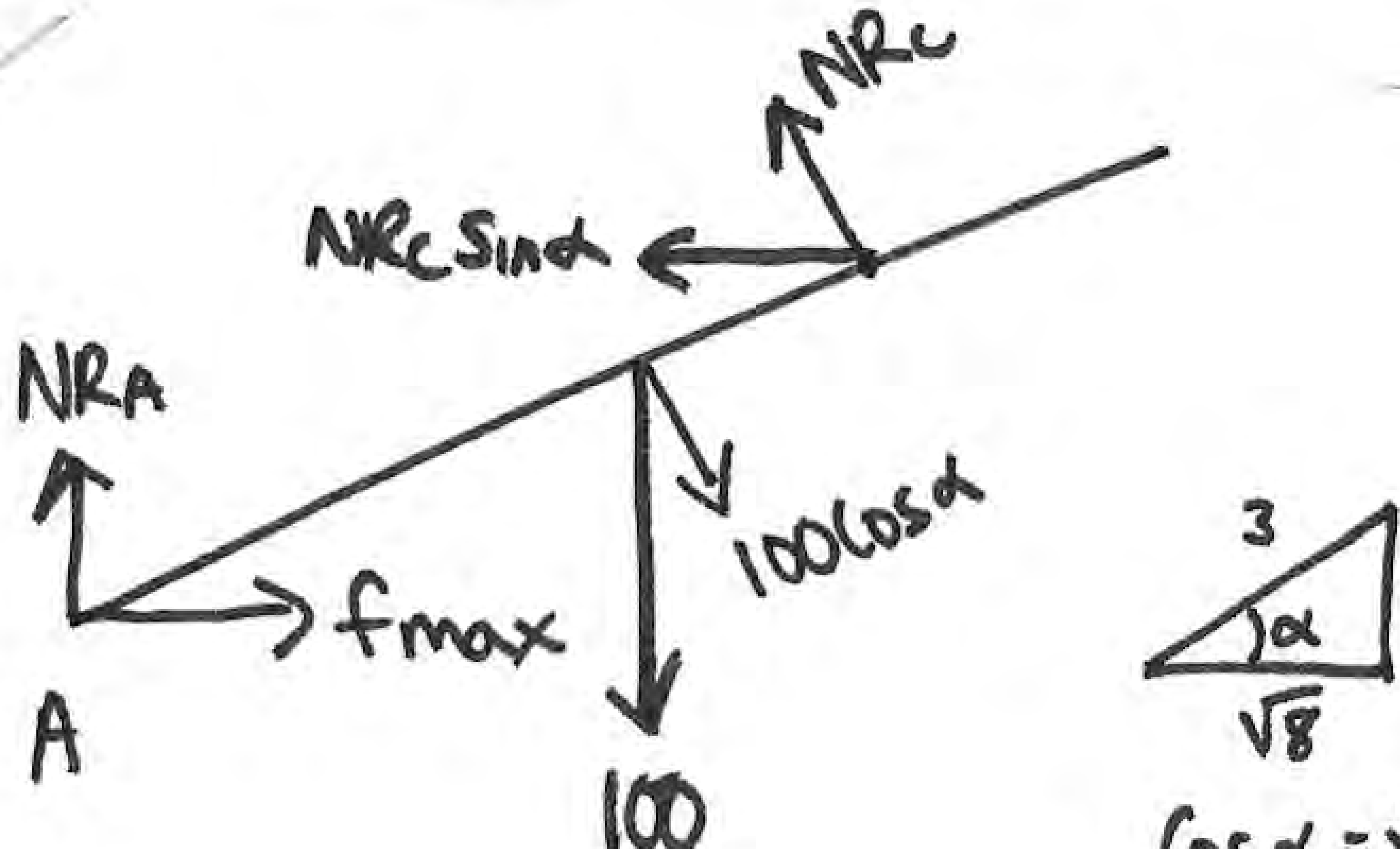


Figure 4

A uniform plank AB, of weight 100 N and length 4 m, rests in equilibrium with the end A on rough horizontal ground. The plank rests on a smooth cylindrical drum. The drum is fixed to the ground and cannot move. The point of contact between the plank and the drum is C, where $AC = 3$ m, as shown in Figure 4. The plank is resting in a vertical plane which is perpendicular to the axis of the drum, at an angle α to the horizontal, where $\sin\alpha = \frac{1}{3}$. The coefficient of friction between the plank and the ground is μ. Modelling the plank as a rod, find the least possible value of μ.

(10)



$\cos\alpha = \frac{\sqrt{8}}{3}$

$A\circlearrowright \quad 100 \times \frac{\sqrt{8}}{3} \times 2 = 3NRc \Rightarrow NRc = 62.85...$

$\rightarrow = \leftarrow \quad NRc\sin\alpha = fmax \Rightarrow fmax = 20.95..$

$\uparrow = \downarrow \quad NRA + NRc\cos\alpha = 100 \Rightarrow NRA = 40.74..$

$fmax = \mu \times NRA \Rightarrow 20.95... = 40.74..\mu$

$\mu = 0.514.$

8. A particle P of mass m kg is moving with speed 6 m s^{-1} in a straight line on a smooth horizontal floor. The particle strikes a fixed smooth vertical wall at right angles and rebounds. The kinetic energy lost in the impact is 64 J. The coefficient of restitution between P and the wall is $\frac{1}{3}$.

(a) Show that $m = 4$.

(6)

After rebounding from the wall, P collides directly with a particle Q which is moving towards P with speed 3 m s^{-1}. The mass of Q is 2 kg and the coefficient of restitution between P and Q is $\frac{1}{3}$.

(b) Show that there will be a second collision between P and the wall.

(7)

a) $e = \frac{sep}{app} = \frac{V}{6} = \frac{1}{3} \Rightarrow \overleftarrow{V} = 2ms^{-1}$

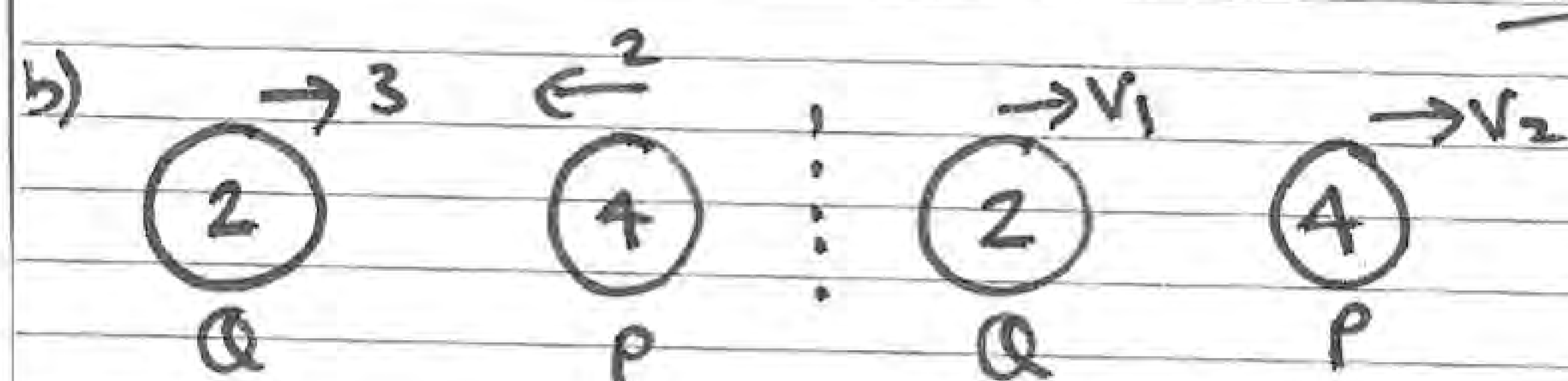
$KE_{before} = \frac{1}{2}mV^2 = 18m$

$KE_{after} = \frac{1}{2}mV^2 = 2m$

$\Rightarrow loss = 16m = 64$

$m = 4kg$

b)



$2 \times 3 + 4 \times \bar{2} = 2V_1 + 4V_2$

$-2 = 2V_1 + 4V_2 \quad (\times 3)$

$\frac{V_2 - V_1}{5} = \frac{1}{3} \quad -3V_1 + 3V_2 = 5 \ (\times 2)$

$6V_1 + 12V_2 = -6$

$-6V_1 + 6V_2 = 10$ +

$18V_2 = 4$

$V_2 = \frac{2}{9} \quad V_1 = \frac{-13}{9}$

$V_2 > 0$, will hit wall