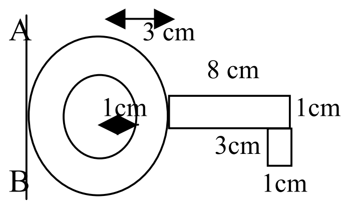
MECHANICS (C) UNIT 2 TEST PAPER 7

Take $g = 9{\cdot}8$ ms^{-2} and give all answers correct to 3 significant figures where necessary.

1. A particle of mass 0·6 kg moves in a horizontal circle with constant angular speed 1·5 radians per second. If the force directed towards the centre of the circle has magnitude 0·27 N, find the radius of the circular path. [3]

2. 

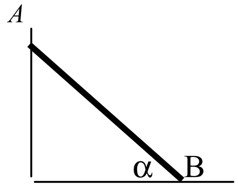
A key is modelled as a lamina which consists of a circle of radius 3 cm, with a circle of radius 1 cm removed from its centre, attached to a rectangle of length 8 cm and width 1 cm, with a rectangle measuring 3 cm by 1 cm fixed to its end as shown.
Calculate the distance of the centre of mass of the key from the line marked *AB*. [5]

3. A van of mass 1600 kg is moving with constant speed **down** a straight road inclined at 7^0 to the horizontal. The non-gravitational resistance to the van's motion has a constant magnitude of 2000 N and the engine of the van is working at a rate of 1·5 kW. Find
(i) the constant speed of the van, [4]
(ii) the acceleration of the van if the resistance is suddenly reduced to 1900 N. [2]

4. A body of mass 1 kg moves in a plane under the action of a constant force of magnitude ž41 N. The body moves from the point *P* with coordinates (–3, –15) to the point *Q* with position vector (9, 0), the unit of distance on each axis being 1 metre.
(i) Find the work done by the force in moving the body from *P* to *Q*. [4]
(ii) Given that the body started from rest at *P*, find its speed when it is at *Q*. [3]

5. Two railway trucks *A* and *B*, whose masses are 6*m* and 5*m* respectively, are moving in the same direction along a straight track with speeds 5*u* and 3*u* respectively, and collide directly. Immediately after this impact, the speeds of *A* and *B* are *v* and *kv* respectively, in the same direction as before. The coefficient of restitution between *A* and *B* is *e*.
Modelling the trucks as particles,
(i) show that (a) $v = \dfrac{45u}{5k+6}$, (b) $v = \dfrac{2eu}{k-1}$. [7]
(ii) Use the fact that $0 < e < 1$ to deduce the range of possible values of *k*. [4]

6. A piece of lead and a table tennis ball are dropped together from a point *P* near the top of the Leaning Tower of Pisa. The lead hits the ground after 3·3 seconds.
(i) Calculate the height above ground from which the lead was dropped. [2]
According to a simple model, the ball hits the ground at the same time as the lead.
(ii) State why this may not be true in practice and describe a refinement to the model which could lead to a more realistic solution. [2]
The piece of lead is now thrown again from *P*, with speed 7 ms^{-1} at an angle of 30^0 to the horizontal.
(iii) Find expressions in terms of *t* for *x* and *y*, the horizontal and vertical displacements respectively of the piece of lead from *P* at time *t* seconds after it is thrown. [4]
(iv) Deduce that $y = \dfrac{\sqrt{3}x}{3} - \dfrac{2x^2}{15}$ [2]
(v) Find the speed of the piece of lead when it has travelled 10 m horizontally from *P*. [4]

7. 

A uniform ladder AB, of mass m kg and length $2a$ m, rests with its upper end A in contact with a smooth vertical wall and its lower end B in contact with a fixed peg on horizontal ground. The ladder makes an angle α with the ground, where $\tan \alpha = {}^3/_4$.

(i) Show that the magnitude of the resultant force acting on the ladder at B is $\dfrac{\sqrt{13}}{3}\text{mg}$. [6]

(ii) Find, to the nearest degree, the direction of this resultant force at B. [2]

The peg will break when the horizontal force acting on it exceeds $2mg$ N.

A painter of mass $6m$ kg starts to climb the ladder from B.

(iii) Find, in terms of a, the greatest distance up the ladder that the painter can safely climb. [6]

<u>MECHANICS 2 (C) TEST PAPER 7 : ANSWERS AND MARK SCHEME</u>

1. $0{\cdot}27 = mr\omega^2 = 0{\cdot}6r(1{\cdot}5^2)$ $\quad r = 0{\cdot}2$ m — M1 A1 A1 — 3

2. $8\pi(3) + 8(10) + 3(13{\cdot}5) = (11 + 8\pi)\bar{x}$ — M1 A1
   $\bar{x} = (24\pi + 120{\cdot}5) \div (8\pi + 11) = 5{\cdot}42$ cm — M1 A1 A1 — 5

3. (i) Net resisting force $= 2000 - 1600g \sin 7^0 = 89{\cdot}1$ N — M1 A1
   $1500 = 89{\cdot}1v$ $\quad v = 16{\cdot}8\ \text{ms}^{-1}$ — M1 A1
   (ii) Now accelerating force $= 100$ N $= 1600a$ $\quad a = 0{\cdot}0625\ \text{ms}^{-2}$ — M1 A1 6

4. (i) Displacement $= \sqrt{(12^2 + 15^2)} = 3\sqrt{41}$ — M1 A1
   F = ž41, so work done $= 3\sqrt{41} \times \sqrt{41} = 123$ J — M1 A1
   (ii) Work = change in K.E., so $\frac{1}{2} v^2 = 123$ $\quad v = \sqrt{246} = 15{\cdot}7\ \text{ms}^{-1}$ — M1 A1 A1 — 7

5. (i) (a) Momentum : $30mu + 15mu = 6mv + 5mkv$ — M1 A1
   $45u = (6 + 5k)v$ $\quad v = \dfrac{45u}{5k+6}$ — A1
   (b) Elasticity : $(kv - v) / (3u - 5u) = -e$ — M1 A1
   $(k - 1)v = (-2u)(-e)$ $\quad v = \dfrac{2eu}{k-1}$ — M1 A1
   (ii) $\dfrac{45u}{5k+6} = \dfrac{2eu}{k-1}$ $\quad e = \dfrac{45(k-1)}{2(5k+6)}$ — M1 A1
   $0 < e < 1$, so $0 < 45k - 45 < 10k + 12$ $\quad k - 1 \neq 0$, so $1 < k < {}^{57}/_{35}$ — M1 A1 — 11

6. (i) $s = gt^2 = \text{ x } 9{\cdot}8 \text{ x } 3{\cdot}3^2 = 53{\cdot}4$ m — M1 A1

(ii) Ball, being lighter, may be affected by air resistance : include this — B1 B1

(iii) $x = (7\cos 30^0)t = \dfrac{7\sqrt{3}}{2}\,t \qquad y = (7\sin 30^0)t - \tfrac{1}{2}gt^2 = {}^7/_2 t - 4{\cdot}9t^2$ — M1 A1 M1 A1

(iv) $t = \dfrac{2x}{7\sqrt{3}} \qquad y = \dfrac{x}{\sqrt{3}} - \dfrac{4.9(2x)^2}{\left(7\sqrt{3}\right)^2} = \dfrac{\sqrt{3}x}{3} - \dfrac{2x^2}{15}$ — M1 A1

(v) When $x = 10,\ y = -7{\cdot}56$ — B1

$\tfrac{1}{2}m(49) + 7{\cdot}56mg = \tfrac{1}{2}mv^2 \qquad v = 14{\cdot}0 \text{ ms}^{-1}$ — M1 A1 A1 — 14

7. (i) $R = mg,\ F = S \qquad$ M(B) : $mga\cos\alpha = 2a\,S\sin\alpha$ — B1 M1 A1

$S = mg / 2\tan\alpha = \dfrac{2mg}{3} = F$ — A1

Resultant force at $B = \sqrt{(mg)^2 + (\dfrac{2mg}{3})^2}\ ] = \dfrac{\sqrt{13}mg}{3}$ — M1 A1

(ii) Angle $= \tan^{-1}({}^3/_2) = 56^0$ to horizontal — M1 A1

(iii) M(B) : $mga\cos\alpha + 6mgx\cos\alpha = 2a\,S\sin\alpha$ — M1 A1

$S = \dfrac{2mg(a+6x)}{3a} \qquad$ When $S = 2mg,\ \ a + 6x = 3a$ — A1 M1 A1

$6x = 2a \qquad x = \tfrac{1}{3}a$ — A1 — 14