M2 JAN 06

1a) KE lost $= \frac{1}{2}mu^2 = \frac{1}{2}(3)(8)^2 = \underline{96\text{J}}$

b) KE lost = Wd against friction

$\Rightarrow 96 = f_{max} \times 12 \Rightarrow \mu \times NR = 8 \Rightarrow \mu = \frac{8}{3g} = 0.27\ (2\text{s.f.})$

2) $\text{vel} = \frac{d\underline{r}}{dt} = (2t+4)\underline{i} + (3-3t^2)\underline{j}$

$t=3,\ \underline{v} = 10\underline{i} - 24\underline{j} \Rightarrow \text{speed} = \sqrt{10^2+24^2} = \underline{13\ ms^{-1}}$

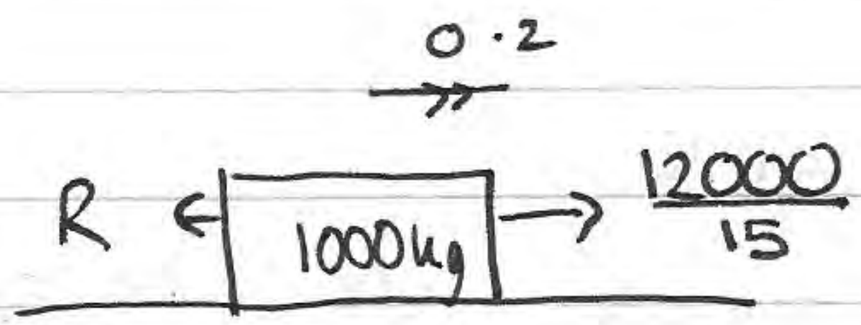
b) Initial momentum $= mu = 0.4(10\underline{i} - 24\underline{j}) = 4\underline{i} - 9.6\underline{j}$

$+ \text{Impulse} = 8\underline{i} - 12\underline{j}$

$= \text{final mom} = mv = 12\underline{i} - 21.6\underline{j}$

$\div 0.4 \quad \underline{v} = \underline{30\underline{i} - 54\underline{j}}$

3)

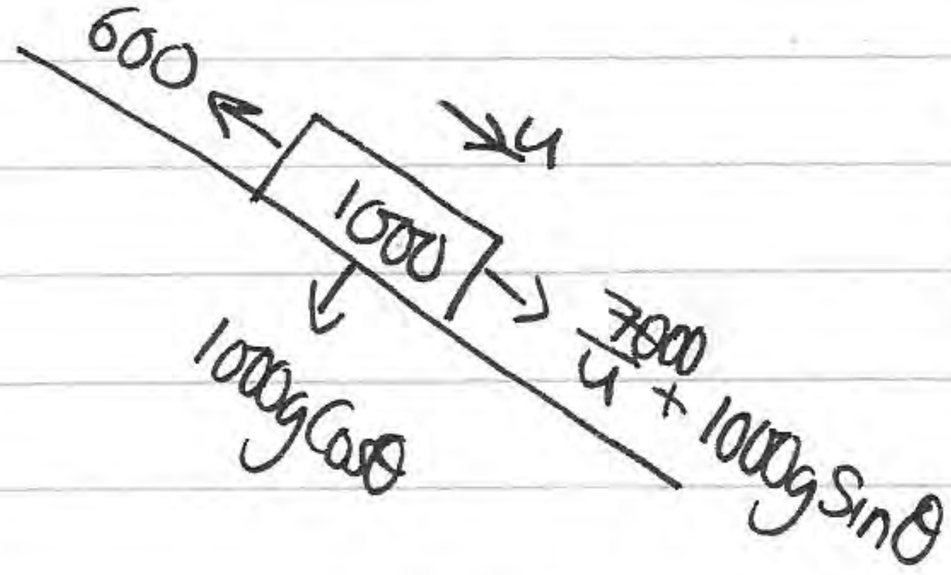


$\overrightarrow{Rf} = ma$

$\Rightarrow \frac{12000}{15} - R = 1000 \times 0.2$

$\Rightarrow 800 - R = 200 \Rightarrow R = 600\text{N}$ #

b)



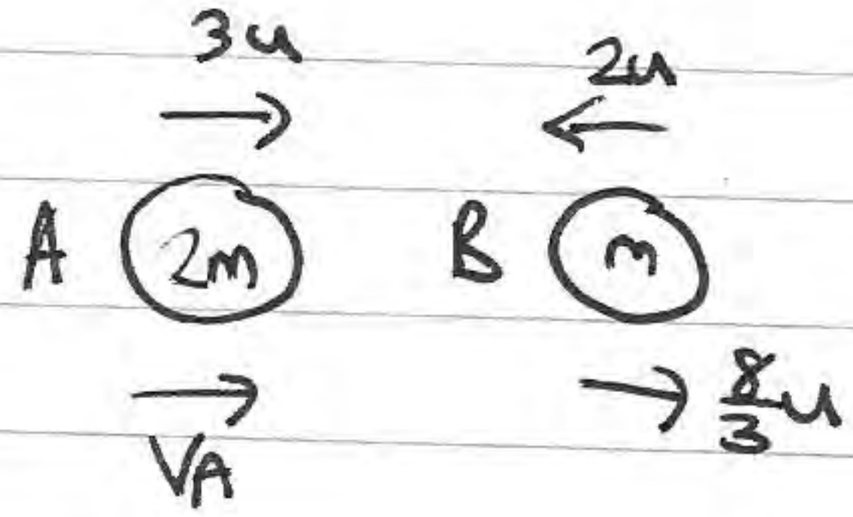
$R\nearrow = 0$

$\Rightarrow \frac{7000}{v} + 1000g \times \frac{1}{40} = 600$

$\Rightarrow \frac{7000}{v} = 600 - 25g$

$\Rightarrow v = \frac{7000}{600 - 25g} \Rightarrow \underline{v = 19.7\ ms^{-1}\ (3\text{s.f.})}$

4)



CLM $\Rightarrow 2m(3u) + m(-2u)$
$= 2m(v_A) + m(\frac{8}{3}u)$

$\Rightarrow 6mu - 2mu = 2mv_A + \frac{8}{3}mu$

$\Rightarrow \frac{4}{3}u = 2v_A \Rightarrow v_A = \frac{2}{3}u$

$e = \frac{sep}{app} = \frac{2u}{5u} \Rightarrow e = \frac{2}{5}$

b) total KE before $= \frac{1}{2}(2m)(3u)^2 + \frac{1}{2}m(2u)^2$
$= 11u^2$

total KE after $= \frac{1}{2}(2m)(\frac{2}{3}u)^2 + \frac{1}{2}(m)(\frac{8}{3}u)^2$
$= \frac{4}{9}u^2 + \frac{32}{9}u^2 = 4u^2$

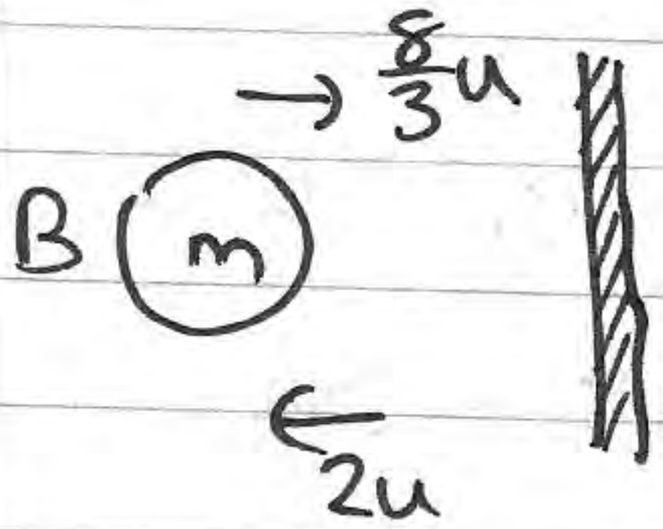
KE lost $= 11u^2 - 4u^2 = 7u^2$

3c) Initial mom $= m(\frac{8}{3}u) = \frac{8}{3}mu$

$\pm$Impulse $= \frac{14}{3}mu$ (must be $-$)

final mom $= -\frac{6}{3}mu = -2mu$

$\Rightarrow mv = -2mu \Rightarrow v = -2u$



$e = \frac{sep}{app} = \frac{2u}{\frac{8}{3}u} = \frac{6}{8}$

$\Rightarrow e = \frac{3}{4}$

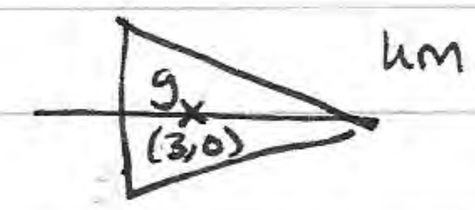
5) 

$4mg \times 0 + 2mg \times 0 + 6mg \times 9 = 12mg\,\bar{x} \quad \Rightarrow \bar{x} = \frac{54}{12}$

$\bar{x} = 4.5$

$\rightarrow\!+\!\curvearrowright$ $2mg \times -4 + 4mg \times 4 + 6mg \times 0 = 12mg\,\bar{y}$

$\bar{y} = \frac{8}{12} = \frac{2}{3} \qquad (4.5, \frac{2}{3})$



b) 

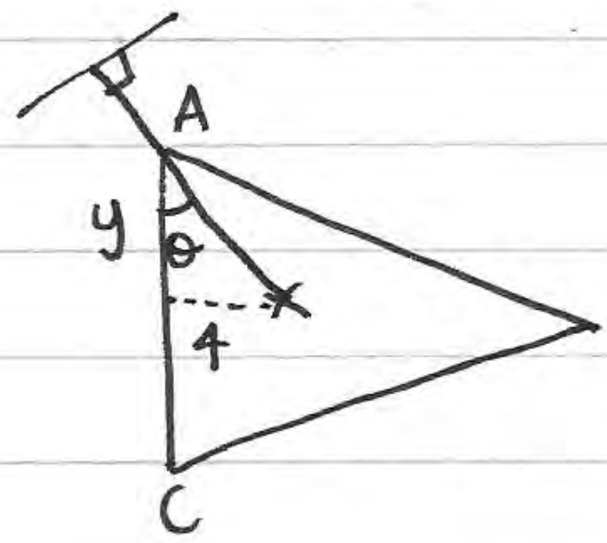
$12mg \times 4.5 + 4mg \times 3 = (12+4)mg \times 4$

$\Rightarrow 54 + 3k = 4(12+k) \Rightarrow 54 + 3k = 48 + 4k$

$\Rightarrow \underline{k = 6}$ #

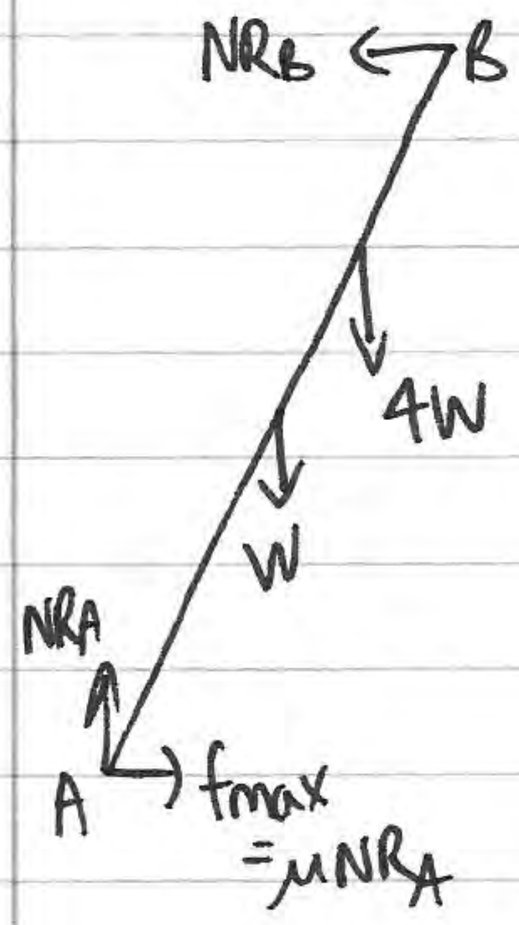
c) $\uparrow\!+\!\curvearrowright$ $12mg \times \frac{2}{3} + 4mg \times 0 = (12+4)mg \times \lambda$

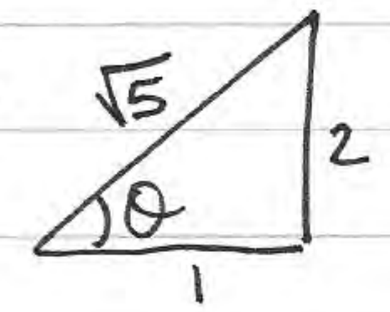
$\Rightarrow 8 = (12+6)\lambda \Rightarrow \lambda = \frac{8}{18} = \frac{4}{9}$

d) 

$y = 4 - \frac{4}{9}$

$\Rightarrow \theta = \tan^{-1}\left(\frac{4}{4 - \frac{4}{9}}\right) \Rightarrow \theta = \tan^{-1}\left(\frac{9}{2}\right) = 83.7^\circ$

6) 



$\tan\theta = \frac{2}{1} \qquad \sin\theta = \frac{2}{\sqrt{5}} \qquad \cos\theta = \frac{1}{\sqrt{5}}$

$R\!f\uparrow = 0 \Rightarrow NRA = 5W$

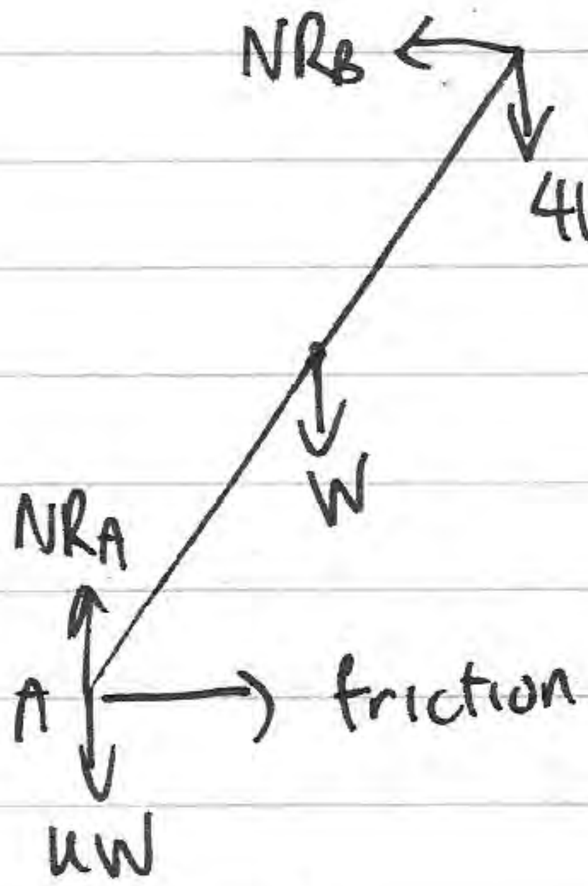
$\overrightarrow{R\!f} = 0 \quad f_{max} = NRB \Rightarrow \mu NRA = NRB \Rightarrow \mu = \frac{NRB}{5W}$

A2 $W \times 2a \times \left(\frac{1}{\sqrt{5}}\right) + 4W \times 3a \times \left(\frac{1}{\sqrt{5}}\right) = NRB \times 4a \times \left(\frac{2}{\sqrt{5}}\right)$

$2W + 12W = 8NRB \qquad NRB = \frac{14}{8}W$

$\therefore \quad \mu = \dfrac{\frac{14}{8}W}{5W} = \dfrac{14}{40} = \dfrac{7}{20}$ #

b)



$Rf\uparrow = 0 \Rightarrow NRA = (5+k)W$

$\overrightarrow{Rf} = 0 \Rightarrow$ friction $= NRB$

A2 $W \times 2a\left(\frac{1}{\sqrt{5}}\right) + 4W \times 4a\left(\frac{1}{\sqrt{5}}\right) = NRB \times 4a\left(\frac{2}{\sqrt{5}}\right)$

$\Rightarrow 2W + 16W = 8NRB \quad \Rightarrow NRB = \frac{9}{4}W$

$f_{max} = \mu NRA = \frac{7}{20}(5+k)W \qquad$ friction $= NRB = \frac{9}{4}W$

friction $\leq f_{max} \Rightarrow \frac{9}{4}W \leq \frac{7}{20}(5+k)W$

$\Rightarrow \frac{45}{7} \leq 5+k \Rightarrow k \geq \frac{10}{7}$

7) (H) $Vel = 11\cos 30 \quad dist = 10 \Rightarrow t = \dfrac{10}{11\frac{\sqrt{3}}{2}} = 1.05$

b) (V↑) $u\uparrow = 11\sin 30 = 5.5$
$a\uparrow = -9.8$
$t = \frac{20\sqrt{3}}{33}$

$S = ut + \frac{1}{2}at^2$
$S = 5.5\left(\frac{20\sqrt{3}}{33}\right) - 4.9\left(\frac{20\sqrt{3}}{33}\right)^2$

$S = 0.374\ldots$ above A

$\Rightarrow C = 1 - 0.374\ldots$ below T

$\therefore C$ is $0.63m$ below T

c) $u\uparrow = V\sin 30 = \frac{1}{2}V$
$a = -9.8$
$S = 1$ (above A)

$\Rightarrow 1 = \left(\frac{1}{2}V\right)\left(\frac{20\sqrt{3}}{3V}\right) - 4.9\left(\frac{20\sqrt{3}}{3V}\right)^2$

$\vec{H}$ $Vel = V\cos 30 \quad x = 10$

$\Rightarrow t = \dfrac{10}{V\frac{\sqrt{3}}{2}} = \dfrac{20\sqrt{3}}{3V}$

$$\Rightarrow 1 - \frac{10\sqrt{3}}{3} = \frac{1960}{3V^2}$$

$$\Rightarrow (3 - 10\sqrt{3})V^2 = 1960 \quad \Rightarrow V = \sqrt{\frac{1960}{3-10\sqrt{3}}}$$

$$V = 11.7 \text{ ms}^{-1} \quad (3sf)$$

d) we have ignored wind, air resistance, spin. We have also considered the ball to be a particle, so v could be increased and decreased but still hit the target as the ball might hit the target at the top or bottom of the ball.