JAN 12

ANSEY

1. A tennis ball of mass 0.1 kg is hit by a racquet. Immediately before being hit, the ball has velocity $30\mathbf{i}$ m s^{-1}. The racquet exerts an impulse of $(-2\mathbf{i}-4\mathbf{j})$ N s on the ball. By modelling the ball as a particle, find the velocity of the ball immediately after being hit.

(4)

mom before $= 0.1\begin{pmatrix}30\\0\end{pmatrix} = \begin{pmatrix}3\\0\end{pmatrix}$

+ Impulse =

mom after $= \begin{pmatrix}3\\0\end{pmatrix} + \begin{pmatrix}-2\\-4\end{pmatrix} = \begin{pmatrix}1\\-4\end{pmatrix} = mV$

$\therefore V = \begin{pmatrix}10\\-40\end{pmatrix}$

2. A particle P is moving in a plane. At time t seconds, P is moving with velocity $\mathbf{v}$ m s^{-1}, where $\mathbf{v} = 2t\mathbf{i} - 3t^2\mathbf{j}$.

Find

(a) the speed of P when $t = 4$

(2)

(b) the acceleration of P when $t = 4$

(3)

Given that P is at the point with position vector $(-4\mathbf{i} + \mathbf{j})$ m when $t = 1$,

(c) find the position vector of P when $t = 4$

(5)

a) $v = \begin{pmatrix} 2t \\ -3t^2 \end{pmatrix}$ $t=4$ $v = \begin{pmatrix} 8 \\ -48 \end{pmatrix}$ speed $= \sqrt{8^2+48^2}$

$= \underline{48.7\,\text{ms}^{-1}}$ (3sf)

b) $a = \frac{dv}{dt} = \begin{pmatrix} 2 \\ -6t \end{pmatrix}$ $t=4$ $a = \begin{pmatrix} 2 \\ -24 \end{pmatrix}$

$a = \sqrt{2^2+24^2} = \underline{24.1}\ \text{ms}^{-2}$

c) $s = \int v\,dt = \begin{pmatrix} t^2 + c_1 \\ -t^3 + c_2 \end{pmatrix}$ $t=1$ $s = \begin{pmatrix} 1+c_1 \\ -1+c_2 \end{pmatrix} = \begin{pmatrix} -4 \\ 1 \end{pmatrix}$

$\therefore\ c_1 = -5 \quad c_2 = 2$

$s = \begin{pmatrix} t^2 - 5 \\ 2 - t^3 \end{pmatrix}$ $t = 4 \Rightarrow s = \begin{pmatrix} 11 \\ -62 \end{pmatrix}$

3. A cyclist and her cycle have a combined mass of 75 kg. The cyclist is cycling up a straight road inclined at 5° to the horizontal. The resistance to the motion of the cyclist from non-gravitational forces is modelled as a constant force of magnitude 20 N. At the instant when the cyclist has a speed of 12 m s^{-1}, she is decelerating at 0.2 m s^{-2}.

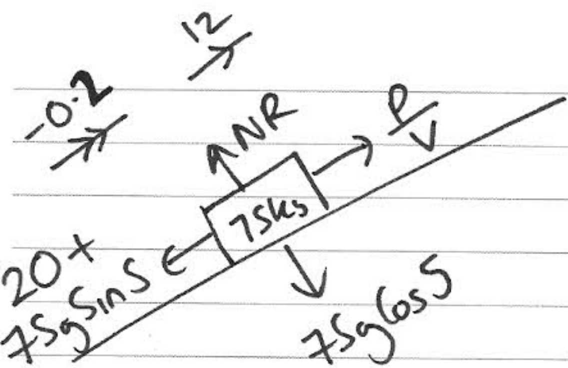
(a) Find the rate at which the cyclist is working at this instant.

(5)

When the cyclist passes the point *A* her speed is 8 m s^{-1}. At *A* she stops working but does not apply the brakes. She comes to rest at the point *B*.
The resistance to motion from non-gravitational forces is again modelled as a constant force of magnitude 20 N.

(b) Use the work-energy principle to find the distance *AB*.

(5)



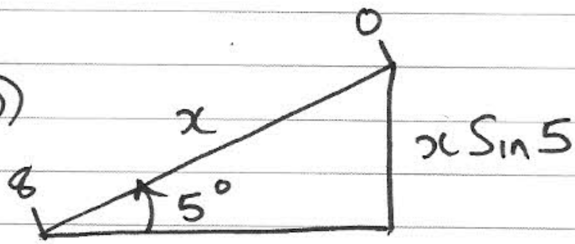
$\nearrow$ RF = ma

$\frac{P}{12} - 20 - 75g\sin 5 = 75 \times -0.2$

$\frac{P}{12} = 69.059.. \quad P = 828.7\text{w}$

$\therefore P = 0.83\text{kW}$ (2sf)

b)

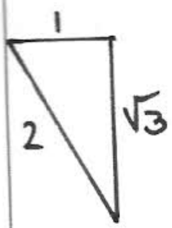


$\boxed{KE_A} = \boxed{\begin{array}{c} PE\ B \\ \hline WdR \end{array}}$

$\frac{1}{2}(75)8^2 = 75g\,x\sin 5 + 20x$

$2400 = (75g\sin 5 + 20)x \quad \therefore x = \underline{28.6\text{m}}$ (3sf)

4.



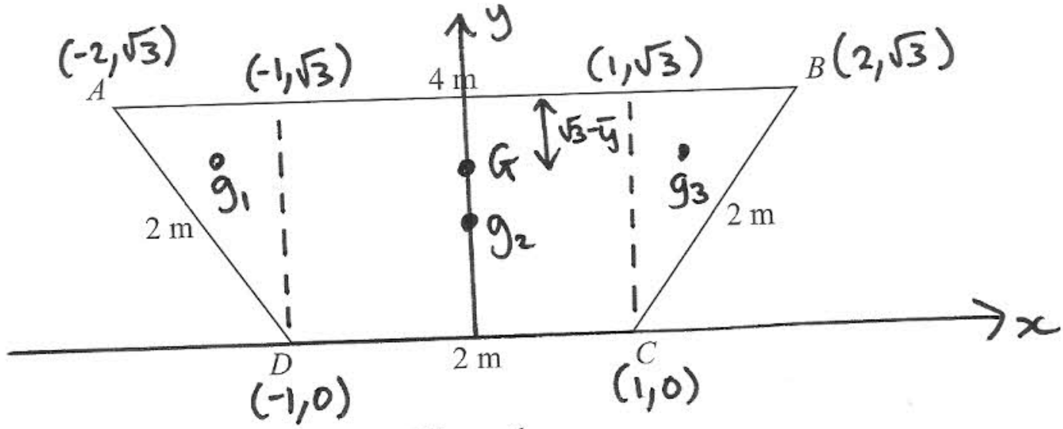


Figure 1

The trapezium $ABCD$ is a uniform lamina with $AB = 4$ m and $BC = CD = DA = 2$ m, as shown in Figure 1.

(a) Show that the centre of mass of the lamina is $\frac{4\sqrt{3}}{9}$ m from AB. **(5)**

The lamina is freely suspended from D and hangs in equilibrium.

(b) Find the angle between DC and the vertical through D. **(5)**

$g_1\left(-\frac{4}{3}, \frac{2}{3}\sqrt{3}\right)$ $M_1 = \frac{\sqrt{3}}{2}k$ $\qquad \circlearrowleft x$

$g_2\left(0, \frac{1}{2}\sqrt{3}\right)$ $M_2 = 2\sqrt{3}k$ $\qquad 2\left(\frac{\sqrt{3}}{2}k\cancel{g}\times\frac{2\sqrt{3}}{3}\right)+2\sqrt{3}k\cancel{g}\times\frac{\sqrt{3}}{2}$

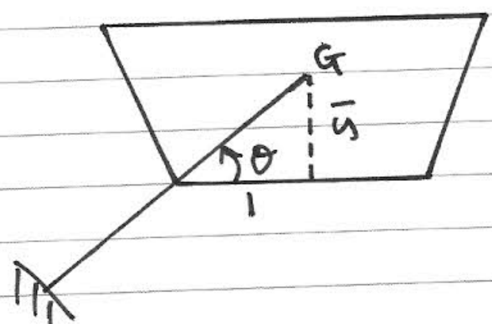
$g_3\left(\frac{4}{3}, \frac{2}{3}\sqrt{3}\right)$ $M_3 = \frac{\sqrt{3}}{2}k$ $\qquad = 3\sqrt{3}k\cancel{g}\times\bar{y}$

$G(0, \bar{y})$ $M = 3\sqrt{3}k$ $\qquad \Rightarrow 2+3 = 3\sqrt{3}\,\bar{y}$

mass per unit area $= k$ $\qquad \bar{y} = \frac{5}{3\sqrt{3}} = \frac{5\sqrt{3}}{9}$

$\therefore$ from AB $= \sqrt{3}-\bar{y} = \frac{4\sqrt{3}}{9}$ #

b)



$\theta = \tan^{-1}\left(\frac{\bar{y}}{1}\right)$

$= \tan^{-1}\left(\frac{5\sqrt{3}}{9}\right) = 43.9^\circ$ (3sf)

5.

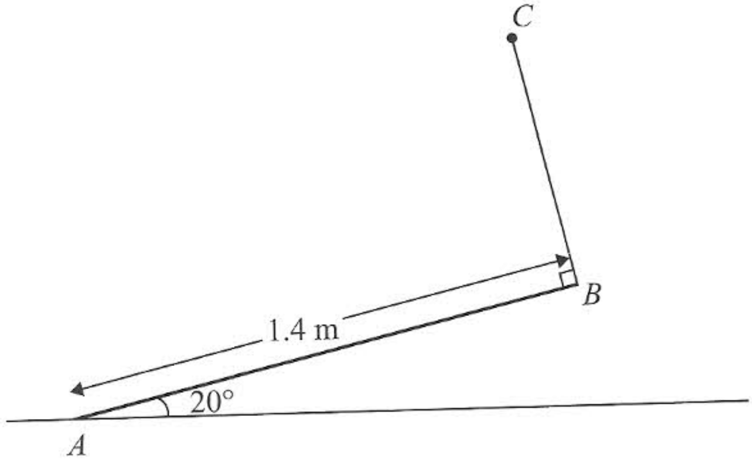


Figure 2

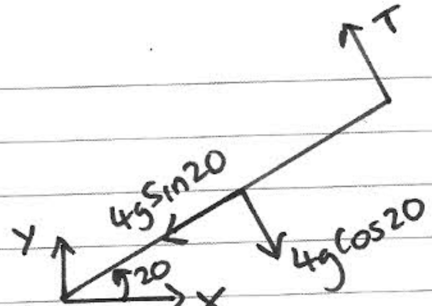
A uniform rod AB has mass 4 kg and length 1.4 m. The end A is resting on rough horizontal ground. A light string BC has one end attached to B and the other end attached to a fixed point C. The string is perpendicular to the rod and lies in the same vertical plane as the rod. The rod is in equilibrium, inclined at 20° to the ground, as shown in Figure 2.

(a) Find the tension in the string. **(4)**

Given that the rod is about to slip,

(b) find the coefficient of friction between the rod and the ground. **(7)**

a)

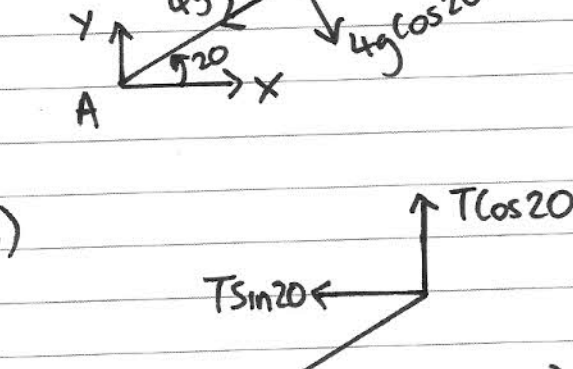


$A\circlearrowright$ $4g\cos 20 \times 0.7 = T \times 1.4$

$T = 18.41797...$

$T = 18.4N$ (3sf)

b)



$RF\uparrow = 0$

$NR_A = 4g - T\cos 20$

$\overrightarrow{RF} = 0 \Rightarrow T\sin 20 = f_{max} = \mu NR_A$

$\therefore T\sin 20 = \mu(4g - T\cos 20)$

$\therefore \mu = \dfrac{T\sin 20}{4g - T\cos 20} = 0.288$ (3sf)

6. Three identical particles, A, B and C, lie at rest in a straight line on a smooth horizontal table with B between A and C. The mass of each particle is m. Particle A is projected towards B with speed u and collides directly with B. The coefficient of restitution between each pair of particles is $\frac{2}{3}$.

(a) Find, in terms of u,

(i) the speed of A after this collision,

(ii) the speed of B after this collision.

(7)

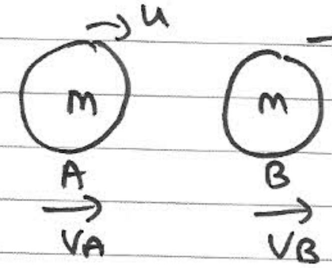
(b) Show that the kinetic energy lost in this collision is $\frac{5}{36}mu^2$

(4)

After the collision between A and B, particle B collides directly with C.

(c) Find, in terms of u, the speed of C immediately after this collision between B and C.

(4)



$e = \frac{2}{3}$ $\quad \frac{V_B - V_A}{u} = \frac{2}{3} \Rightarrow 3V_B - 3V_A = 2u$

CLM ⇒ $\cancel{m}u = \cancel{m}V_A + \cancel{m}V_B$

$u = V_A + V_B$ (×3)

$\therefore 3V_B + 3V_A = 3u$

$3V_B - 3V_A = 2u$ −

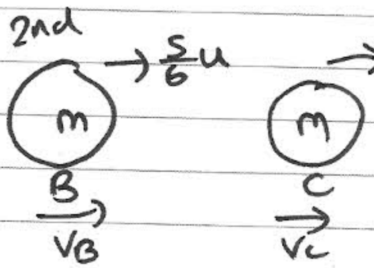
$6V_A = u \quad \therefore V_A = \frac{1}{6}u$

$\Rightarrow V_B = \frac{5}{6}u$

b) $KE_{lost} = \frac{1}{2}mu^2 - \frac{1}{2}m(V_A^2 + V_B)^2$

$= \frac{1}{2}m\left[u^2 - \frac{26}{36}u^2\right] = \frac{5}{36}mu^2$

c) 2nd



$e = \frac{2}{3}$ $\quad \frac{V_C - V_B}{\frac{5}{6}u} = \frac{2}{3} \Rightarrow 18V_C - 18V_B = 10u$

CLM $\quad \cancel{m}\frac{5}{6}u = \cancel{m}V_B + \cancel{m}V_C$

$5u = 6V_B + 6V_C$ (×3)

$15u = 18V_B + 18V_C$

$$18V_C + 18V_B = 15u$$
$$\underline{18V_C - 18V_B = 10u} \; +$$
$$36V_C = 25u$$
$$\Rightarrow V_C = \frac{25}{36}u$$

7. [*In this question, the unit vectors* **i** *and* **j** *are horizontal and vertical respectively.*]

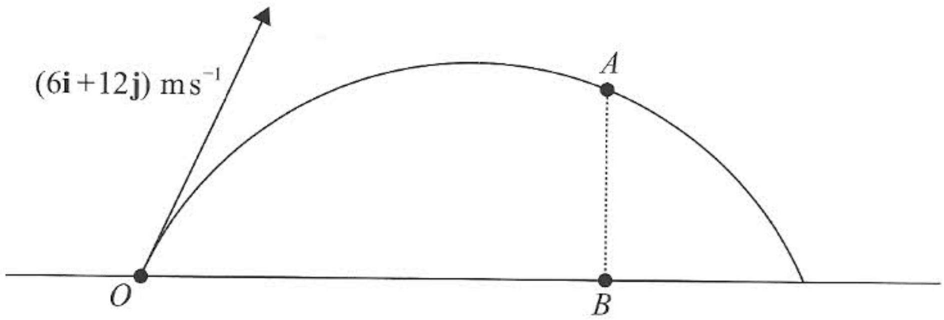


Figure 3

The point O is a fixed point on a horizontal plane. A ball is projected from O with velocity $(6\mathbf{i} + 12\mathbf{j})$ m s^{-1}, and passes through the point A at time t seconds after projection. The point B is on the horizontal plane vertically below A, as shown in Figure 3. It is given that $OB = 2AB$.

Find

(a) the value of t,

(7)

(b) the speed, V m s^{-1}, of the ball at the instant when it passes through A.

(5)

At another point C on the path the speed of the ball is also V m s^{-1}.

(c) Find the time taken for the ball to travel from O to C.

(3)

$\overrightarrow{H}$ Speed $= 6$
dist $= 2x$
dist $=$ speed $\times$ time
$2x = 6t$
$\therefore x = 3t$

$V\uparrow$ $u = 12\uparrow$
$s = x$
$a = -9.8\uparrow$

$s = ut + \frac{1}{2}at^2$
$x = 12t - 4.9t^2$

$\Rightarrow 3t = 12t - 4.9t^2$ $t = \frac{9}{4.9}$
$t = 1.8367 \simeq 1.84$ (3sf)

b) $v\uparrow = u + at = 12 - 9.8t = -6$

$\therefore$ speed $= \sqrt{6^2 + 6^2} = 8.4852.. \simeq 8.49$ (3sf)

c) $V_h\rightarrow = 6$ Same speed $\therefore V\uparrow = 6$

$\uparrow v = u + at$ $6 = 12 - 9.8t$ $t = \frac{6}{9.8}$ $t = 0.612$ (3sf)