M2 JAN 08

1) KE lost $= \frac{1}{2}(2.5)8^2 = \underline{80J}$

b) KE lost = Wd against Res = Res × 20 $\Rightarrow$ Res = $\underline{4N}$

2) $\frac{dp}{dt} = (6t-6)i + (9t^2-4)j$ = vel

b) parallel to i $\Rightarrow$ j = 0 $\Rightarrow 9t^2-4=0 \Rightarrow t^2 = \frac{4}{9} \Rightarrow t = \frac{2}{3}$

c) $t=1 \Rightarrow u = 0i + 5j$ Mom = mv = $\frac{1}{2}(0i+5j) = 0i+2.5j$

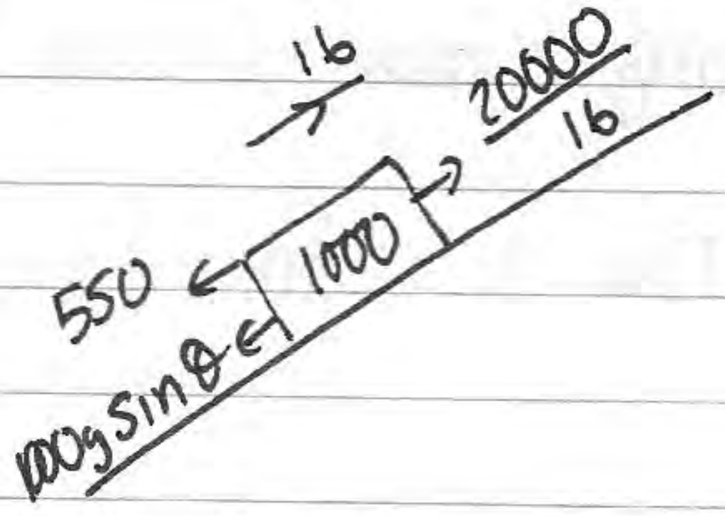
+ Impulse + $2i-6j$

final Mom = $2i-3.5j$

mv = $\frac{1}{2}$v = $2i-3.5j$

final vel = $\underline{4i-7j}$ ms^{-1}

3)



$\uparrow R_f = 0 \Rightarrow \frac{20000}{16} = 550 + 1000g\sin\theta$

$\Rightarrow \sin\theta = \frac{700}{1000g} = \frac{7}{98} = \frac{1}{14}$ #

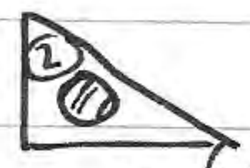
b) KE lost = Gain in PE + Wd against Res

$\frac{1}{2}(1000)16^2 = (1000)g(y\sin\theta) + 550 \times y$

$\Rightarrow 128000 = 700y + 550y = 1250y \Rightarrow y = 102.4$

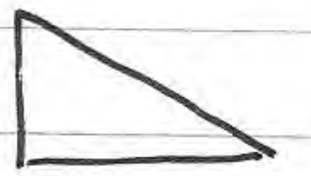
$\underline{y = 102m \text{ (3sf)}}$

4) ① M = 9πk, $g_1(5,5)$



M = (126−9π)k, $g_2(\bar{x},\bar{y})$

$\Rightarrow$



M = 126k, G(7,4)

mass per unit area = k

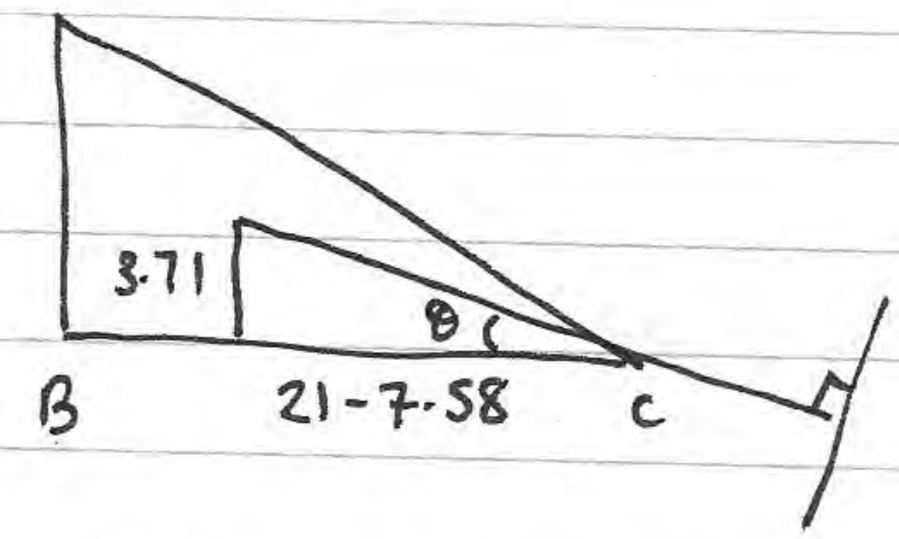
A
B ↑ $9\pi kg \times 5 + (126 - 9\pi)kg \times \bar{x} = 126kg \times 7$

$$\bar{x} = \frac{882 - 45\pi}{126 - 9\pi} = \underline{7.58cm\ (3sf)}$$

B → C $9\pi kg \times 5 + (126 - 9\pi)kg \times \bar{y} = 126kg \times 4$

$$\bar{y} = \frac{504 - 9\pi \times 5}{126 - 9\pi} = 3.71cm\ (3sf)$$

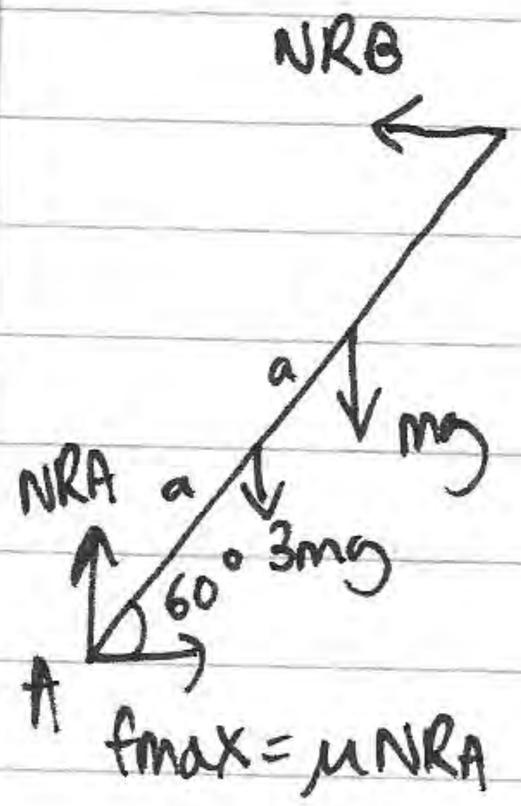
b)



$$\theta = \tan^{-1}\left(\frac{3.71}{13.42}\right)$$

$$\theta = 15°\ (nd)$$

5)



$R\!f\uparrow = 0 \Rightarrow NR_A = 4mg \Rightarrow fmax = \mu 4mg$

$\vec{R}f = 0 \Rightarrow fmax = NR_B \Rightarrow \mu 4mg = NR_B$

$A\circlearrowright$ $3mg \times a\cos 60 + mg \times 2a\cos 60 = NR_B \times 4a\sin 60$

$\Rightarrow \frac{3}{2}mga + mga = \mu \times 8\sqrt{3}\,mga$

$$\mu = \frac{2.5}{8\sqrt{3}} = \frac{5}{16\sqrt{3}} \quad (= \underline{0.18\ (2sf)})$$

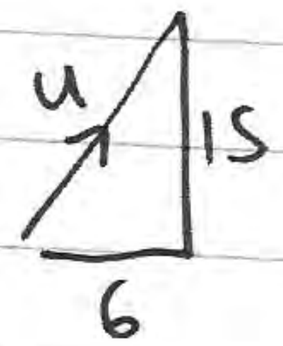
6) (→ H) $x = 30$ $Vel = 2u \Rightarrow t = \frac{30}{2u} = \frac{15}{u}$

(↑ V) $u\uparrow = 5u$ $a = -9.8$ $S = -47.5$ $t = \frac{15}{u}$

$$-47.5 = (5u)\left(\frac{15}{u}\right) - 4.9\left(\frac{15}{u}\right)^2 \Rightarrow 4.9\left(\frac{15^2}{u^2}\right) = 122.5$$

$$\Rightarrow u^2 = 9 \Rightarrow u = 3 \Rightarrow t = \frac{15}{3} = \underline{5sec}$$

6 b) $u=3$ from (a) $\vec{u}=6$ $u\uparrow=15$



$u^2=15^2+6^2$

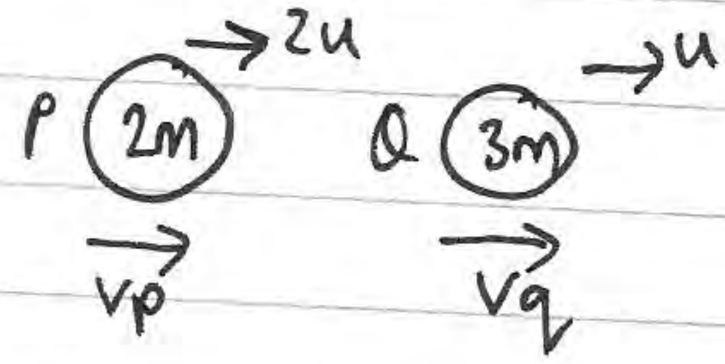
$u^2=261$

c) KE gained = PE lost

$\frac{1}{2}m(v^2-261)=mg(47.5)$

$\Rightarrow v^2-261=931 \Rightarrow v^2=1192 \Rightarrow v=\underline{34.5 ms^{-1}}$ (3sf)

7)



$e=\frac{v_q-v_p}{u}=\frac{1}{2}$

$\Rightarrow 2v_q-2v_p=u$

CLM $\Rightarrow 4mu+3mu=2mv_p+3mv_q$

$\Rightarrow 7mu=m(2v_q-u)+3mv_q$

$\Rightarrow 7mu=5mv_q-mu$

$\Rightarrow 8mu=5mv_q$

$2v_p=2v_q-u$

$\Rightarrow v_q=\frac{8}{5}u$ ##

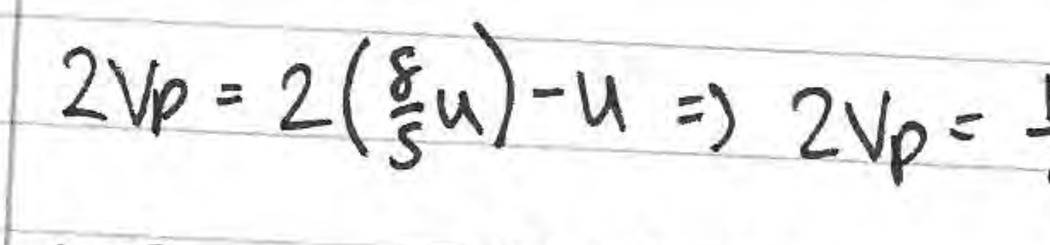
b) KE before $=\frac{1}{2}(2m)(2u)^2+\frac{1}{2}(3m)u^2=4mu^2+\frac{3}{2}mu^2=\frac{11}{2}mu^2$

$2v_p=2(\frac{8}{5}u)-u \Rightarrow 2v_p=\frac{11}{5}u \Rightarrow v_p=\frac{11}{10}u$

KE after $=\frac{1}{2}(2m)(\frac{11}{10}u)^2+\frac{1}{2}(3m)(\frac{8}{5}u)^2=\frac{121}{100}mu^2+\frac{96}{25}mu^2=\frac{101}{20}mu^2$

KE lost $=\frac{11}{2}mu^2-\frac{101}{20}mu^2=\underline{\frac{9}{20}mu^2}$

c)



If Q collides into P again $v_Q<\frac{11}{10}u$

$e_{QR}=\frac{v_R-v_Q}{\frac{8}{5}u}=e$ $v_R-v_Q=\frac{8}{5}ue \Rightarrow v_Q=v_R-\frac{8}{5}ue$

$v_R=v_Q+\frac{8}{5}ue$

CLM $\frac{24}{5}mu+0=3mv_Q+mv_R$

$\Rightarrow 24mu = 15mV_Q + 5V_R m$ $\qquad 5V_R = 5V_Q + 8ue$

$\Rightarrow 24mu = 15mV_Q + (5V_Q + 8ue)m$

$\Rightarrow 24\cancel{m}u = 20\cancel{m}V_Q + 8\cancel{m}eu$

$\Rightarrow 6u = 5V_Q + 2eu$

$\Rightarrow \frac{6u - 2eu}{5} = V_Q \qquad \Rightarrow V_Q = \frac{2}{5}u(3-e) \qquad V_Q < \frac{11}{10}u$

$\frac{2}{5}\cancel{u}(3-e) < \frac{11}{10}\cancel{u} \Rightarrow 3-e < \frac{55}{20} \Rightarrow \frac{5}{20} < e$

$\therefore e > \frac{1}{4} \qquad \frac{1}{4} < e \leq 1$