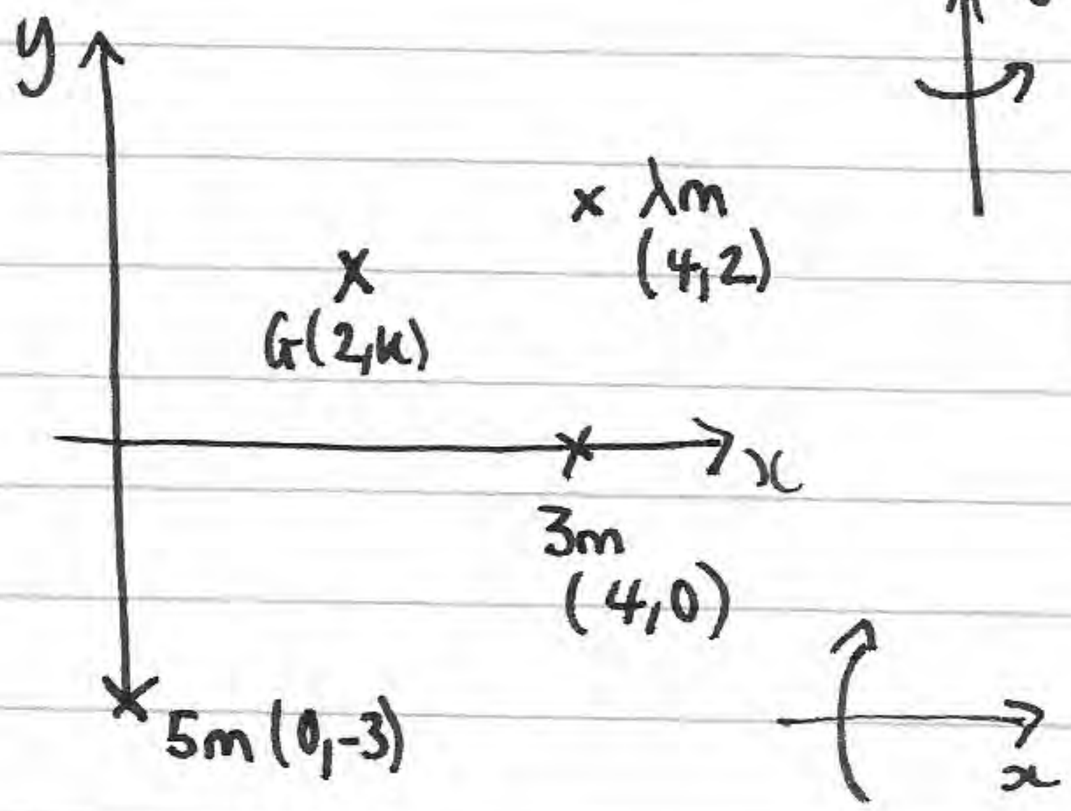
M2 JAN03

1)



$3mgx4 + \lambda mgx4 = (8+\lambda)mgx2$

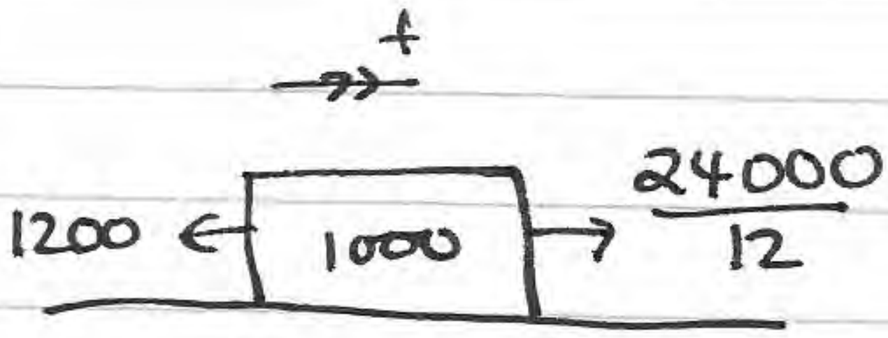
$(12+4\lambda)mg = (8+\lambda)x2mg$

$12+4\lambda = 16+2\lambda \Rightarrow 2\lambda = 4 \Rightarrow \underline{\lambda = 2}$

$5mgx-3 + 2mgx2 = 10mgxk$

$-11mg = 10mgxk \quad \underline{k = -1.1}$

2)



$\overrightarrow{Rf} = ma \Rightarrow 2000-1200 = 1000f$

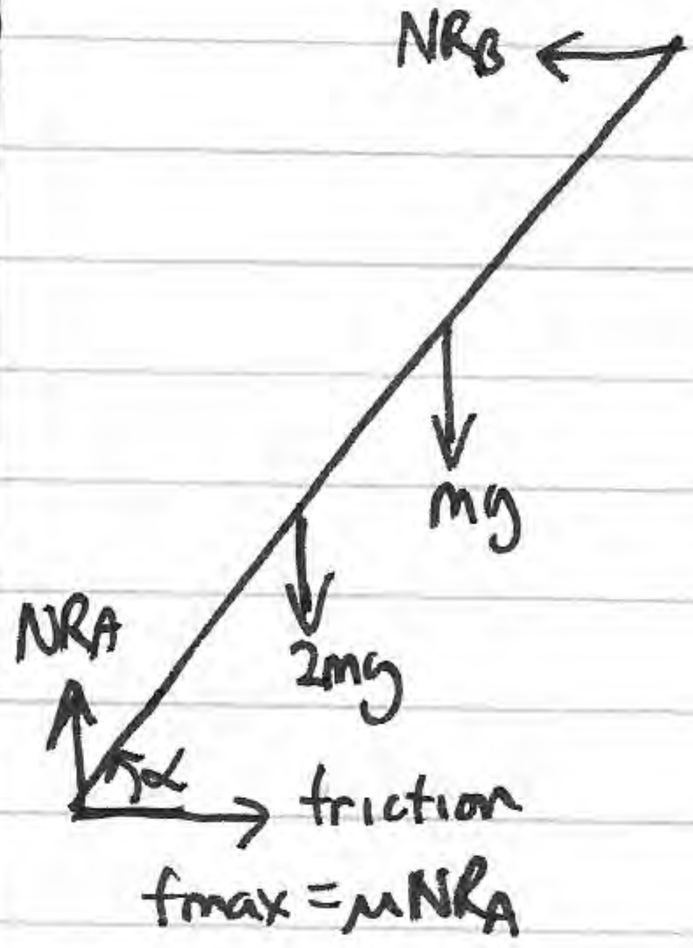
$\Rightarrow f = \underline{0.8ms^{-2}}$

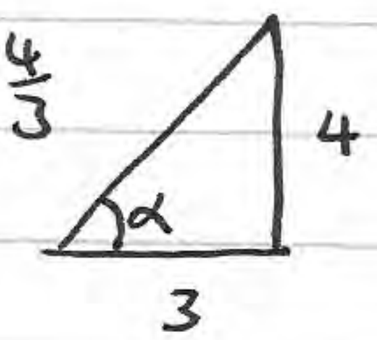
b) KE lost = Wd against Res

$\frac{1}{2}(1000)14^2 = 1200d \quad \Rightarrow d = \frac{245}{3}$

c) Resistance would vary with speed.

3)





$\tan\alpha = \frac{4}{3} \quad \sin\alpha = \frac{4}{5} \quad \cos\alpha = \frac{3}{5}$

A↻ $2mgx\frac{1}{2}a\left(\frac{3}{5}\right) + mgxa\left(\frac{3}{5}\right) = NRx2a\left(\frac{4}{5}\right)$

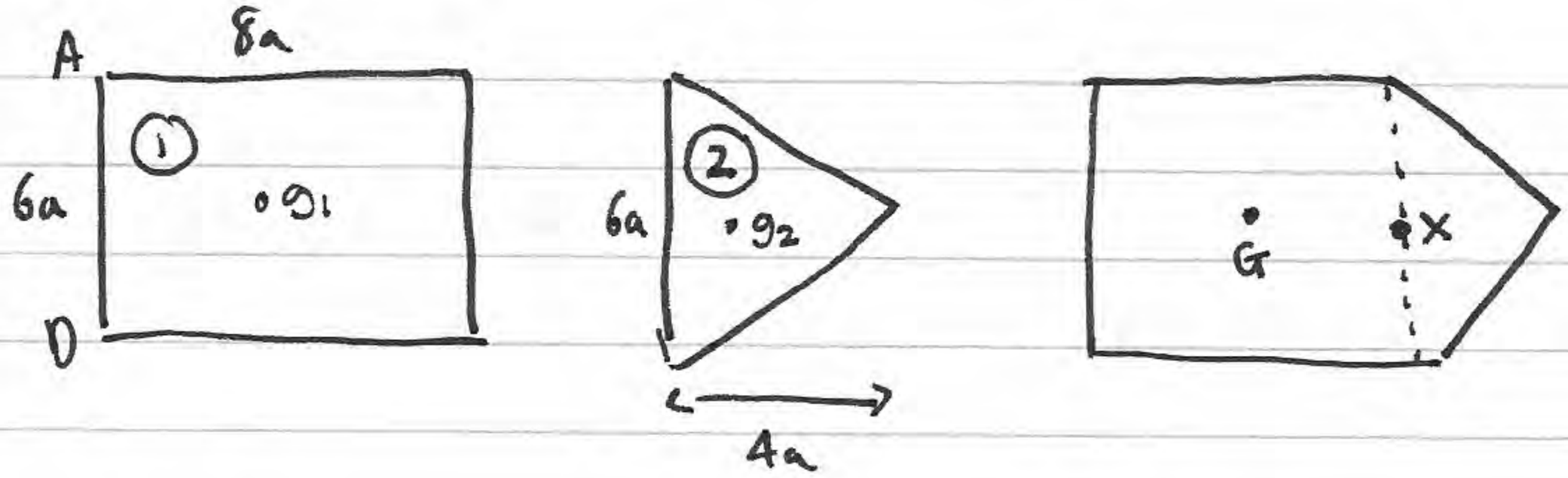
$\Rightarrow 3mg + 3mg = 8NR \Rightarrow \underline{NR_B = \frac{3}{4}mg}$

$\overrightarrow{RF} = 0 \Rightarrow NR_B = f_{max} = \mu NR_A \quad \mu x NR_A = \frac{3}{4}mg$

$RF\uparrow = 0 \Rightarrow NR_A = 3mg \quad \Rightarrow \mu x 3mg = \frac{3}{4}mg \quad \Rightarrow \underline{\mu = \frac{1}{4}}$

$friction \leq f_{max} \Rightarrow f_{max} \geq \frac{1}{4}$

4)



M_1 $48a^2k$ $M_2 = 12a^2k$ $M = 60a^2k$

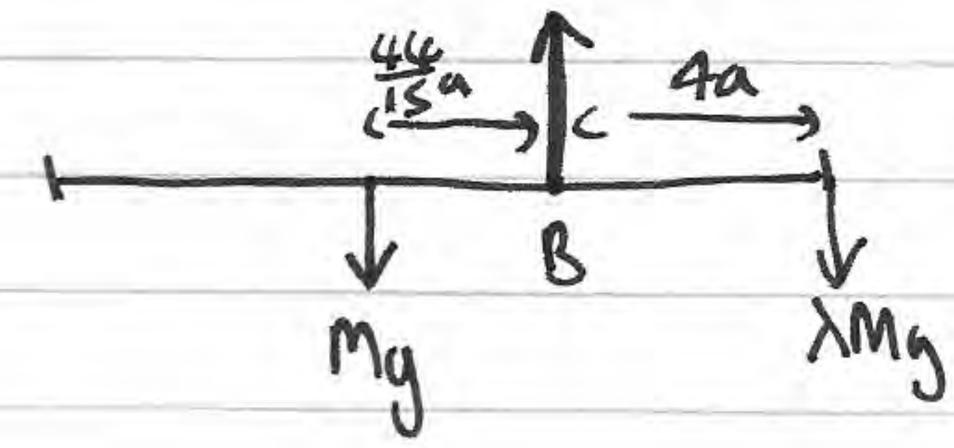
$g_1(4a, 3a)$ $g_2\left((8+\frac{4}{3})a, 3a\right)$ $M(\bar{x}, 3a)$

A ↑ D ↻ $48a^2kg \times 4a + 12a^2kg \times \frac{28}{3}a = 60a^2kg\,\bar{x}$

$192a + 112a = 60\bar{x} \Rightarrow \bar{x} = \frac{76}{15}a$

$GX = 8a - \frac{76}{15}a = \frac{44}{15}a$ #

b)



B↻ $Mg \times \frac{44}{15}\alpha = \lambda Mg \times 4\alpha$

$\frac{44}{15} = 4\lambda \Rightarrow \lambda = \frac{11}{15}$

5) $a = 4t - 8$

$V = \int a\,dt = 2t^2 - 8t + c$ $V=6, t=0 \Rightarrow c=6$

$\Rightarrow V = 2t^2 - 8t + 6$

b) at rest $\Rightarrow V = 0$ $2t^2 - 8t + 6 = 0 \Rightarrow t^2 - 4t + 3 = 0$

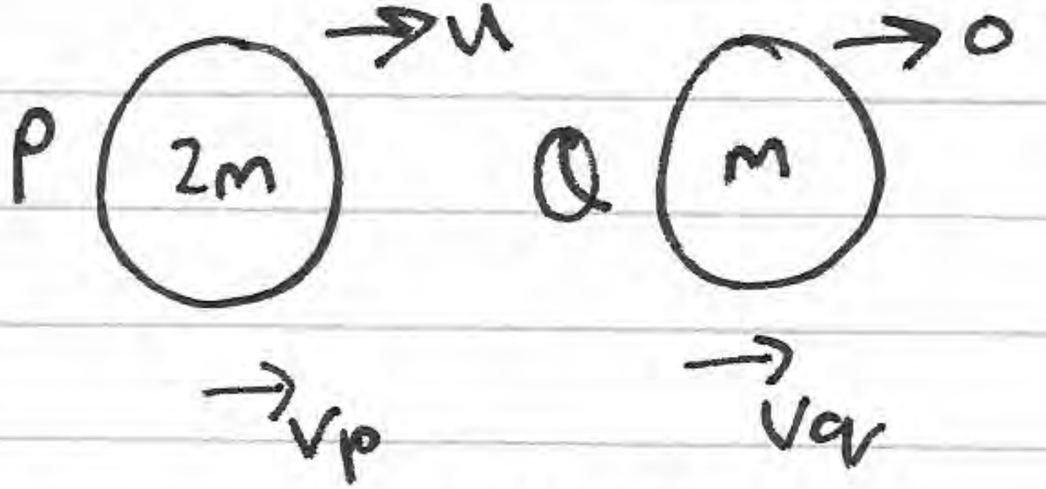
$(t-3)(t-1) = 0$

$t=3, t=1$

$\int_1^3 v\,dt = \int_1^3 2t^2 - 8t + 6\,dt$

$= \left[\frac{2}{3}t^3 - 4t^2 + 6t\right]_1^3 = 0 - \frac{8}{3} = -\frac{8}{3} \Rightarrow$ dist $= \frac{8}{3}$ m

6)



$v_Q > v_P$

$e = \frac{1}{3} = \frac{\text{Sep}}{\text{app}} \Rightarrow \frac{v_Q - v_P}{u} = \frac{1}{3}$

$\Rightarrow 3v_Q - 3v_P = u$

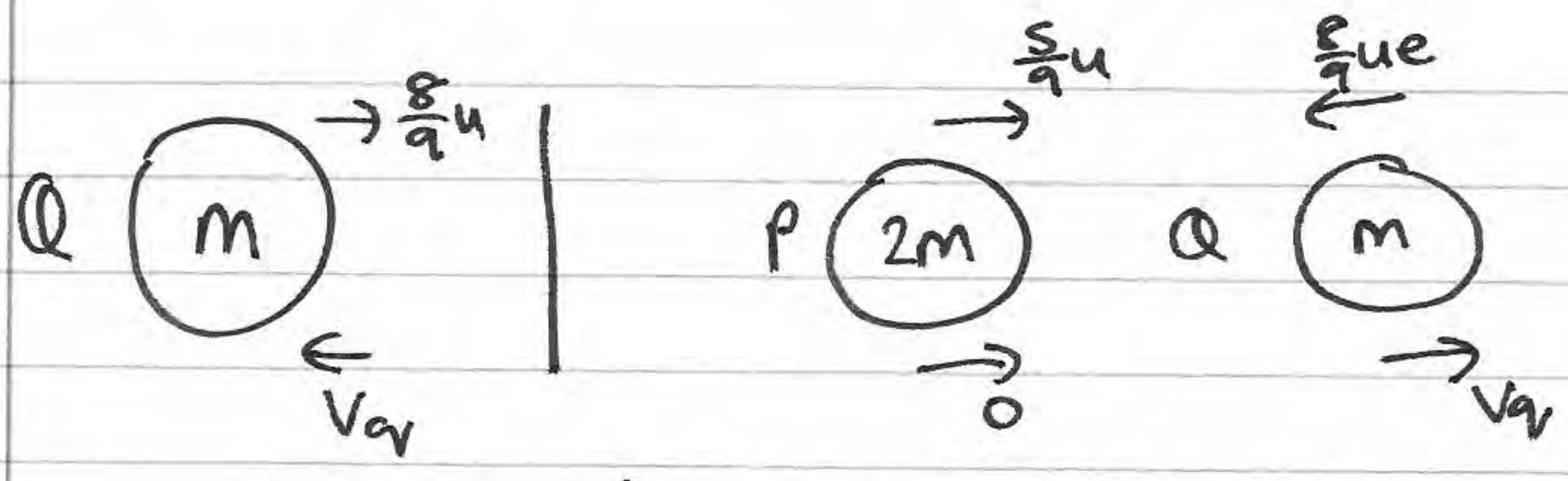
$3v_Q = u + 3v_P$

CLM $\Rightarrow 2mu = 2mv_P + mv_Q$ (x3) $6u = 6v_P + 3v_Q$

$\Rightarrow 6u = 9v_P + u$

$\Rightarrow v_P = \frac{1}{9}(6u - u) \Rightarrow v_P = \frac{5}{9}u$

$v_Q = \frac{1}{3}(u + 3v_P) = \frac{1}{3}\left(u + \frac{15}{9}u\right) = \frac{1}{3}\left(\frac{24}{9}u\right) = \frac{8}{9}u$ #



$e = \frac{v_{QP}}{\frac{8}{9}u}$ ⋮ $e = \frac{1}{3} = \frac{v_{QP}}{\frac{5}{9}u + \frac{8}{9}ue}$ $= V_Q = \frac{5}{27}u + \frac{8}{27}ue$

$\Rightarrow V_Q = \frac{8}{9}ue$; CLM $\Rightarrow \frac{10}{9}mu - \frac{8}{9}mue = mV_Q$

$\Rightarrow \frac{5}{27}u + \frac{8}{27}ue = \frac{10}{9}u - \frac{8}{9}ue$

$(\times 27)$ $5 + 8e = 30 - 24e \Rightarrow 32e = 25 \Rightarrow e = \frac{25}{32}$

Q is now moving towards the wall again, and will bounce back towards P which is at rest.

7) Mom B before $= 0.4(-20i + 4j) = -8i + 1.6j$ Ns
Mom B after $= 0.4(15i + 16j) = 6i + 6.4j$ Ns

$\Rightarrow$ Impulse $= 14i + 4.8j$ Ns

$|\text{Impulse}| = \sqrt{14^2 + 4.8^2} = 14.8$ Ns

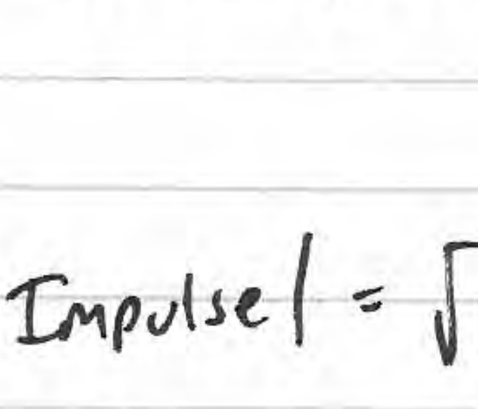
b) $u = 15i + 16j = \sqrt{481}$

Initial KE $= \frac{1}{2}(0.4)(\sqrt{481})^2 = 96.2$ J
Initial PE $= (0.4)g(1.2) = 4.704$ J

$\Rightarrow$ Final KE $= 100.904$ J $= \frac{1}{2}(0.4)v^2$

$\Rightarrow v = 22.4615\ldots \approx 22.5$ (3sf)

c)



$\theta = \cos^{-1}\left(\frac{15}{22.4615\ldots}\right)$

e) air resistance, windy, spin of ball.

$\theta = 48.1°$ below horizontal