A-LEVEL

Mathematics

Mechanics 2B – MM2B
Mark scheme

6360
June 2015

Version/Stage: Version 1.0: Final

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts: alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this Mark Scheme are available from aqa.org.uk

Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
–*x* EE	deduct *x* marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Q	Solution	Mark	Total	Comment
1 (a) (i)	$\mathbf{a} = \frac{dv}{dt}$ $\mathbf{a} = -8 \sin 2t\, \mathbf{i} + 3 \cos t\, \mathbf{j}$ Using $\mathbf{F} = m\mathbf{a}$ $\mathbf{F} = 4 \times \{-8 \sin 2t\, \mathbf{i} + 3 \cos t\, \mathbf{j}\}$ $= -32 \sin 2t\, \mathbf{i} + 12 \cos t\, \mathbf{j}$	B1 M1 A1	3	All correct Multiplying their a by 4 [must be a vector with at least one trig term] CAO
(ii)	When $t = \pi$, $F = -12\, \mathbf{j}$ Magnitude of $\mathbf{F}$ is 12	B1 B1	2	CAO CAO
(b)	$\mathbf{r} = 2 \sin 2t\, \mathbf{i} - 3\cos t\, \mathbf{j} + \mathbf{c}$ When $t = 0$, $\mathbf{r} = 2\mathbf{i} - 14\mathbf{j}$, $\therefore \mathbf{c} = 2\, \mathbf{i} - 11\, \mathbf{j}$ $\therefore \mathbf{r} = (2 \sin 2t + 2)\mathbf{i} - (3\cos t + 11)\, \mathbf{j}$	M1 A1 m1 A1 A1	5	M1 one term correct A1 another term correct Condone lack of + c m1 use of + c [$c \neq 0$] A1 CAO CAO [accept uncollected form and ISW [condone lack of brackets but must have – 11j]
	Total		**10**	

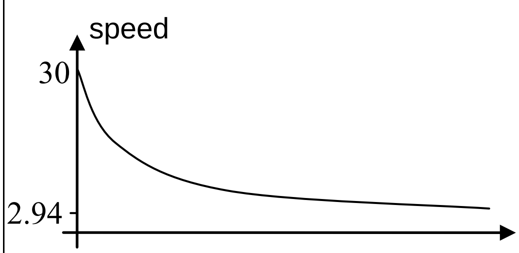
Q	Solution	Mark	Total	Comment
2	Resolve vertically $R = 3g + 4g + 5g + 8g$ $R = 20g$ Taking moments about A $3 \times 4g + AC \times 8g + 6 \times 5g = 4.3 \times 20g$ $42\ g + AC \times 8g = 86g$ $AC = \frac{44}{8}$ Distance AC is 5.5 m	B1 M1 A1 A1	4	**Or** using $\bar{x} \sum m_i = \sum x_i m_i$ $\sum m_i = 20$ B1 **or** moments about any point need 4 non zero terms; could have 20 incorrect all terms either with/without g A1 for all terms correct CAO
	Total		**4**	

Q	Solution	Mark	Total	Comment
3 (a) (i)	P is 2 metres above QR. KE = change in PE $= mgh = 32 \times 9.8 \times 2$ $= 64$ g or 627.2 J $= 627$ J	B1 M1 A1	 3	Do not accept unsimplified expression Correct terms, any value of h used CAO AWRT
(ii)	Speed of Simon is $\sqrt{\dfrac{627.2}{\frac{1}{2} \times 32}}$ $= 6.26$ ms $^{-1}$	M1 A1	 2	Ft from their a CAO [AWRT] Accept square root 4g or 2 root g
(b)	Work done travelling Q to R is $F \times 5$ R = 32 g Work done = change in energy $\mu \times 32g \times 5 = 64$ g or 627.2 $\mu = 0.4$	B1 B1 M1 A1	 4	Needs F times 5 CAO [or 313.6] Ft their 32g and their 64g [from a] condone incorrect distance [eg, 7, 9, 4, 2] **CAO** **Or** if constant acceleration; B1 for 32 g B1for acceleration = ± 2g/5 or ± 3.92 M1 for $\mu g = 2g/5$ A1 for 0.4
	Total		**9**	

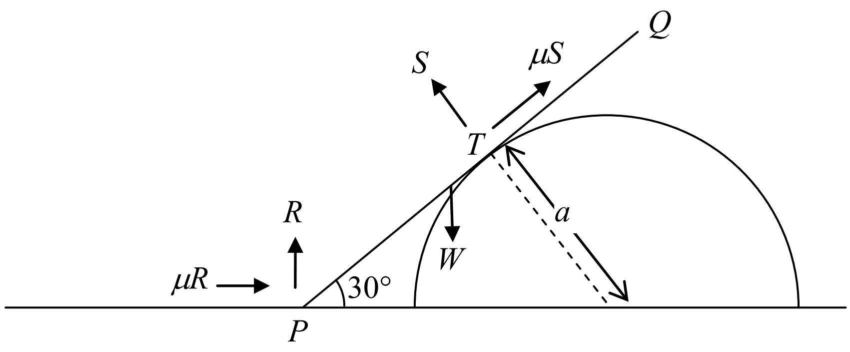
Q	Solution	Mark	Total	Comment
4 (a)	Resolve vertically $T_{AP}\cos 20 = 5g$ $T_{AP} = 52.1$ N	M1A1 A1	 3	M1 could be sin 20 A1 correct CAO AWRT
(b)	Resolve horizontally $T_{AP}\sin 20 + T_{BP} = m\frac{v^2}{r}$ $T_{BP} = 5\frac{v^2}{0.6} - \frac{5g}{\cos 20}\sin 20$ $= \frac{25}{3}v^2 - 5g\tan 20$ **AG**	M1 A1 A1	 3	Needs all the terms, could be cos 20 Needs sin 20 or cos 70
(c)	$T_{AP} = T_{BP}$ $\frac{25}{3}v^2 - 5g\tan 20 = 52.1$ or $\frac{5g}{\cos 20°}$ $\frac{25}{3}v^2 = 69.9$ $v^2 = 8.388$ or 8.3975 $v = 2.90$	M1A1 A1 A1	 4	ft from (a) **CAO PI** Or 2.896.. or 2.8978 CAO 2.9 not accepted
	Total		**10**	

Q	Solution	Mark	Total	Comment
5	900 rpm $= 900 \times \frac{2\pi}{60}$ radians per second $= 30\pi$ radians per second	M1 A1		Use of $\frac{2\pi}{60}$ CAO or for v = 9π or 28.27 or 28.3
	Minimum reactive force is $m\omega^2 r - mg$	M1		Needs both terms and correct signs could be using v
	$= 0.8 \times (30\pi)^2 \times 0.3 - 0.8g$ $= 2131.83 - 7.84$ Minimum magnitude is 2123.99 $= 2120$	A1		CAO AWRT
	Maximum reactive force is $m\omega^2 r + mg$	M1		Needs both terms and correct signs
	$= 0.8 \times (30\pi)^2 \times 0.3 + 0.8g$ $= 2131.83 + 7.84$ Maximum magnitude is 2139.67 $= 2140$	A1		CAO AWRT [must be clear which is min/max unless in this order]
	Total		**6**	

Q	Solution	Mark	Total	Comment
6	Gravitational force is $mg\sin\theta$ $= 1400 \times g \times \sin\theta$	M1 A1		Could accept cos θ CAO
	Accelerating force is *ma* $= 1400 \times 0.2 = 280$	B1		CAO [do not need the 280]
	Total force exerted by engine is $1400 \times g \times \sin\theta + 280 + 4000$ $= 1400 \times g \times \sin\theta + 4280$	B1		Need 3 terms [gravity, acc force, 4000 could be wrong sign]; CAO
	Power $= 91100$ $= (1400 \times g \times \sin\theta + 4280) \times 20$ $1400 \times g \times \sin\theta + 4280 = 4555$	M1 M1 A1		Needs force [ft] times 20 M1 for equation need 4 terms 3 correct or Total force exerted by engine is 91100/20 M1 = 4555 A1
	$1400 \times g \times \sin\theta = 275$			or using F = ma 1400×0.2= 91100/20 – 4000 -1400gsinθ need 4 terms 3 correct [ignore signs] B1for 1400x0.2;91100/20M1A1 1400gsinθM1A1;form equation M1A1
	$\sin\theta = 0.0200$ $\theta = 1.15°$	**A1** **A1**	**9**	CAO CAO
	Total		**9**	

Q	Solution	Mark	Total	Comment
7(a);	Using $F = ma$			
	$72\dfrac{dv}{dt} = 72g - 240v$	M1		CAO
	$-\dfrac{3}{10}\dfrac{dv}{dt} = v - 2.94$	A1	2	AG; Needs M1 above
(b)	Hence $\displaystyle\int \frac{1}{v-2.94}\,dv = -\frac{10}{3}\int dt$			
	$\ln(v - 2.94) = -\dfrac{10}{3}t + c$	M1A1		M1 for either side integrated correctly A1 for all correct
		m1		m1 for + c
	$v - 2.94 = Ce^{-\frac{10}{3}t}$ $t = 0,\ v = 30$ $\therefore C = 27.06$	A1		CAO condone 1353/50 accept c = ln 27.06
	$\therefore v = 2.94 + 27.06e^{-\frac{10}{3}t}$	A1	5	CAO condone 27.1m
(c)		B2	2	B1 for starting at 30 and basic shape B1 for asymptote of 2.94
	Total		9	

Q	Solution	Mark	Total	Comment
8 (a)	When $x \geq 26$, KE is $\frac{1}{2} \times 70 \times v^2$ EPE is $\frac{1456 \times (x-26)^2}{2 \times 26}$ Change in PE is $70 \times g \times x$			
	Conservation of energy : $\frac{1}{2} \times 70 \times v^2 + \frac{1456 \times (x-26)^2}{2 \times 26} = 70\,g \times x$	M1A1 A1		M1 for 3 terms of correct items A1 for 2 of the 3 types of energy are correct [ignore signs] [treat all GPE terms as one term] A1 for all terms correct [70g is 686] Accept 4 terms if PE is on both sides
	$35v^2 + 28(x-26)^2 = 70gx$ $5v^2 + 4(x-26)^2 = 98x$ $5v^2 = 306x - 4x^2 - 2704$	A1	4	CAO
(b)	If x is not greater than 26, cord is not stretched. Hence EPE cannot be used unless x is greater than 26.	B1	1	Either statement, or cord not taut no EPE
(c)	At maximum value of x, $v = 0$ $\therefore 4x^2 - 306x + 2704 = 0$ $x = 66.3$	M1 A1	 2	Correct use of v = 0 CAO [bod if give 2 values]
(d)(i)	When speed is a maximum, $a = 0$ tension = gravitational force $\frac{1456 \times (x-26)}{26} = 70\,g$ $x - 26 = 12.25$ $x = 38.25$	 M1 A1	 2	 or differentiating (a) 306 – 8x = 0 Accept 38.2 or 38.3 Could be seen with no working
(ii)	Using (a) and (d)(i) for maximum speed $5v^2 = 11704.5 - 5852.25 - 2704$ $v^2 = 629.65$ Maximum speed is 25.1 ms $^{-1}$	B1	1	CAO
	Total		**10**	

Q	Solution	Mark	Total	Comment
9				
	a / PT = tan 30 $PT = \frac{a}{\tan 30}$	B1		
	Resolve vertically R + S cos 30 + μS sin 30 = W (1)	M1A1		**Or** resolve along the rod μS+ Rsin30 + μRcos 30 =W sin 30 M1 for any 4 terms; must include at least 1 friction term and a trig term
	Resolve horizontally μR + μS cos 30 = S sin 30 (2)	M1A1		Resolve perpendicular to rod S + R cos 30 =μRsin 30 +W cos30 M1 for any 4 terms; must include at least 1 friction term and a trig term If resolve horizontally M1 for any 3 terms; must include a trig term
	Moments about P PT × S = W × a cos 30 $\frac{a\cos 30}{\sin 30}$ × S = W × a cos 30 S = W sin 30 or = $\frac{1}{2}$W	B1		Allow,bod, if moments taken about another point
	(2)→μR = W ($\sin^2 30$–μ$\sin 30\cos 30$) or μR = W ($\frac{1}{4}$ – $\frac{\sqrt{3}}{4}$ μ)			R(sin30 +μ cos 30) = Wsin 30 (1 - μ)
	(1)→ μR + μScos30 +μ^2S sin30 = μW W ($\frac{1}{4}$–$\frac{\sqrt{3}}{4}$μ) + μ $\frac{1}{2}$ W$\frac{\sqrt{3}}{2}$+μ^2 $\frac{1}{2}$W $\frac{1}{2}$= μW μ = $\sin^2 30$ + $\mu^2\sin^2 30$ **or** μ = $\frac{1}{4}$ + $\frac{1}{4}$ μ^2 $\mu^2 - 4\mu + 1 = 0$ μ = 2 – √3 or 0.268	m1 A1	8	R(cos30 –μ sin 30) =W(cos30 –sin 30) Dividing $\frac{\sin 30+\mu\cos 30}{\cos 30-\mu\sin 30} = \frac{\sin 30\,(1-\mu)}{\cos 30-\sin 30}$ m1 for simplifying into a quadratic Dependent on both M1 above condone μ = 2 + √3
	Total		**8**	