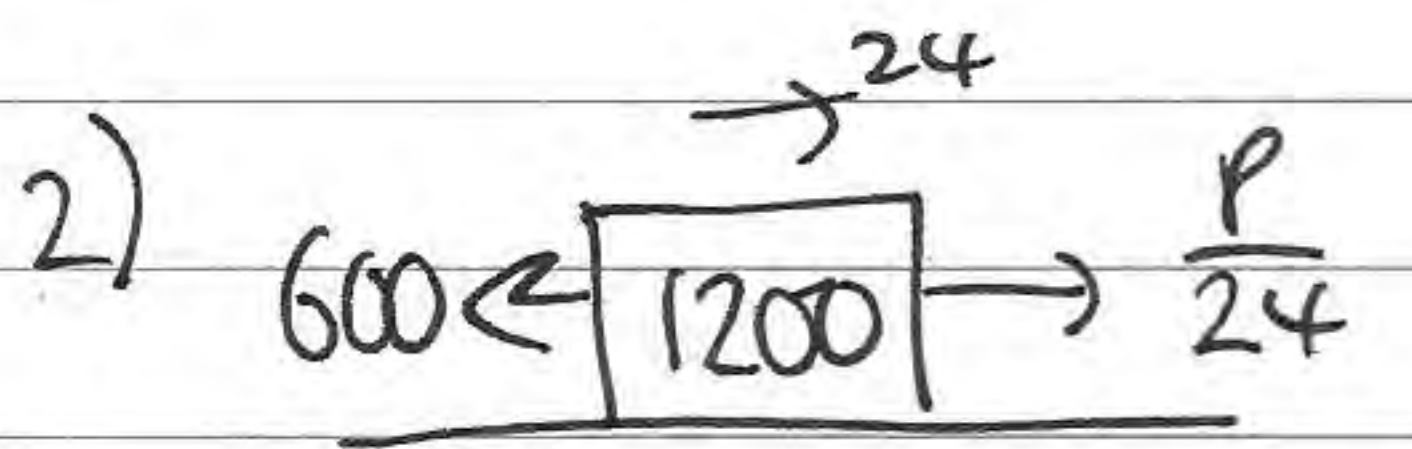


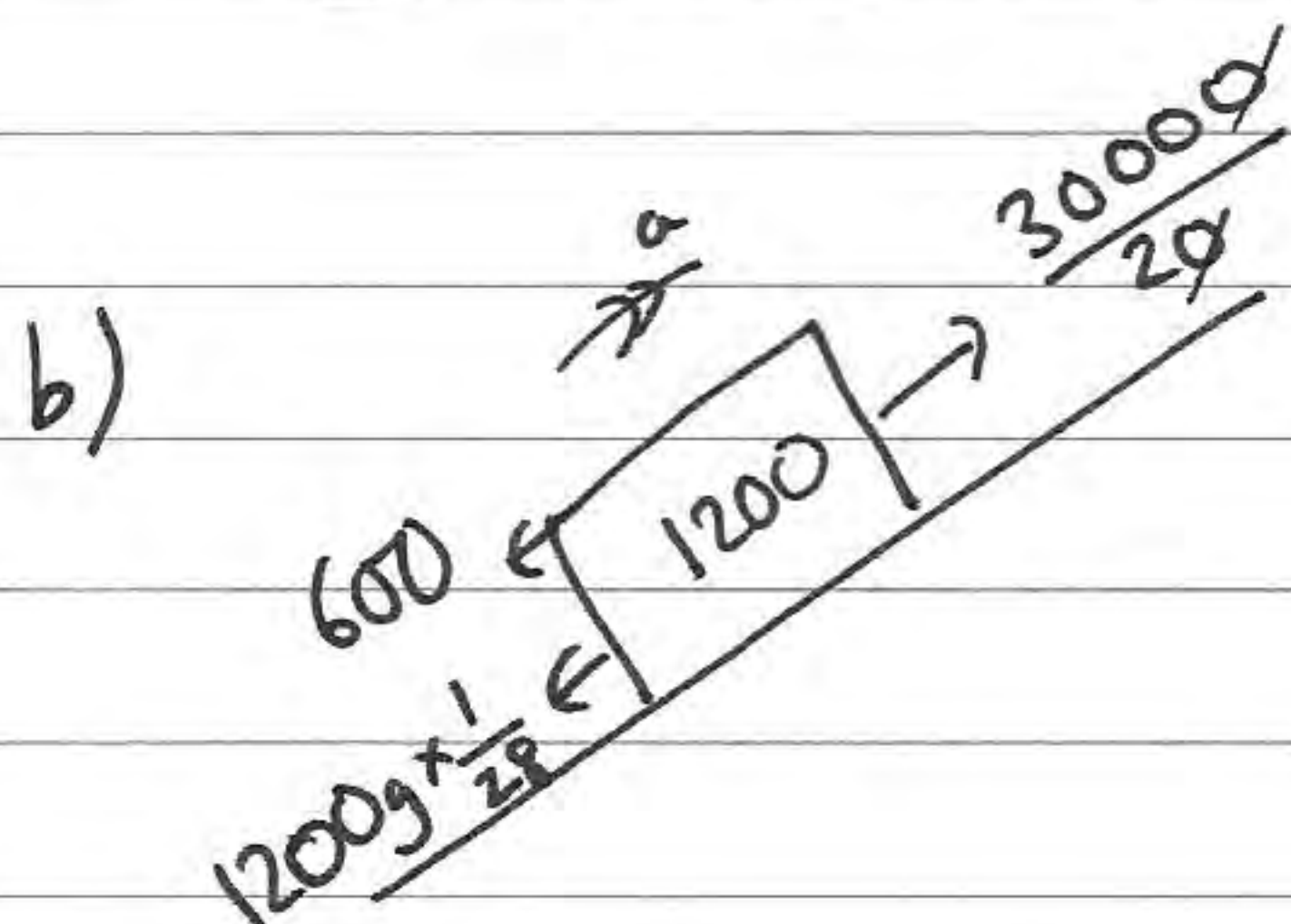
M2 JUNE 06

1) $acc = 5 - 2t$

$v = \int a dt = 5t - t^2 + C$ $t=0, v=6$ $6=C \Rightarrow v = 5t - t^2 + 6$

at rest when $v=0 \Rightarrow 5t - t^2 + 6 = 0, t^2 - 5t - 6 = 0$
 $(t-6)(t+1) = 0 \Rightarrow t = \underline{6 \text{ sec}}$

2)  $\Rightarrow \frac{P}{24} = 600 \Rightarrow P = 14400 \text{ W}$
 $\Rightarrow P = \underline{14.4 \text{ kW}} \quad (3 \text{ s.f.})$

b)  $R_f \uparrow = ma \Rightarrow 1500 - 600 - \frac{1200g}{28} = 1200a$
 $\Rightarrow a = \frac{480}{1200} \Rightarrow a = \underline{0.4 \text{ ms}^{-2}}$

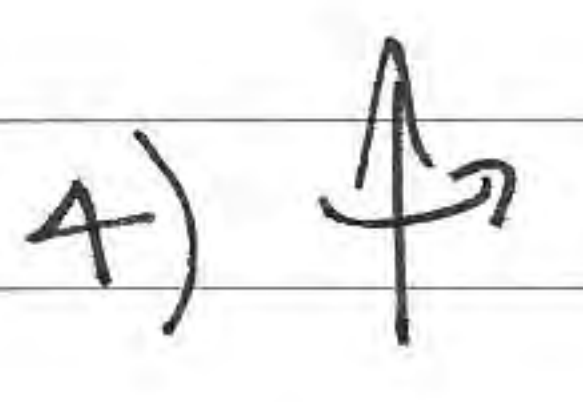
3) Momentum before = $mu = 0.5(-30i) = -15i$ \Rightarrow Impulse
Momentum after = $mv = 0.5(16i + 20j) = 8i + 10j$ $= 23i + 10j$

$|\text{Impulse}| = \sqrt{23^2 + 10^2} = \underline{25.1 \text{ N s}} \quad (3 \text{ s.f.})$

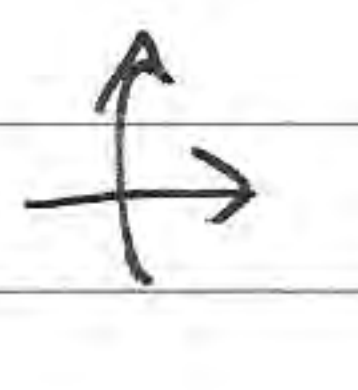
b) $r = 16ti + (20t - 5t^2)j$

$\frac{dr}{dt} = 16i + (20 - 10t)j$ $t=3 \Rightarrow v = 16i - 10j$

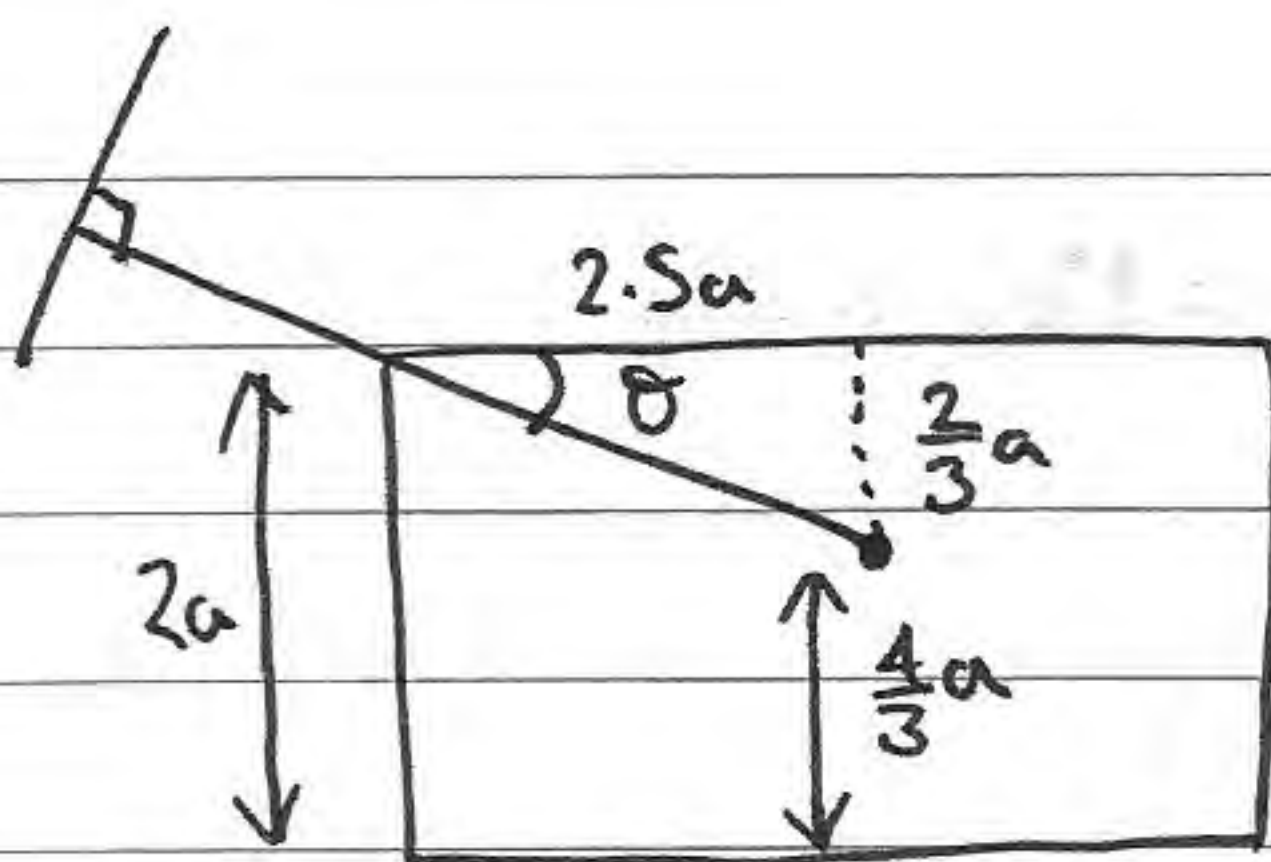
Speed = $\sqrt{16^2 + 10^2} = \underline{18.9 \text{ ms}^{-1}}$

4)  $mg \times 1.5a + mg \times 1.5a + mg \times 3a + 6mg \times 3a + 2mg \times 3a = 12mg \bar{x}$

$30mga = 12mg \bar{x} \quad \bar{x} = \frac{30}{12}a = \underline{2.5a}$

 $mg \times a + mg \times a + mg \times 2a + 6mg \times 2a = 12mg \bar{y} \quad \bar{y} = \frac{16a}{12} = \underline{\frac{4}{3}a}$

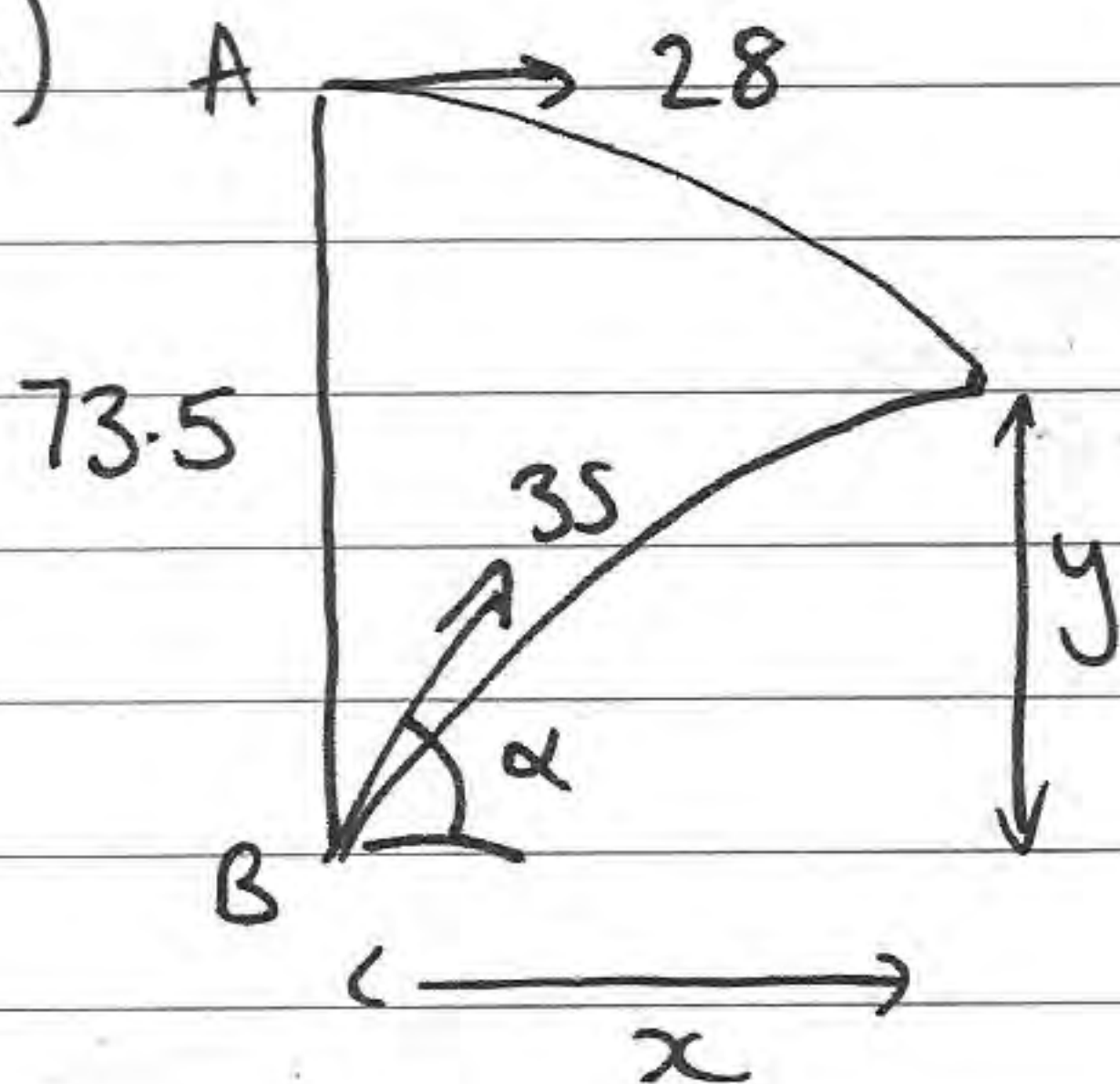
4b)



$$\theta = \tan^{-1} \left(\frac{\frac{2}{3}a}{2.5a} \right)$$

$$\theta = \tan^{-1} \left(\frac{4}{15} \right) = \underline{14.9^\circ} \quad (3sf)$$

5)

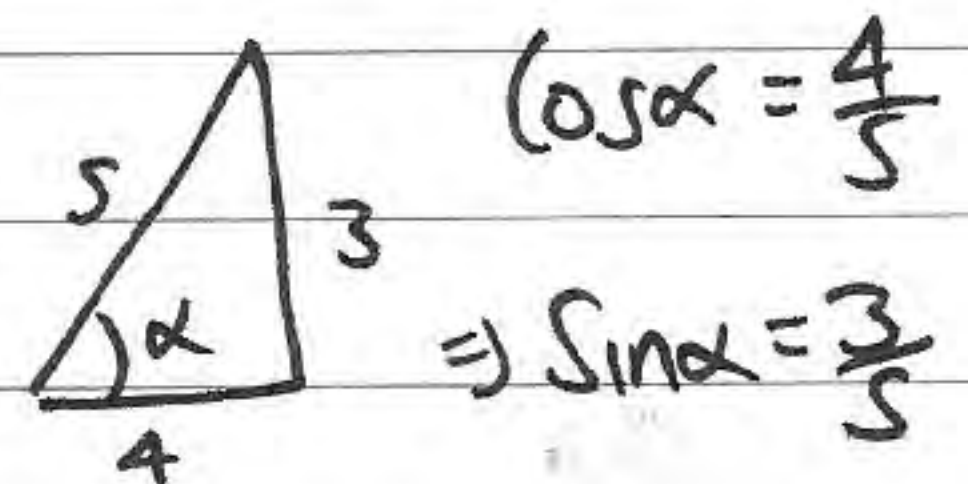


$$\textcircled{A} \quad \text{Vel} = 28 \Rightarrow x = 28t$$

$$\textcircled{B} \quad \text{Vel} = 35 \cos \alpha \Rightarrow x = 35 \cos \alpha \times t$$

$$\Rightarrow 28t = 35 \cos \alpha \times t \Rightarrow \cos \alpha = \frac{28}{35} = \frac{4}{5} \#$$

$$\textcircled{A} \downarrow \quad \begin{aligned} u_{\downarrow} &= 0 \\ a_{\downarrow} &= 9.8 \\ s_{\downarrow} &= 73.5 - y \end{aligned} \Rightarrow 73.5 - y = 4.9t^2$$



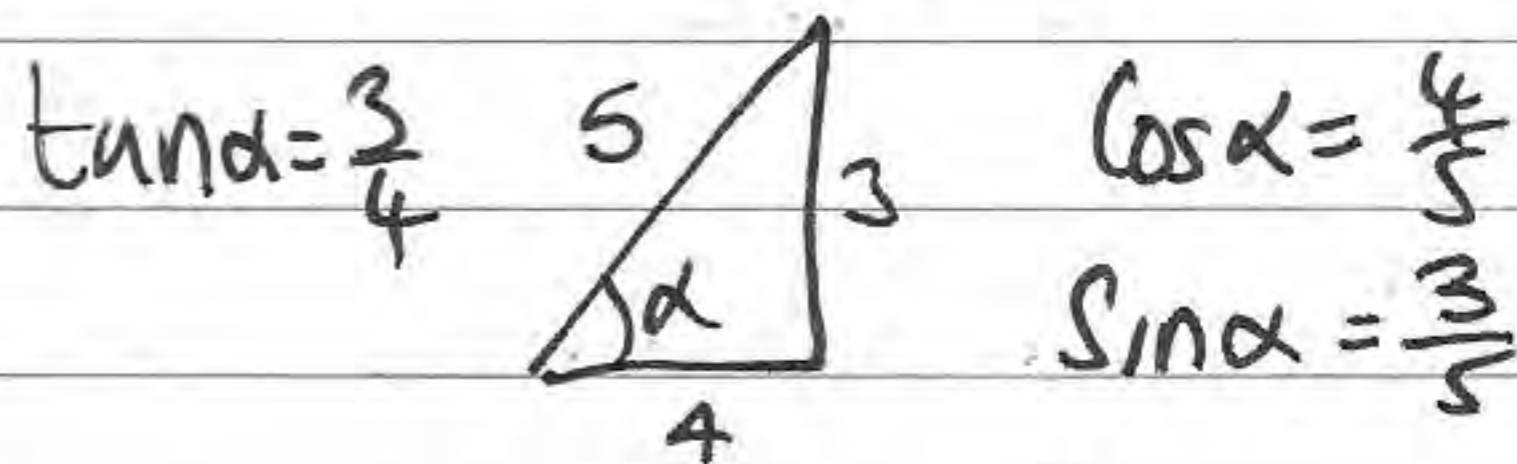
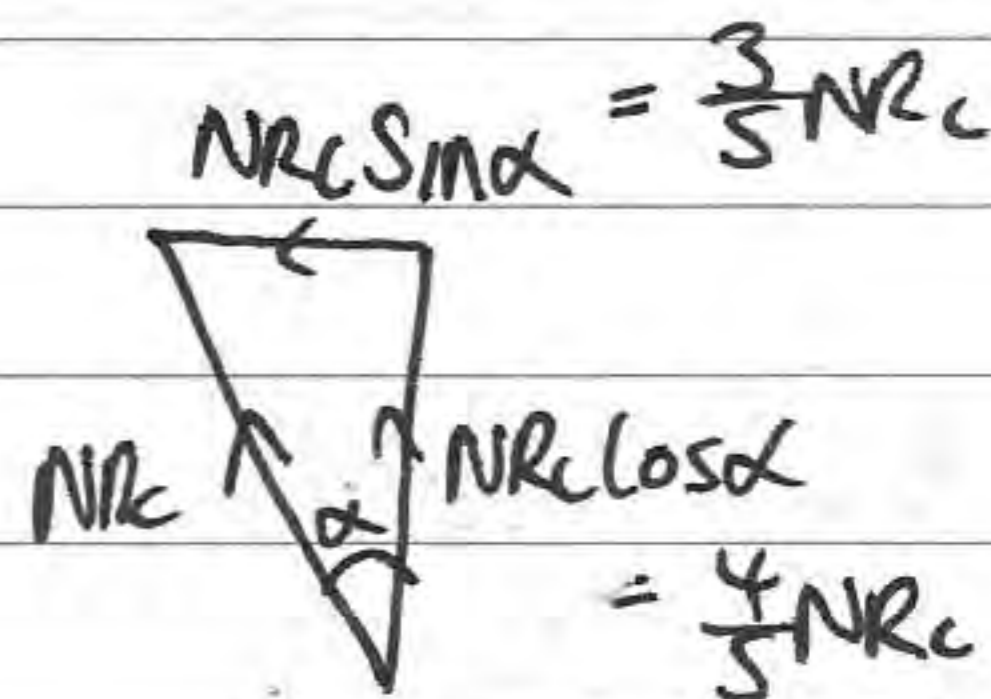
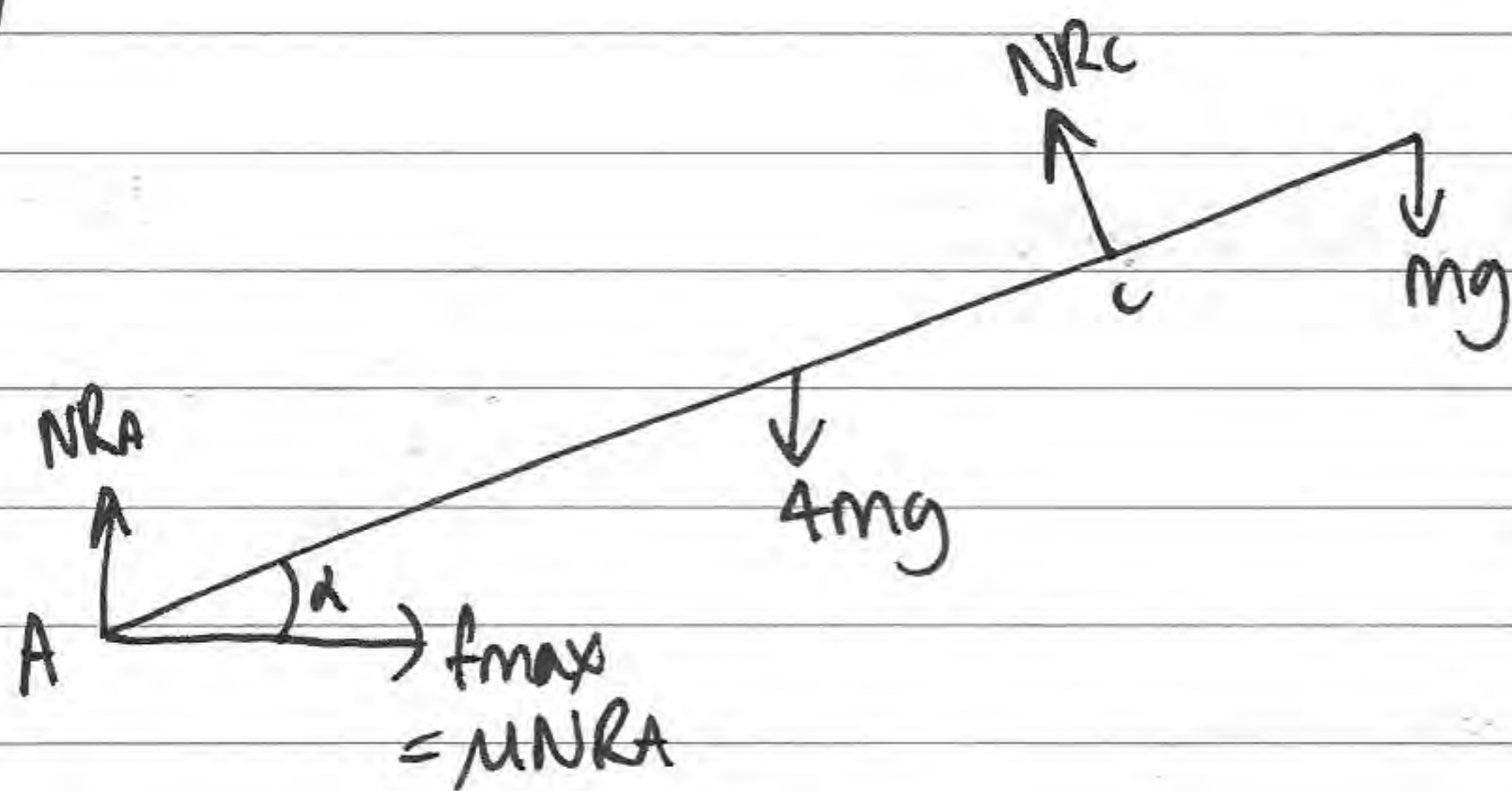
$$\textcircled{B} \uparrow \quad \begin{aligned} u_{\uparrow} &= 35 \sin \alpha = 35 \times \frac{3}{5} = 21 \\ a_{\uparrow} &= -9.8 \\ s_{\uparrow} &= y \end{aligned}$$

$$y = 2t - 4.9t^2 \quad \text{sub in } \textcircled{A}$$

$$73.5 - (2t - 4.9t^2) = 4.9t^2$$

$$\Rightarrow 73.5 - 2t + 4.9t^2 = 4.9t^2 \Rightarrow 2t = 73.5 \Rightarrow t = \underline{3.5 \text{ sec}}$$

6)



$$\uparrow = \downarrow \Rightarrow N_{RA} + \frac{4}{5}N_{Rc} = 5mg \Rightarrow N_{RA} = 5mg - \frac{4}{5}N_{Rc}$$

$$\rightarrow = \leftarrow \frac{3}{5}N_{Rc} = f_{\max} \Rightarrow \frac{3}{5}N_{Rc} = \mu N_{RA}$$

$$A_2 \quad 4mg \times 2 \cos \alpha + mg \times 4 \cos \alpha = N_{Rc} \times 3 \Rightarrow \frac{32}{5}mg + \frac{16}{5}mg = 3N_{Rc}$$

$$\Rightarrow 3N_{Rc} = \frac{48}{5}mg \Rightarrow N_{Rc} = \frac{48}{15}mg \Rightarrow N_{Rc} = \frac{16}{5}mg \neq$$

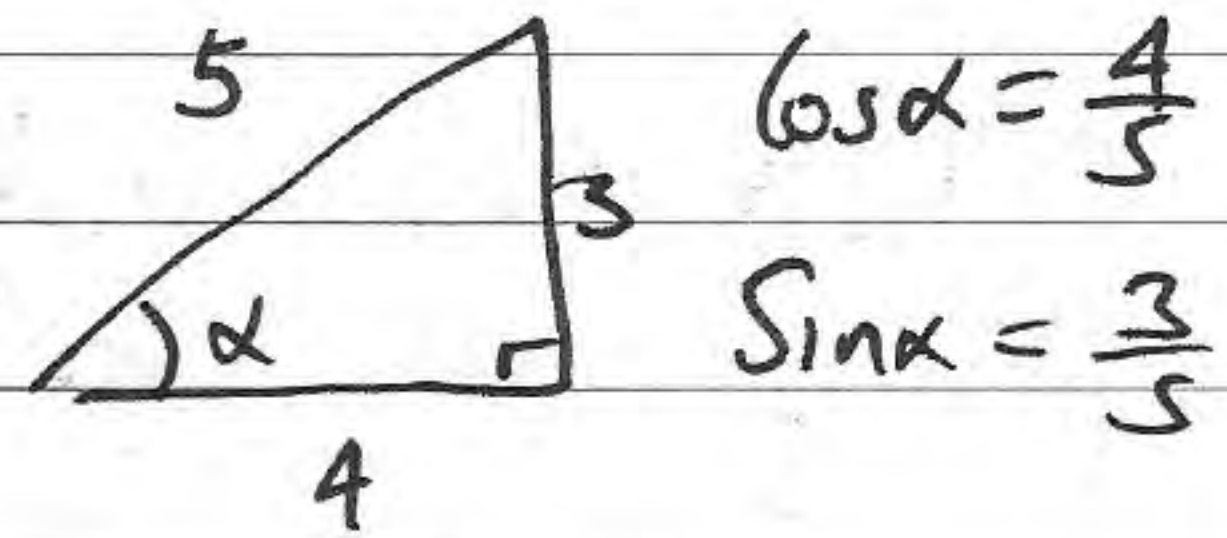
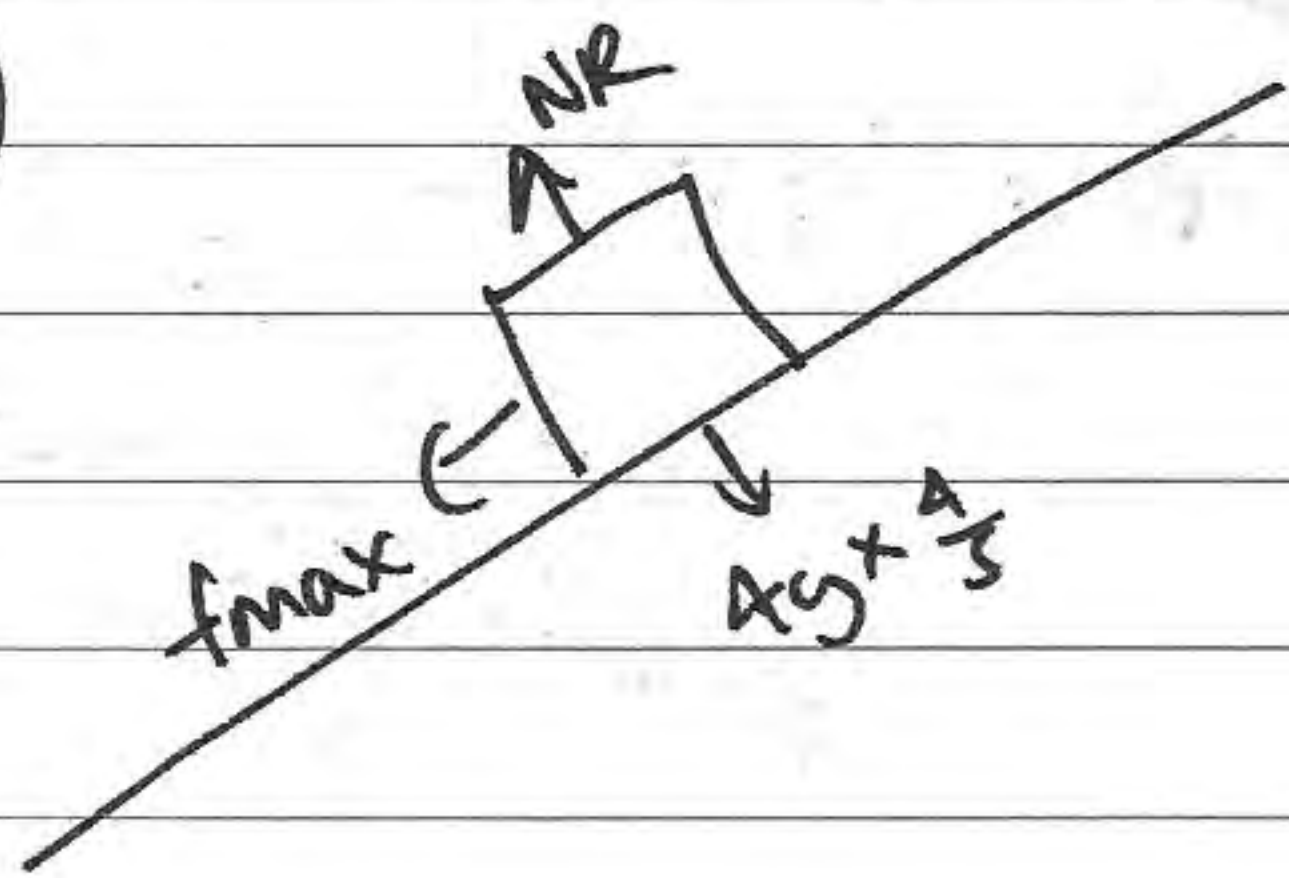
$$b) \quad N_{RA} = 5mg - \frac{4}{5}\left(\frac{16}{5}mg\right) = \frac{61}{25}mg \quad \frac{3}{5}\left(\frac{16}{5}mg\right) = \mu \left(\frac{61}{25}mg\right)$$

$$\mu = \frac{48}{61}$$

at limiting equilibrium $\Rightarrow \mu \geq \frac{48}{61}$
 friction = f_{\max} friction $\leq f_{\max}$.

c) No frictional forces at Peg.

7)



$$\mu = \frac{2}{7}$$

$$f_{\max} = \mu NR = \mu \frac{16}{5} g = \frac{32}{35} g$$

$$\text{Wd against friction} = f_{\max} \times 2.5 = \frac{32}{35} g \times \frac{5}{2} = \frac{16}{7} g \text{ J}$$

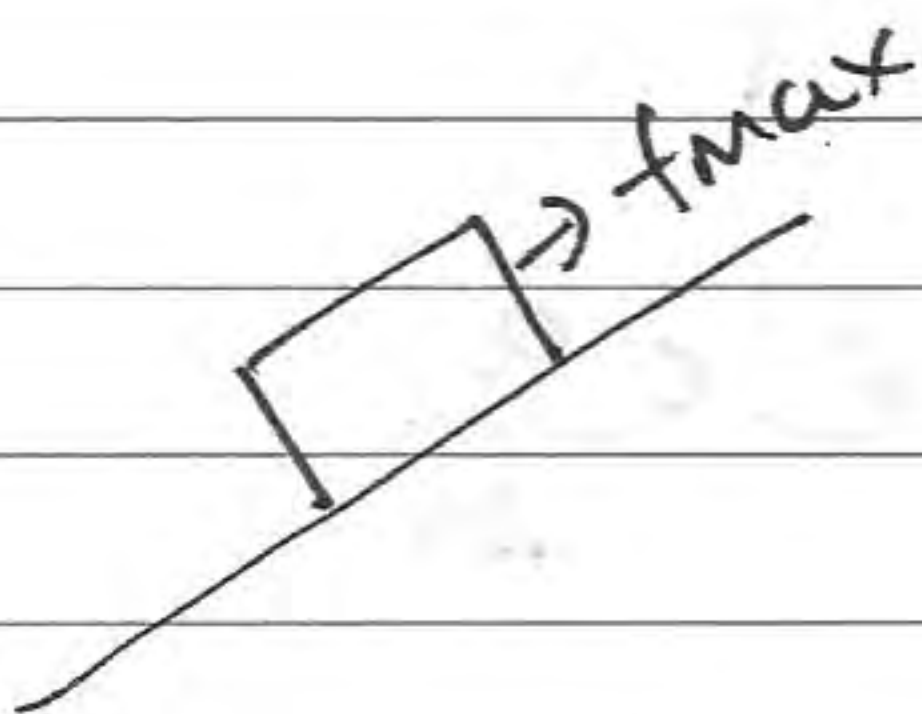
b) $KE_A = \text{Wd against friction} = PE_{\text{gain at B}}$

$$\frac{1}{2}(4)u^2 - \frac{16}{7}g = 4g(2.5 \times \frac{3}{5})$$

$$\Rightarrow 2u^2 = 6g + \frac{16}{7}g \Rightarrow 2u^2 = \frac{58}{7}g \Rightarrow u^2 = \frac{29}{7}g$$

$$\Rightarrow u = \underline{6.4 \text{ m s}^{-1}} \text{ (2sf)}$$

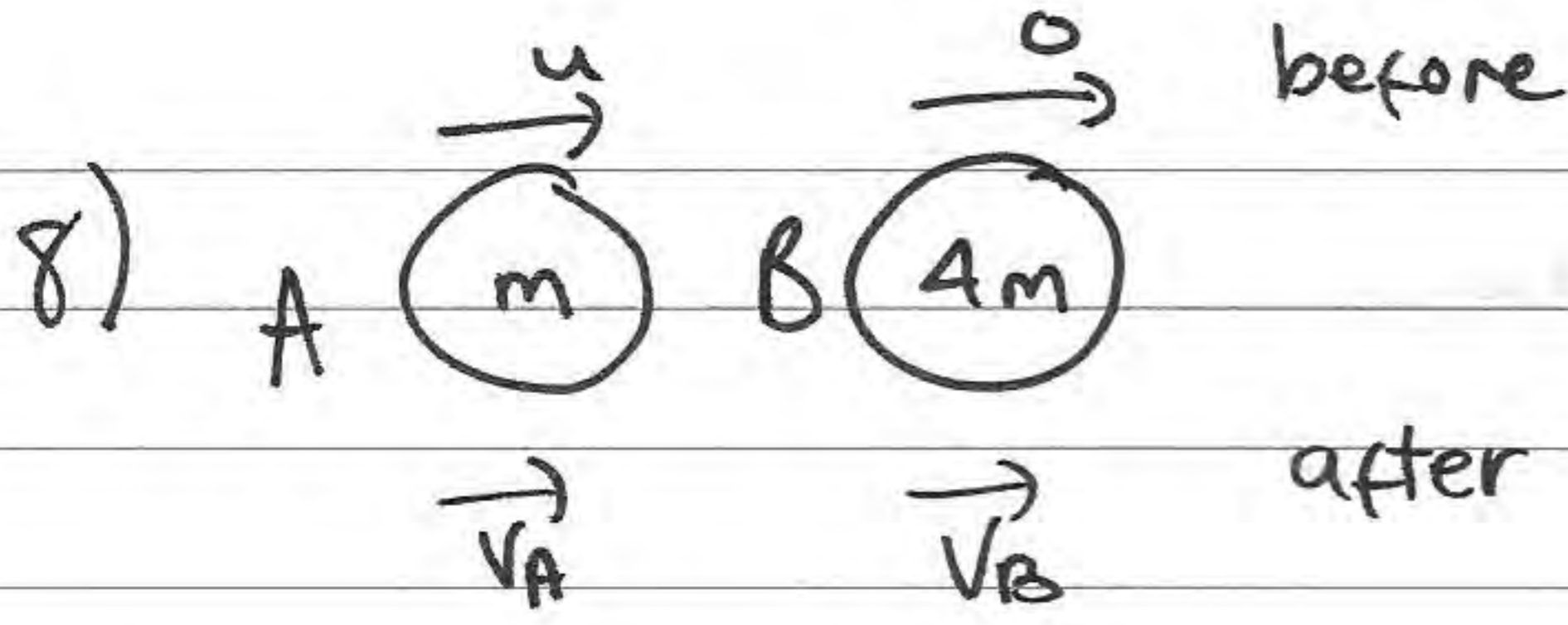
c)



$$PE_B - \text{Wd against friction} = KE_A$$

$$4g(2.5 \times \frac{3}{5}) - \frac{16}{7}g = \frac{1}{2} m v^2$$

$$\Rightarrow \frac{26}{7}g = 2v^2 \Rightarrow v^2 = \frac{13}{7}g \Rightarrow v = \underline{4.3 \text{ m s}^{-1}} \text{ (2sf)}$$



$$e = \frac{v_B - v_A}{u}$$

$$eu = v_B - v_A$$

$$v_B = v_A + eu$$

CLM $\Rightarrow mu = mv_A + 4mv_B$

$$\Rightarrow mu = mv_A + 4m(v_A + eu)$$

$$\Rightarrow u = v_A + 4v_A + 4eu$$

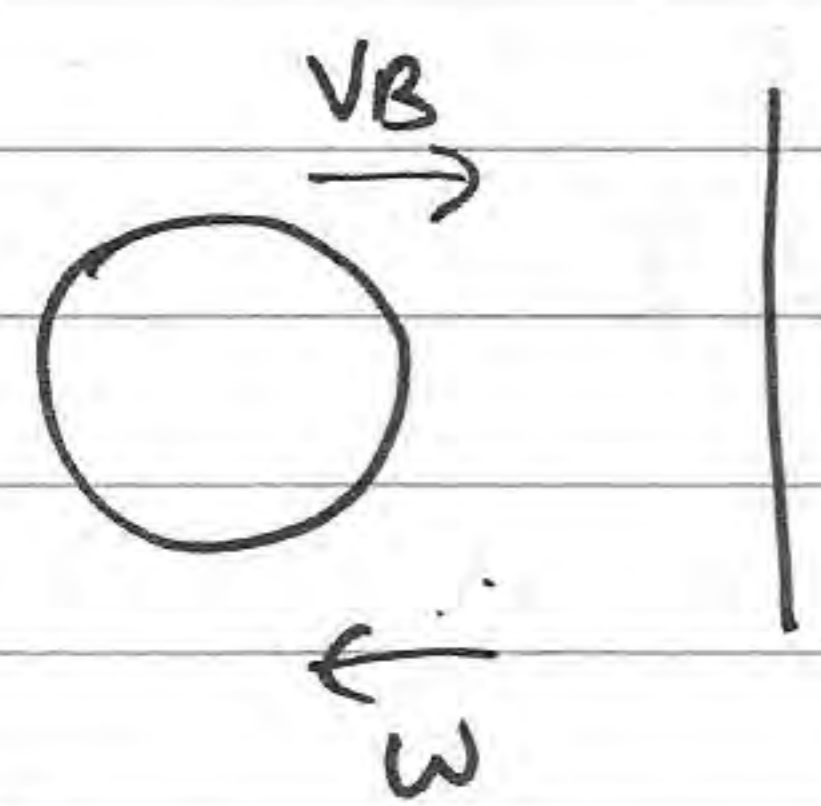
$$\Rightarrow u = 4eu = 5v_A$$

$$\Rightarrow v_A = \frac{1}{5}u(1 - 4e)$$

A is reversed
 $\Rightarrow \text{speed}_A = \frac{1}{5}u(4e - 1)$

$$v_B = \frac{1}{5}u - \frac{4}{5}eu + eu$$

$$\Rightarrow v_B = \frac{1}{5}u(1 + e)$$



$$e = \frac{\omega}{v_B} = \frac{4}{5} \Rightarrow 4v_B = 5\omega$$

$$\Rightarrow \frac{4}{5}v_B = \omega$$

Since B and A collide $\omega > \frac{1}{5}u(4e - 1)$

$$\frac{4}{5}v_B > \frac{1}{5}u(4e - 1) \Rightarrow \frac{4}{5} \left(\frac{1}{5}u(1 + e) \right) > \frac{1}{5}u(4e - 1)$$

$$\Rightarrow \frac{4}{5} + \frac{4}{5}e > 4e - 1$$

$$\Rightarrow \frac{9}{5} > \frac{16}{5}e \Rightarrow e < \frac{9}{16}$$

$$v_A = \frac{1}{5}u(1 - 4e) \quad \text{A is reversed} \Rightarrow v_A < 0$$

Since $u > 0 \Rightarrow 1 - 4e < 0 \quad 1 < 4e \Rightarrow \frac{1}{4} < e \Rightarrow e > \frac{1}{4}$

$$\therefore \frac{1}{4} < e < \frac{9}{16} \quad \#$$

$$c) \quad v_A = \frac{1}{5}u(1-4e) \quad e = \frac{1}{2} \Rightarrow v_A = -\frac{1}{5}u$$

$$v_B = \frac{1}{5}u(1+e) \quad e = \frac{1}{2} \Rightarrow v_B = \frac{3}{10}u.$$

$$KE_{\text{before}} = \frac{1}{2}(m)u^2 = \frac{1}{2}mu^2$$

$$KE_{\text{after}} = \frac{1}{2}m\left(-\frac{1}{5}u\right)^2 + \frac{1}{2}(4m)\left(\frac{3}{10}u\right)^2 = \frac{1}{50}mu^2 + \frac{9}{50}mu^2 \\ = \frac{1}{5}mu^2$$

$$KE_{\text{lost}} = \frac{1}{2}mu^2 - \frac{1}{5}mu^2 = \underline{\underline{\frac{3}{10}mu^2}}$$