MECHANICS (C) UNIT 2 **TEST PAPER 8**

Take $g = 9{\cdot}8$ ms^{-2} and give all answers correct to 3 significant figures where necessary.

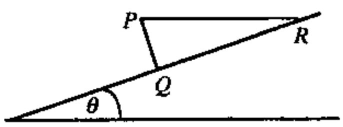
1. A heavy ball, of mass 2 kg, rolls along a horizontal surface. It strikes a vertical wall at a speed of 4 ms^{-1} and rebounds. The coefficient of restitution between the ball and the wall is 0·4.
Find the kinetic energy lost in the impact. [4]

2. A uniform plank of wood XY, of mass 1·4 kg, rests with its upper end X against a rough vertical wall and its lower end Y on rough horizontal ground. The coefficient of friction between the plank and both the wall and the ground is μ. The plank is in limiting equilibrium at both ends and the vertical component of the force exerted on the plank by the ground has magnitude 12 N.
Find the value of μ, to 2 decimal places. [7]

3. A motor-cycle and its rider have a total mass of 460 kg. The maximum rate at which the cycle's engine can work is 25 920 W and the maximum speed of the cycle on a horizontal road is 36 ms^{-1}. A variable resisting force acts on the cycle and has magnitude kv^2, where v is the speed of the cycle in ms^{-1}.
(i) Show that $k = \frac{5}{9}$. [3]
(ii) Find the acceleration of the cycle when it is moving at 25 ms^{-1} on the horizontal road, with its engine working at full power. [4]

4. PQR is a triangular lamina with $PQ = 18$ cm, $QR = 24$ cm and $PR = 30$ cm.
(i) Verify that angle PQR is a right angle and find the distances of the centre of mass of the lamina from (a) PQ, (b) QR. [4]

The lamina is held in a vertical plane and placed on a line of greatest slope of a rough plane inclined at an angle θ to the horizontal, as shown.



(ii) Find the largest value of θ for which equilibrium cannot be broken by toppling. [3]

5. Two smooth spheres A and B, of equal radius and masses $9m$ and $4m$ respectively, are moving towards each other along a straight line with speeds 4 ms^{-1} and 6 ms^{-1} respectively. They collide, after which the direction of motion of A remains unchanged.

(i) Show that the speed of B after the impact cannot be more than 3 ms^{-1}. [4]

The coefficient of restitution between A and B is e.

(ii) Show that $e < \frac{3}{10}$. [4]

(iii) Find the speeds of A and B after the impact in the case when $e = 0$. [3]

6. An aeroplane, travelling horizontally at a speed of 55 ms^{-1} at a height of 600 metres above horizontal ground, drops a sealed packet of leaflets. Find

(i) the time taken by the packet to reach the ground, [2]

(ii) the horizontal distance moved by the packet during this time. [2]

The packet will split open if it hits the ground at a speed in excess of 125 ms^{-1}.

(iii) Determine, with explanation, whether the packet will split open. [4]

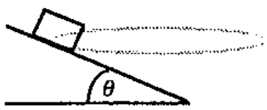
(iv) Find the lowest speed at which the aeroplane could be travelling, at the same height of 600 m, to ensure that the packet will split open when it hits the ground. [2]

One of the leaflets is stuck to the front of the packet and becomes detached as it leaves the aeroplane.

(v) If the leaflet is modelled as a particle, state how long it takes to reach the ground. [1]

(vi) Comment on the model of the leaflet as a particle. [1]

7. A car of mass m kg moves round a curve of radius r m on a road whose cross-section is inclined at an angle θ to the horizontal. The frictional force on the car acts up the slope.



(i) Modelling the car as a particle, draw a diagram to show all the forces acting on the car in the plane of the cross-section. [2]

(ii) Show that, when the speed of the car is $\sqrt{(gr)}$, the sideways frictional force acting on it has magnitude $mg(\sin\theta - \cos\theta)$ N. [10]

MECHANICS 2 (C) TEST PAPER 8 : ANSWERS AND MARK SCHEME

1.	Rebound speed = $0{\cdot}4(4) = 1{\cdot}6$ ms^{-1}	M1 A1	
	K.E. lost = $\frac{1}{2} \times 2 \times (4^2 - 1{\cdot}6^2) = 13{\cdot}4$ J	M1 A1	4
2.	Let R = reaction at wall Resolve horizontally : $R = 12\mu$	M1 A1	
	Resolve vertically : $12 + \mu R = 1{\cdot}4g$	M1 A1	
	Hence $12 + 12\mu^2 = 1{\cdot}4g$ $1 + \mu^2 = 1{\cdot}143$ $\mu = 0{\cdot}38$	M1 A1 A1	7
3.	(i) $25920 = k(36^2)(36)$ $k = 25920 \div 36^3 = \frac{5}{9}$	M1 A1 A1	
	(ii) $25920 = 25(\frac{5}{9}(25)^2 + 460a)$ $a = 1{\cdot}50$ ms^{-2}	M1 A1 M1 A1	7
4.	(i) PQR is a 3, 4, 5 Δ so angle $PQR = 90°$	B1	
	By property of medians, distances are (a) $\frac{1}{3} \times 24 = 8$ cm from PQ	M1 A1	
	(b) $\frac{1}{3} \times 18 = 6$ cm from QR	A1	
	(ii) Equilibrium is about to be broken when G is above Q	M1	
	Then $\tan\theta = 8/6$ $\theta = 53{\cdot}1°$	A1 A1	7
5.	(i) Momentum : $36m - 24m = 9mv_A + 4mv_B$ $9v_A + 4v_B = 12$	M1 A1	
	$v_A > 0$, so $4v_B < 12$ $v_B < 3$	M1 A1	
	(ii) $(v_B - v_A)/(-6 - 4) = -e$ $e = (v_B - v_A) / 10$	M1 A1	
	Now $v_B - v_A < v_B < 3$, so $e < \frac{3}{10}$	M1 A1	
	(iii) If $e = 0$, $v_B = v_A$ $13v_A = 12$ $v_A = v_B = \frac{12}{13}$ ms^{-1}	M1 A1 A1	11
6.	(i) $600 = \frac{1}{2}gt^2$ $t = \sqrt{122.45} = 11{\cdot}1$ s	M1 A1	
	(ii) $x = 55t = 608{\cdot}6$ m	M1 A1	
	(iii) $v_x = 55$, $v_y = gt = 108{\cdot}4$ $v = \sqrt{(v_x^2 + v_x^2)} = \sqrt{14785} = 121{\cdot}6$	M1 A1 A1	
	$121{\cdot}6 < 125$ so packet does not split open	A1	
	(iv) Need $v_x^2 + 108{\cdot}4^2 = 125^2 = 15625$ so $v_x = 62{\cdot}2$ ms^{-1}	M1 A1	
	(v) 11·1 s, as in (a)	A1	
	(vi) Likely to drift due to wind, so particle model not appropriate	B1	12
7.	(i) Diagram showing weight mg acting down, reaction R perp. to	B1	
	slope, friction F acting up slope	B1	
	(ii) $R\cos\theta + F\sin\theta = mg$, $R\sin\theta - F\cos\theta = mv^2/r = mg$	M1 A1 M1 A1	
	$(mg - F\sin\theta)/\cos\theta = (mg + F\cos\theta)/\sin\theta$	M1 A1	
	$mg\sin\theta - F\sin^2\theta = mg\cos\theta + F\cos^2\theta$	M1 A1	
	$F(\sin^2\theta + \cos^2\theta) = F = mg(\sin\theta - \cos\theta)$	M1 A1	12