General Certificate of Education
Advanced Level Examination
June 2011

Mathematics MM2B

Unit Mechanics 2B

Monday 20 June 2011 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Take $g = 9.8\ \mathrm{m\,s^{-2}}$, unless stated otherwise.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

1 In an Olympic diving competition, Kim, who has mass 58 kg, dives from a fixed platform, 10 metres above the surface of the pool. She leaves the platform with a speed of $2\,\text{m}\,\text{s}^{-1}$.

Assume that Kim's weight is the only force that acts on her after she leaves the platform. Kim is to be modelled as a particle which is initially 1 metre above the platform.

(a) Calculate Kim's initial kinetic energy. *(2 marks)*

(b) By using conservation of energy, find Kim's speed when she is 6 metres below the platform. *(5 marks)*

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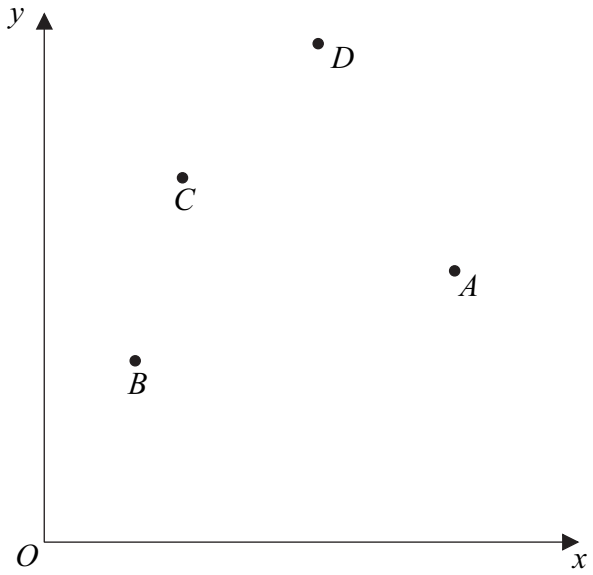
2 The diagram shows four particles, A, B, C and D, which are fixed in a horizontal plane which contains the x- and y-axes, as shown.

Particle A has mass 2 kg and is attached at the point $(9, 6)$.
Particle B has mass 3 kg and is attached at the point $(2, 4)$.
Particle C has mass 8 kg and is attached at the point $(3, 8)$.
Particle D has mass 7 kg and is attached at the point $(6, 11)$.



Find the coordinates of the centre of mass of the four particles. *(5 marks)*

3 A particle moves in a horizontal plane under the action of a single force, **F** newtons. The unit vectors **i** and **j** are directed east and north respectively. At time t seconds, the velocity of the particle, $\mathbf{v}\,\text{m}\,\text{s}^{-1}$, is given by

$$\mathbf{v} = 4\text{e}^{-2t}\,\mathbf{i} + (6t - 3t^2)\mathbf{j}$$

(a) Find an expression for the acceleration of the particle at time t. *(3 marks)*

(b) The mass of the particle is 5 kg.

(i) Find an expression for the force **F** acting on the particle at time t. *(2 marks)*

(ii) Find the magnitude of **F** when $t = 0$. *(2 marks)*

(c) Find the value of t when **F** acts due west. *(2 marks)*

(d) When $t = 0$, the particle is at the point with position vector $(6\mathbf{i} + 5\mathbf{j})$ m.

Find the position vector, **r** metres, of the particle at time t. *(5 marks)*

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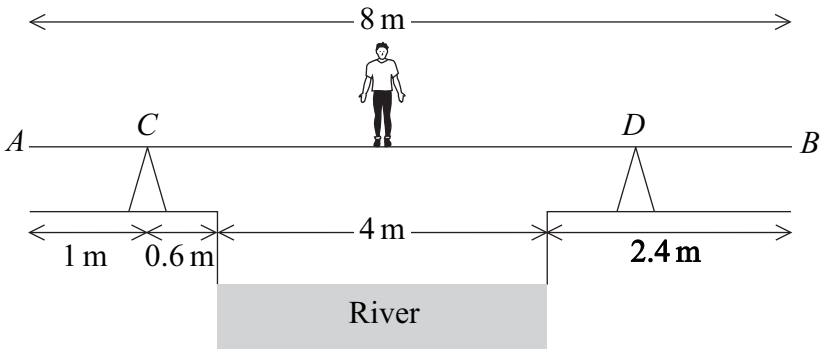
4 Ken is trying to cross a river of width 4 m. He has a uniform plank, AB, of length 8 m and mass 17 kg. The ground on both edges of the river bank is horizontal. The plank rests at two points, C and D, on fixed supports which are on opposite sides of the river. The plank is at right angles to both river banks and is horizontal. The distance AC is 1 m, and the point C is at a horizontal distance of 0.6 m from the river bank. Ken, who has mass 65 kg, stands on the plank directly above the middle of the river, as shown in the diagram.



(a) Draw a diagram to show the forces acting on the plank. *(2 marks)*

(b) Given that the reaction on the plank at the point D is $44g$ N, find the horizontal distance of the point D from the nearest river bank. *(4 marks)*

(c) State how you have used the fact that the plank is uniform in your solution. *(1 mark)*

5 A train consists of an engine and five carriages. A constant resistance force of 3000 N acts on the engine, and a constant resistance force of 400 N acts on each of the five carriages.

The maximum speed of the train on a horizontal track is $90\,\text{km}\,\text{h}^{-1}$.

(a) Show that this speed is $25\,\text{m}\,\text{s}^{-1}$. *(1 mark)*

(b) Hence find the maximum power output of the engine. Give your answer in kilowatts. *(3 marks)*

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6 A car, of mass m kg, is moving along a straight horizontal road. At time t seconds, the car has speed $v\,\text{m}\,\text{s}^{-1}$. As the car moves, it experiences a resistance force of magnitude $2mv^{\frac{5}{4}}$ newtons. No other horizontal force acts on the car.

(a) Show that

$$\frac{\mathrm{d}v}{\mathrm{d}t} = -2v^{\frac{5}{4}}$$ *(1 mark)*

(b) The initial speed of the car is $16\,\text{m}\,\text{s}^{-1}$.

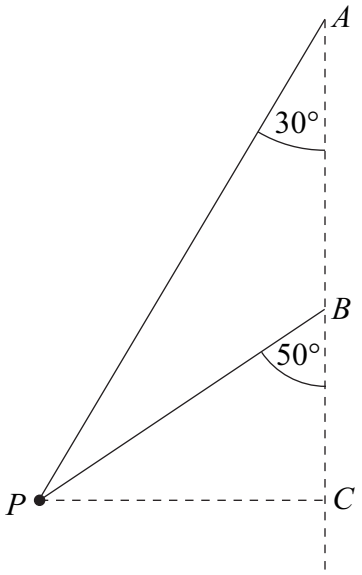
Show that

$$v = \left(\frac{2}{t+1}\right)^4$$ *(5 marks)*

7 Two light inextensible strings each have one end attached to a particle, P, of mass 4 kg. The other ends of the strings are attached to the fixed points A and B. The point A is vertically above the point B.

The particle moves at a constant speed in a horizontal circle. The centre, C, of this circle is directly below the point B. The two strings are inclined at 30° and 50° to the vertical, as shown in the diagram. Both strings are taut.

As the particle moves in the horizontal circle, the tension in the string BP is 20 N.

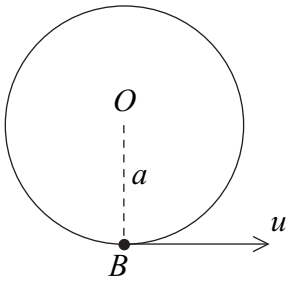


(a) Find the tension in the string AP. *(4 marks)*

(b) The speed of the particle is $5\,\text{m}\,\text{s}^{-1}$.

Find the length of CP, the radius of the horizontal circle. *(4 marks)*

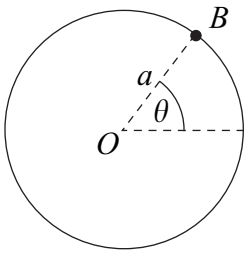
8 A smooth wire is fixed in a vertical plane so that it forms a circle of radius a metres and centre O. A bead, B, of mass 0.3 kg, is threaded on the wire and is set in motion with a speed $u\,\mathrm{m\,s^{-1}}$ at the lowest point of its circular path, as shown in the diagram.



(a) Show that, if the bead is going to make complete revolutions around the wire,

$$u > 2\sqrt{ag}$$ *(3 marks)*

(b) At time t seconds, the angle between OB and the horizontal is θ, as shown in the diagram.



It is given that $u = \sqrt{\frac{9}{2}ag}$.

(i) Find the reaction of the bead on the wire, giving your answer in terms of g and θ. *(5 marks)*

(ii) Find θ when this reaction is zero. *(2 marks)*

9 At a theme park, a light elastic rope is used to bring a carriage to rest at the end of a ride.

The carriage has mass 200 kg and is travelling at $8\,\text{m}\,\text{s}^{-1}$ when the elastic rope is attached to the carriage as it passes over a point O. The other end of the elastic rope is fixed to the point O. The carriage then moves along a horizontal surface until it is brought to rest. The elastic rope is then detached so that the carriage remains at rest.

The elastic rope has natural length 6 m and modulus of elasticity 1800 N. The rope, once taut, remains horizontal throughout the motion.

(a) Calculate the elastic potential energy of the rope when the carriage is 10 m from O. *(3 marks)*

(b) A student's simple model assumes that there are no resistance forces acting on the carriage so that it is brought to rest by the elastic rope alone.

Find the distance of the carriage from O when it is brought to rest. *(3 marks)*

(c) The student improves the model by also including a constant resistance force of 800 N which acts while the carriage is in motion.

Find the distance of the carriage from O when it is brought to rest. *(8 marks)*

END OF QUESTIONS