

MECHANICS (A) UNIT 2**TEST PAPER 6**

Take $g = 9.8 \text{ ms}^{-2}$ and give all answers correct to 3 significant figures where necessary.

1. A ship, of mass 5000 tonnes, is moving through the sea at a constant speed of 15 km h^{-1} .
 - (a) Calculate the momentum of the ship, in the form $a \times 10^n$, where $0 \leq a < 10$ and n is an integer. State the units of your answer. (3 marks)

Given that there is a constant force of magnitude 4000 N acting against the ship due to air and water resistances,

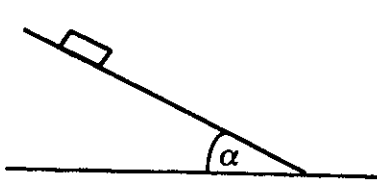
 - (b) find the rate, in kW, at which the ship's engines are working. (3 marks)

2. Two small smooth spheres P and Q are moving along a straight line in opposite directions, with equal speeds, and collide directly. Immediately after the impact, the direction of P 's motion has been reversed and its speed has been halved. The coefficient of restitution between P and Q is e .
 - (a) Express the speed of Q after the impact in the form $au(be + c)$, where a , b and c are constants to be found. (4 marks)
 - (b) Deduce the range of values of e for which the direction of motion of Q remains unaltered. (3 marks)

3. i and j are perpendicular unit vectors in a horizontal plane. At a certain instant, a particle P of mass 1.8 kg is moving with velocity $(24i - 7j) \text{ ms}^{-1}$.
 - (a) Calculate the kinetic energy of P at this instant. (3 marks)

P is now subjected to a constant retardation. After 10 seconds, the velocity of P is $(-12i + 3.5j) \text{ ms}^{-1}$.

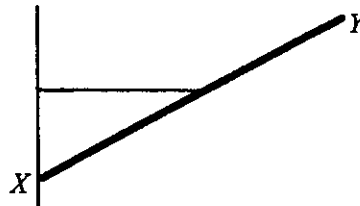
 - (b) Calculate the work done by the retarding force over the 10 seconds. (5 marks)

4. 

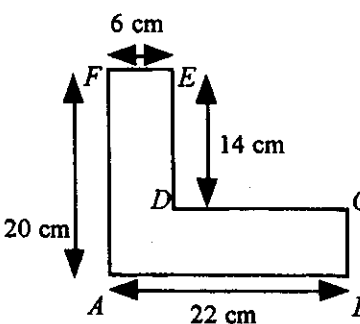
A small block of wood, of mass 0.5 kg , slides down a line of greatest slope of a smooth plane inclined at an angle α to the horizontal, where $\sin \alpha = \frac{2}{5}$. The block is given an initial impulse of magnitude 2 Ns , and reaches the bottom of the plane with kinetic energy 19 J .

 - (a) Find, in J, the change in the potential energy of the block as it moves down the plane. (3 marks)
 - (b) Hence find the distance travelled by the block down the plane. (4 marks)
 - (c) State two modelling assumptions that you have made. (2 marks)

MECHANICS 2 (A) TEST PAPER 6 Page 2

5.  A uniform rod XY , of length $2a$ and mass m , is connected to a vertical wall by a smooth hinge at the end X . A horizontal light inelastic string connects the mid-point of XY to the wall and the rod is in equilibrium in this position.

- (a) Draw a diagram to show all the forces acting on the rod. (4 marks)
- Given that the tension in the horizontal string is of magnitude $2mg$,
- (b) find the angle which XY makes with the vertical. (5 marks)

6.  The diagram shows a uniform lamina $ABCDEF$.

- (a) Calculate the distance of the centre of mass of the lamina from (i) AF , (ii) AB . (6 marks)

The lamina is hung over a smooth peg at D and rests in equilibrium in a vertical plane.

- (b) Find the angle between CD and the vertical. (4 marks)

7. A particle P moves in a straight line so that its displacement s metres from a fixed point O at time t seconds is given by the formula $s = t^3 - 7t^2 + 8t$.
- (a) Find the values of t when the velocity of P equals zero, and briefly describe what is happening to P at these times. (5 marks)
- (b) Find the distance travelled by P between the times $t = 3$ and $t = 5$. (3 marks)
- (c) Find the value of t when the acceleration of P is -2 ms^{-2} . Briefly explain the significance of a negative acceleration at this time. (3 marks)

8. A particle P is projected from a point O with initial velocity $(3.5\mathbf{i} + 12\mathbf{j}) \text{ ms}^{-1}$ and moves under gravity. \mathbf{i} and \mathbf{j} are unit vectors in the horizontal and vertical directions respectively.

- (a) Find the initial speed of P . (2 marks)

- (b) Show that the position vector \mathbf{r} m of P at time t seconds after projection is given by

$$\mathbf{r} = 3.5t\mathbf{i} + (12t - 4.9t^2)\mathbf{j}. \quad (4 \text{ marks})$$

- (c) Find the horizontal distance of P from O at each of the times when it is 4.4 m vertically above the level of O . (5 marks)

In a refined model of the motion of P , the position vector of P at time t seconds is taken to be

$$\mathbf{r} = 3.5t\mathbf{i} + (12t - t^3)\mathbf{j} \text{ m}.$$

- (d) Using this model, find the position vector of the highest point reached by P . (4 marks)