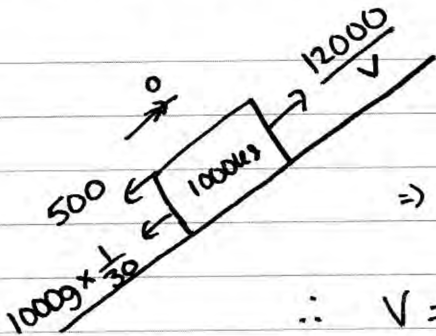
M2 JUNE 11

1. A car of mass 1000 kg moves with constant speed V m s^{-1} up a straight road inclined at an angle θ to the horizontal, where $\sin\theta = \frac{1}{30}$. The engine of the car is working at a rate of 12 kW. The resistance to motion from non-gravitational forces has magnitude 500 N.

Find the value of V.

(5)



$R(\nearrow) = 0$

$\Rightarrow \dfrac{12000}{V} = 500 + \dfrac{100g}{3}$

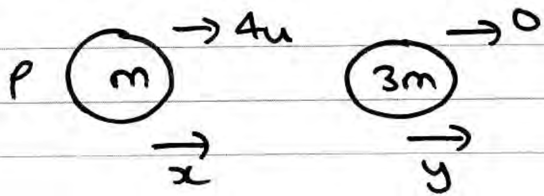
$\therefore V = \dfrac{12000}{500 + \frac{100g}{3}} = 14.516\ldots$

$\simeq 15 \text{ ms}^{-1}$ (2sf)

2. A particle P of mass m is moving in a straight line on a smooth horizontal surface with speed $4u$. The particle P collides directly with a particle Q of mass $3m$ which is at rest on the surface. The coefficient of restitution between P and Q is e. The direction of motion of P is reversed by the collision.

Show that $e > \frac{1}{3}$.

(8)



$$e = \frac{y - x}{4u}$$

$$4eu = y - x$$

$$y = x + 4eu$$

CLM $\Rightarrow 4\cancel{m}u = \cancel{m}x + 3\cancel{m}y$

$\Rightarrow 4u = x + 3x + 12eu$

$\Rightarrow 4u = 4x + 12eu$

$\Rightarrow x = u - 3eu$

$\Rightarrow x = u(1 - 3e)$

Motion P is reversed $\Rightarrow x < 0 \Rightarrow u(1-3e) < 0$

$\Rightarrow 1 - 3e < 0 \quad \Rightarrow 3e > 1 \quad \Rightarrow e > \frac{1}{3}$ #

3. A ball of mass 0.5 kg is moving with velocity $12\mathbf{i}$ m s^{-1} when it is struck by a bat. The impulse received by the ball is $(-4\mathbf{i}+7\mathbf{j})$ N s. By modelling the ball as a particle, find

(a) the speed of the ball immediately after the impact,

(4)

(b) the angle, in degrees, between the velocity of the ball immediately after the impact and the vector $\mathbf{i}$,

(2)

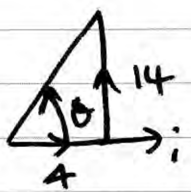
(c) the kinetic energy gained by the ball as a result of the impact.

(2)

final Mom = Initial mom + Impulse $= \frac{1}{2}(12i) + (-4i+7j)$

$\Rightarrow \frac{1}{2}v = 2i + 7j \Rightarrow v = 4i + 14j \Rightarrow$ speed $= \sqrt{4^2+14^2}$

$\therefore$ Speed $= \underline{14.6\,ms^{-1}}$ (3sf)

b) 

$\theta = \tan^{-1}\left(\frac{14}{4}\right) = \underline{74.1^\circ}$ (3sf)

c) final KE − Initial KE $= \frac{1}{2}(0.5)(14.6...)^2 - \frac{1}{2}(0.5)(12)^2$

$= \underline{17J}.$

4.

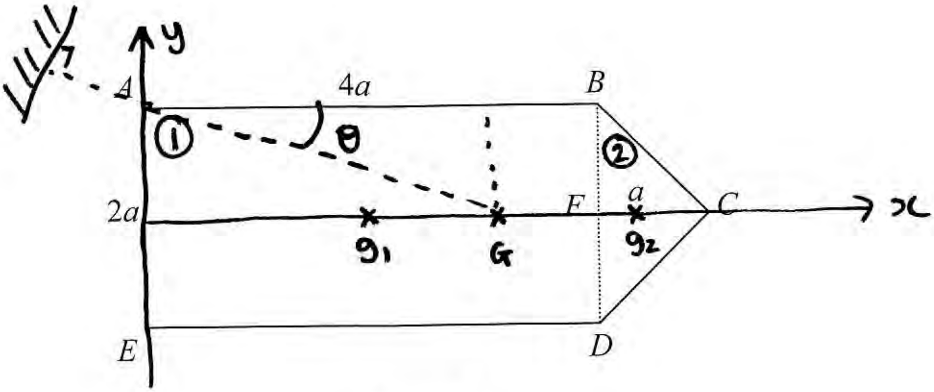


Figure 1

Figure 1 shows a uniform lamina *ABCDE* such that *ABDE* is a rectangle, $BC = CD$, $AB = 4a$ and $AE = 2a$. The point *F* is the midpoint of *BD* and $FC = a$.

(a) Find, in terms of *a*, the distance of the centre of mass of the lamina from *AE*.

(4)

The lamina is freely suspended from *A* and hangs in equilibrium.

(b) Find the angle between *AB* and the downward vertical.

(3)

$$① \quad m_1 = 8a^2k \quad g_1(2a, 0) \qquad m_2 = a^2k \quad g_2\left(\tfrac{13}{3}a, 0\right)$$

$$\therefore \; 8\cancel{a^2}\cancel{kg} \times 2a + \cancel{a^2}\cancel{kg} \times \tfrac{13}{3}a = 9\cancel{a^2}\cancel{kg} \times \bar{x}$$

$$\Rightarrow \quad \frac{61}{3}a = 9\bar{x} \quad \Rightarrow \bar{x} = \underline{\frac{61}{27}a}$$

$$b) \quad \theta = \tan^{-1}\left(\frac{a}{\frac{61}{27}a}\right) \Rightarrow \theta = \tan^{-1}\left(\frac{27}{61}\right) = 23.875\ldots$$

$$\theta \simeq 23.9^\circ \text{ (3sf)}$$

5.

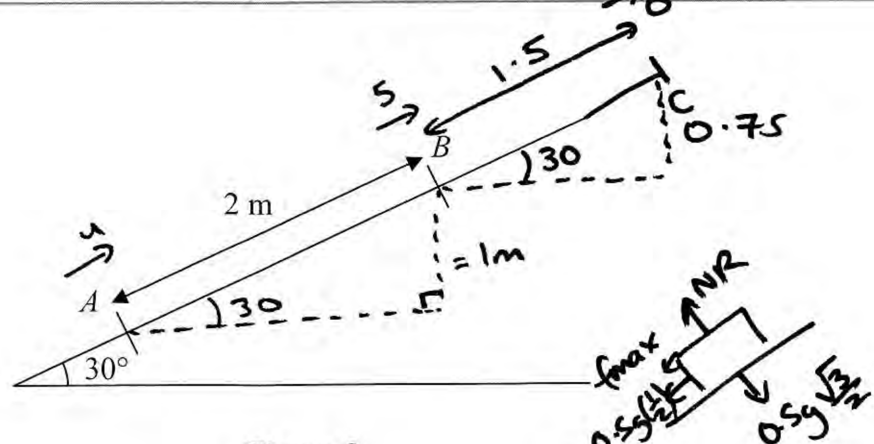


Figure 2

A particle P of mass 0.5 kg is projected from a point A up a line of greatest slope AB of a fixed plane. The plane is inclined at 30° to the horizontal and $AB = 2$ m with B above A, as shown in Figure 2. The particle P passes through B with speed 5 m s^{-1}. The plane is smooth from A to B.

(a) Find the speed of projection.

(4)

The particle P comes to instantaneous rest at the point C on the plane, where C is above B and $BC = 1.5$ m. From B to C the plane is rough and the coefficient of friction between P and the plane is μ.

By using the work-energy principle,

(b) find the value of μ.

(6)

a) Loss in KE = Gain in PE $\Rightarrow \frac{1}{2}(\cancel{0.5})(u^2 - 5^2) = \cancel{0.5}g(1)$

$\Rightarrow u^2 = 2g + 25 \quad \Rightarrow u = 6.678.. = \underline{6.7 \text{ ms}^{-1}}$ (2sf)

b) KE_B - Wd against friction = PE_C (Gain $\vec{BC}$)

$\frac{1}{2}(0.5)5^2 - f_{max} \times 1.5 = (0.5)g(0.75)$

$\Rightarrow 1.5 f_{max} = \frac{25}{4} - \frac{3}{8}g \quad \Rightarrow f_{max} = \frac{2.575}{1.5} = 1.71\dot{6}$

$f_{max} = \mu NR \qquad \mu \times 0.5g\left(\frac{\sqrt{3}}{2}\right) = 1.71\dot{6}$

$\Rightarrow \mu = 0.4045... \quad \Rightarrow \mu = \underline{0.40}$ (2sf)

6. A particle P moves on the x-axis. The acceleration of P at time t seconds is $(t-4)$ m s^{-2} in the positive x-direction. The velocity of P at time t seconds is v m s^{-1}. When $t = 0$, $v = 6$.

Find

(a) v in terms of t,

(4)

(b) the values of t when P is instantaneously at rest,

(3)

(c) the distance between the two points at which P is instantaneously at rest.

(4)

a) $a = t - 4$ $\quad v = \int a\,dt = \frac{1}{2}t^2 - 4t + c$

$t = 0, v = 6 \Rightarrow c = 6$ $\quad v = \frac{1}{2}t^2 - 4t + 6$

b) at rest $\Rightarrow v = 0$ $\quad \frac{1}{2}t^2 - 4t + 6 = 0 \Rightarrow t^2 - 8t + 12 = 0$

$(t-6)(t-2) = 0 \Rightarrow \underline{t = 2}, \underline{t = 6}$

c) $S = \int v\,dt = \frac{1}{6}t^3 - 2t^2 + 6t + c$

$t = 6 \quad S = 0 + c$
$t = 2 \quad S = \frac{16}{3} + c$
$\Rightarrow$ dist between $= \underline{\frac{16}{3}\text{ m}}$

7.

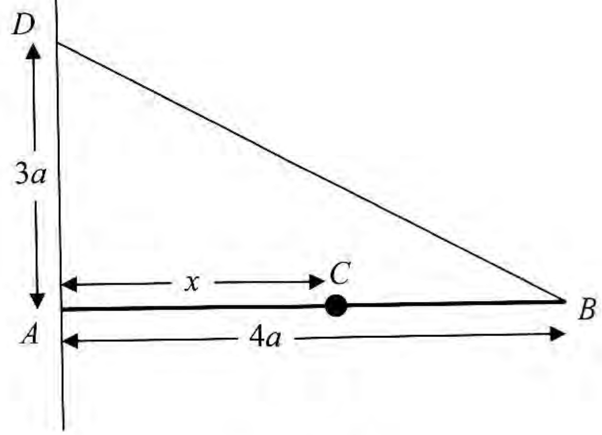


Figure 3

A uniform rod AB, of mass $3m$ and length $4a$, is held in a horizontal position with the end A against a rough vertical wall. One end of a light inextensible string BD is attached to the rod at B and the other end of the string is attached to the wall at the point D vertically above A, where $AD = 3a$. A particle of mass $3m$ is attached to the rod at C, where $AC = x$. The rod is in equilibrium in a vertical plane perpendicular to the wall as shown in Figure 3. The tension in the string is $\frac{25}{4}mg$.

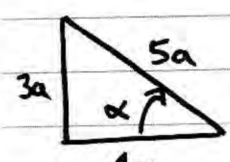
Show that

(a) $x = 3a$, **(5)**

(b) the horizontal component of the force exerted by the wall on the rod has magnitude $5mg$. **(3)**

The coefficient of friction between the wall and the rod is μ. Given that the rod is about to slip,

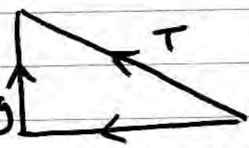
(c) find the value of μ. **(5)**



$\sin\alpha = \frac{3}{5}$

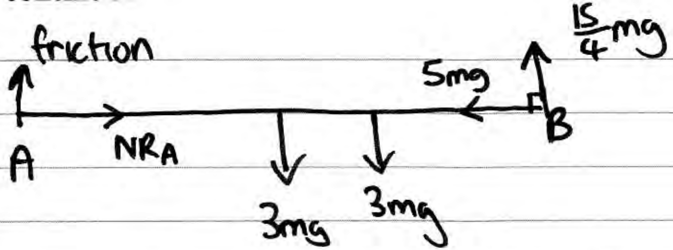
$\cos\alpha = \frac{4}{5}$

$\frac{3}{5}T = \frac{15}{4}mg$



$\frac{4}{5}T = 5mg$

a)



$A \circlearrowright \quad 3m\cancel{g} \times 2a + 3m\cancel{g} \times x = \frac{15}{4} m\cancel{g} \times 4a$

$\Rightarrow \quad 6a + 3x = 15a \Rightarrow 3x = 9a \Rightarrow x = 3a$ #

b) $\overrightarrow{Rf} = 0 \Rightarrow NR_A = 5mg$ #

c) $Rf \uparrow = 0 \Rightarrow \text{friction} = 6mg - \frac{15}{4}mg = \frac{9}{4}mg$

about to slip $\Rightarrow f_{max} = \frac{9}{4}mg = \mu \times NR_A$

$\Rightarrow \frac{9}{4}m\cancel{g} = \mu \times 5m\cancel{g} \Rightarrow \mu = \underline{\frac{9}{20}}$

8. A particle is projected from a point O with speed u at an angle of elevation α above the horizontal and moves freely under gravity. When the particle has moved a horizontal distance x, its height above O is y.

(a) Show that

$$y = x\tan\alpha - \frac{gx^2}{2u^2\cos^2\alpha}$$

(4)

A girl throws a ball from a point A at the top of a cliff. The point A is 8 m above a horizontal beach. The ball is projected with speed 7 m s^{-1} at an angle of elevation of 45°. By modelling the ball as a particle moving freely under gravity,

(b) find the horizontal distance of the ball from A when the ball is 1 m above the beach.

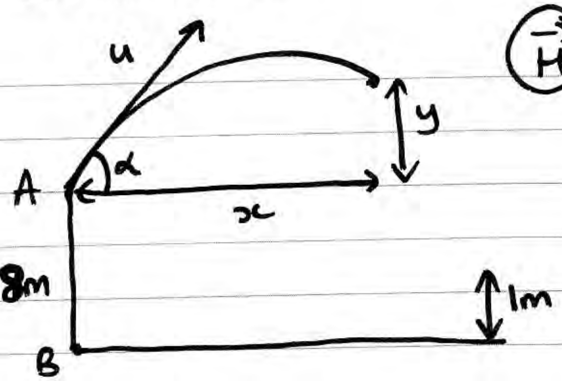
(5)

A boy is standing on the beach at the point B vertically below A. He starts to run in a straight line with speed $v \text{ m s}^{-1}$, leaving B 0.4 seconds after the ball is thrown.

He catches the ball when it is 1 m above the beach.

(c) Find the value of v.

(4)



$\overrightarrow{H}$ vel $= u\cos\alpha$ $\quad S = x$

dist = vel × time

$\Rightarrow t = \dfrac{x}{u\cos\alpha}$

$V\uparrow$ $\uparrow u = u\sin\alpha$ $\qquad S = ut + \frac{1}{2}at^2 \Rightarrow y = u\sin\alpha\left(\dfrac{x}{u\cos\alpha}\right) - \dfrac{g}{2}\left(\dfrac{x}{u\cos\alpha}\right)^2$

$\uparrow S = y$

$\uparrow a = -9.8$ $\qquad \Rightarrow y = \left(\dfrac{\sin\alpha}{\cos\alpha}\right)x - \dfrac{gx^2}{2u^2\cos^2\alpha}$

$\therefore\ y = x\tan\alpha - \dfrac{gx^2}{2u^2\cos^2\alpha}$

b) $y = -7$ $\quad \alpha = 45 \Rightarrow \tan 45 = 1$ $\quad \cos\alpha = \frac{\sqrt{2}}{2}$ $\quad \cos^2\alpha = \frac{1}{2}$

$\Rightarrow -7 = x - \dfrac{gx^2}{2(7)^2(\frac{1}{2})}$ $\quad \Rightarrow \dfrac{g}{49}x^2 - x - 7 = 0$

$\Rightarrow \frac{1}{5}x^2 - x - 7 = 0$

$\Rightarrow x^2 - 5x - 35 = 0$ $\Rightarrow x = \dfrac{5 + \sqrt{5^2 - 4(1)(-35)}}{2}$

$x = 8.92261\ldots = \underline{8.9\text{m}}$ (2sf)

c) $t = \dfrac{x}{u\cos\alpha} = \dfrac{8.9226\ldots}{7\left(\frac{\sqrt{2}}{2}\right)} = 1.80264\ldots$

$\therefore$ the boy takes $1.40264\ldots$ to catch ball

dist = vel × time $V = \dfrac{x}{t} = \dfrac{8.92261\ldots}{1.80264\ldots}$

$V = 6.36129\ldots \simeq \underline{6.4\text{ms}^{-1}}$ (2sf)