



General Certificate of Education
Advanced Level Examination
June 2012

Mathematics

MM2B

Unit Mechanics 2B

Thursday 21 June 2012 1.30 pm to 3.00 pm

For this paper you must have:

- the blue AQA booklet of formulae and statistical tables.
- You may use a graphics calculator.

Time allowed

- 1 hour 30 minutes

Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Take $g = 9.8 \text{ m s}^{-2}$, unless stated otherwise.

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

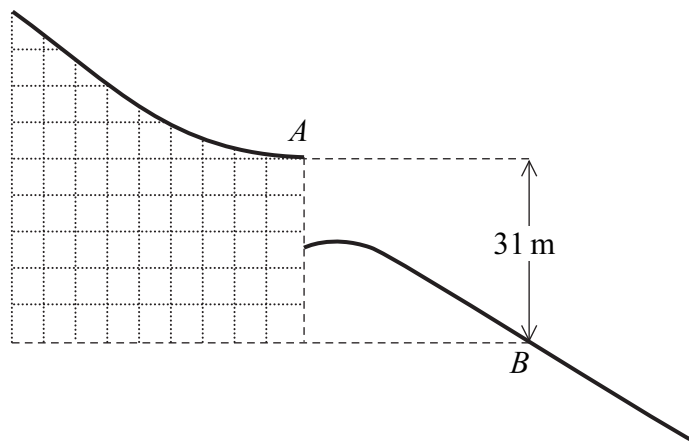
Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

- 1 Alan, of mass 76 kg, performed a ski jump. He took off at the point A at the end of the ski run with a speed of 28 m s^{-1} and landed at the point B .

The level of the point B is 31 metres vertically below the level of the point A , as shown in the diagram.

Assume that his weight is the only force that acted on Alan during the jump.



- (a) Calculate the kinetic energy of Alan when he was at the point A . (2 marks)
- (b) Calculate the potential energy lost by Alan during the jump as he moved from the point A to the point B . (2 marks)
- (c) (i) Find the kinetic energy of Alan when he reached the point B . (2 marks)
- (ii) Hence find the speed of Alan when he reached the point B . (2 marks)

- 2 A particle moves in a straight line. At time t seconds, it has velocity $v \text{ m s}^{-1}$, where

$$v = 6t^2 - 2e^{-4t} + 8$$

and $t \geq 0$.

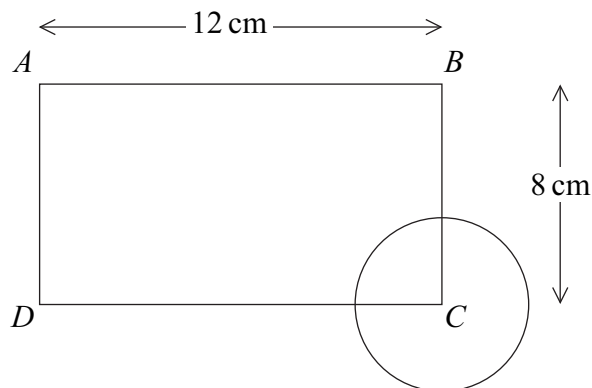
- (a) (i) Find an expression for the acceleration of the particle at time t . (2 marks)
- (ii) Find the acceleration of the particle when $t = 0.5$. (2 marks)
- (b) The particle has mass 4 kg.
- Find the magnitude of the force acting on the particle when $t = 0.5$. (1 mark)
- (c) When $t = 0$, the particle is at the origin.

Find an expression for the displacement of the particle from the origin at time t . (4 marks)



- 3 A uniform rectangular lamina $ABCD$, of mass 1.6 kg, has side AB of length 12 cm and side BC of length 8 cm.

To create a logo, a uniform circular lamina, of mass 0.4 kg, is attached. The centre of the circular lamina is at the point C , as shown in the diagram.



- (a) Find the distance of the centre of mass of the logo:
- (i) from the line AB ; (3 marks)
- (ii) from the line AD . (3 marks)
- (b) The logo is suspended in equilibrium, with AB horizontal, by two vertical strings. One string is attached at the point A and the other string is attached at the point B .
- Find the tension in each of the two strings. (5 marks)
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- 4 A particle moves on a horizontal plane, in which the unit vectors \mathbf{i} and \mathbf{j} are perpendicular.

At time t , the particle's position vector, \mathbf{r} , is given by

$$\mathbf{r} = 4 \cos 3t \mathbf{i} - 4 \sin 3t \mathbf{j}$$

- (a) Prove that the particle is moving on a circle, which has its centre at the origin. (2 marks)
- (b) Find an expression for the velocity of the particle at time t . (2 marks)
- (c) Find an expression for the acceleration of the particle at time t . (2 marks)

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- (d) The acceleration of the particle can be written as

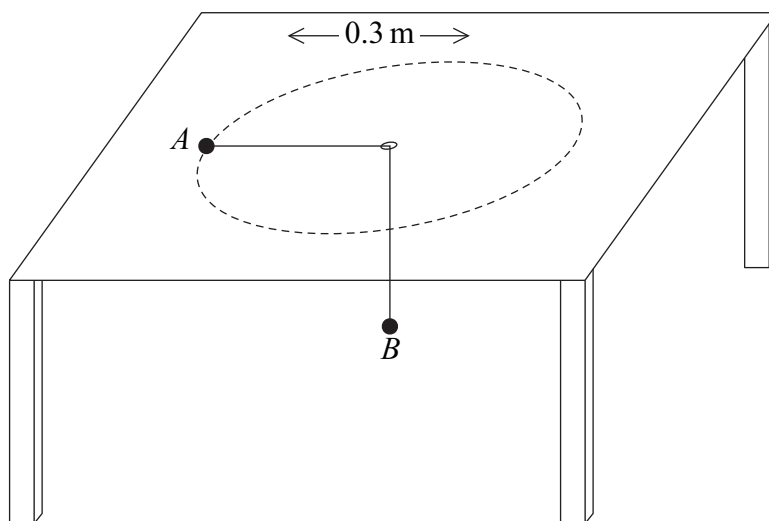
$$\mathbf{a} = k\mathbf{r}$$

where k is a constant.

Find the value of k . (2 marks)

- (e) State the direction of the acceleration of the particle. (1 mark)

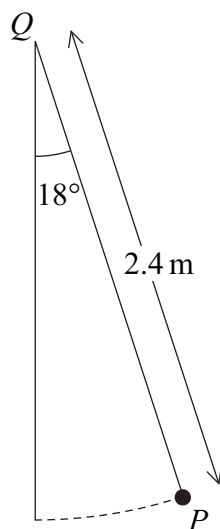
- 5 Two particles, A and B , are connected by a light inextensible string which passes through a hole in a smooth horizontal table. The edges of the hole are also smooth. Particle A , of mass 1.4 kg, moves, on the table, with constant speed in a circle of radius 0.3 m around the hole. Particle B , of mass 2.1 kg, hangs in equilibrium under the table, as shown in the diagram.



- (a) Find the angular speed of particle A . (4 marks)
- (b) Find the speed of particle A . (2 marks)
- (c) Find the time taken for particle A to complete one full circle around the hole. (2 marks)



- 6** Simon, a small child of mass 22 kg, is on a swing. He is swinging freely through an angle of 18° on both sides of the vertical. Model Simon as a particle, P , of mass 22 kg, attached to a fixed point, Q , by a light inextensible rope of length 2.4 m.



- (a) Find Simon's maximum speed as he swings. (4 marks)
- (b) Calculate the tension in the rope when Simon's speed is a maximum. (3 marks)

- 7** A stone, of mass 5 kg, is projected vertically downwards, in a viscous liquid, with an initial speed of 7 m s^{-1} .

At time t seconds after it is projected, the stone has speed $v \text{ m s}^{-1}$ and it experiences a resistance force of magnitude $9.8v$ newtons.

- (a) When $t \geq 0$, show that

$$\frac{dv}{dt} = -1.96(v - 5) \quad (2 \text{ marks})$$

- (b) Find v in terms of t . (5 marks)

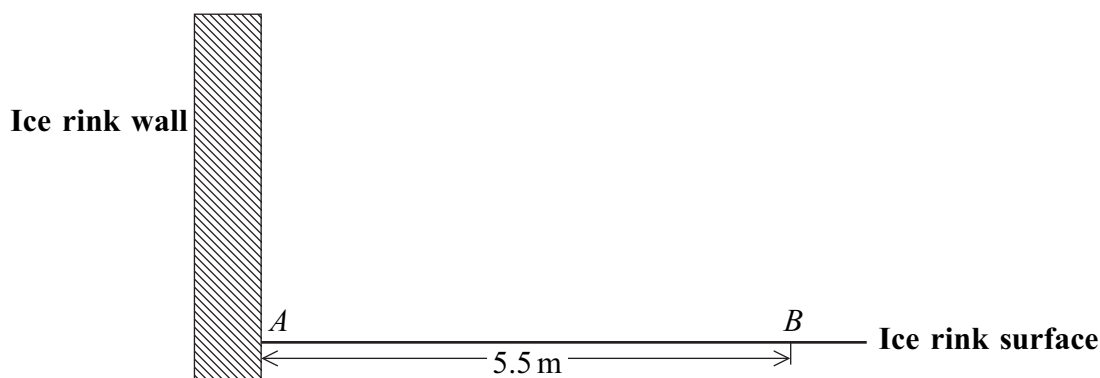
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- 8 Zoë carries out an experiment with a block, which she places on the horizontal surface of an ice rink. She attaches one end of a light elastic string to a fixed point, A , on a vertical wall at the edge of the ice rink at the height of the surface of the ice rink.

The block, of mass 0.4 kg , is attached to the other end of the string. The string has natural length 5 m and modulus of elasticity 120 N .

The block is modelled as a particle which is placed on the surface of the ice rink at a point B , where AB is perpendicular to the wall and of length 5.5 m .



The block is set into motion at the point B with speed 9 m s^{-1} directly towards the point A . The string remains horizontal throughout the motion.

- (a) Initially, Zoë assumes that the surface of the ice rink is smooth.

Using this assumption, find the speed of the block when it reaches the point A .

(4 marks)

- (b) Zoë now assumes that friction acts on the block. The coefficient of friction between the block and the surface of the ice rink is μ .

- (i) Find, in terms of g and μ , the speed of the block when it reaches the point A .

(6 marks)

- (ii) The block rebounds from the wall in the direction of the point B . The speed of the block immediately after the rebound is half of the speed with which it hit the wall.

Find μ if the block comes to rest just as it reaches the point B .

(6 marks)

