General Certificate of Education
June 2009
Advanced Level Examination

MATHEMATICS **MM2B**
Unit Mechanics 2B

Monday 15 June 2009 1.30 pm to 3.00 pm

For this paper you must have:
- an 8-page answer book
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MM2B.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Take $g = 9.8\ \text{m s}^{-2}$, unless stated otherwise.

Information
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 A particle moves under the action of a force, $\mathbf{F}$ newtons. At time t seconds, the velocity, $\mathbf{v}\ \mathrm{m\,s^{-1}}$, of the particle is given by

$$\mathbf{v} = (t^3 - 15t - 5)\mathbf{i} + (6t - t^2)\mathbf{j}$$

(a) Find an expression for the acceleration of the particle at time t. *(3 marks)*

(b) The mass of the particle is 4 kg.

(i) Show that, at time t,

$$\mathbf{F} = (12t^2 - 60)\mathbf{i} + (24 - 8t)\mathbf{j}$$ *(2 marks)*

(ii) Find the magnitude of $\mathbf{F}$ when $t = 2$. *(4 marks)*

2 A slide at a water park may be modelled as a smooth plane of length 20 metres inclined at 30° to the vertical. Anne, who has a mass of 55 kg, slides down the slide. At the top of the slide, she has an initial velocity of $3\ \mathrm{m\,s^{-1}}$ down the slide.

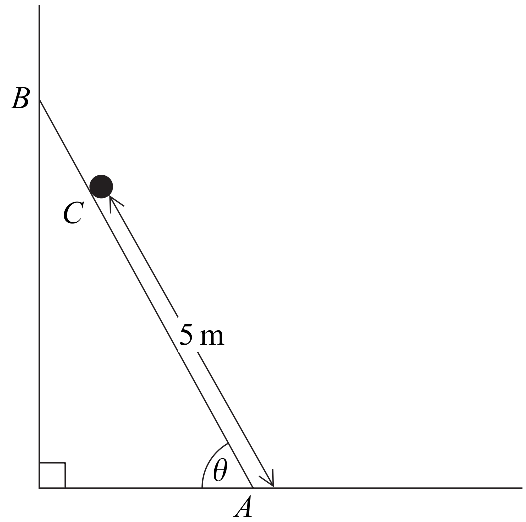
(a) Calculate Anne's initial kinetic energy. *(2 marks)*

(b) By using conservation of energy, find the kinetic energy and the speed of Anne after she has travelled the 20 metres. *(6 marks)*

(c) State one modelling assumption which you have made. *(1 mark)*

3 A uniform ladder, of length 6 metres and mass 22 kg, rests with its foot, A, on a rough horizontal floor and its top, B, leaning against a smooth vertical wall. The vertical plane containing the ladder is perpendicular to the wall, and the angle between the ladder and the floor is θ.

A man, of mass 90 kg, is standing at point C on the ladder so that the distance AC is 5 metres. With the man in this position, the ladder is on the point of slipping. The coefficient of friction between the ladder and the horizontal floor is 0.6. The man may be modelled as a particle at C.

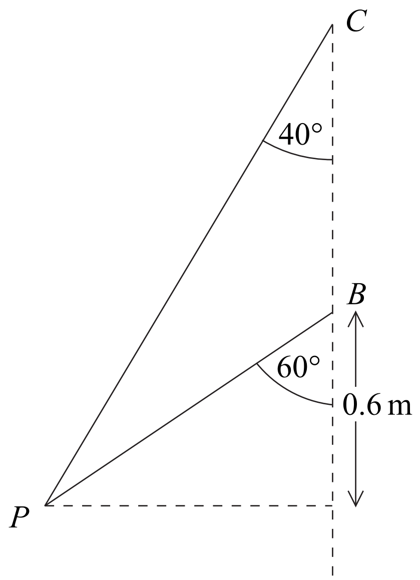


(a) Show that the magnitude of the frictional force between the ladder and the horizontal floor is 659 N, correct to three significant figures. *(4 marks)*

(b) Find the angle θ. *(5 marks)*

Turn over for the next question

4 Two light inextensible strings each have one end attached to a particle, P, of mass 6 kg. The other ends of the strings are attached to the fixed points B and C. The point C is vertically above the point B. The particle moves, at constant speed, in a horizontal circle, with centre 0.6 m below point B, with the strings inclined at 40° and 60° to the vertical, as shown in the diagram. Both strings are taut.



(a) As the particle moves in the horizontal circle, the tensions in the two strings are equal.

Show that the tension in the strings is 46.4 N, correct to three significant figures. *(4 marks)*

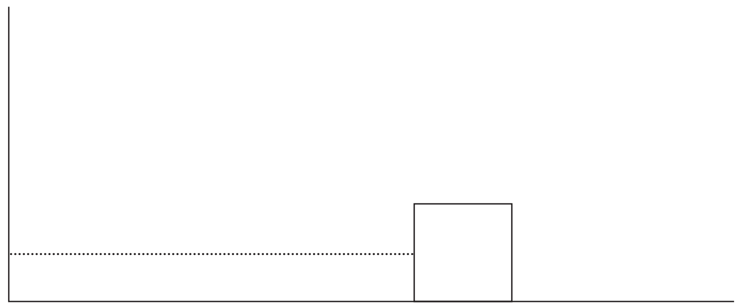
(b) Find the speed of the particle. *(4 marks)*

5 A train, of mass 600 tonnes, travels at constant speed up a slope inclined at an angle θ to the horizontal, where $\sin\theta = \frac{1}{40}$. The speed of the train is $24\,\text{m}\,\text{s}^{-1}$ and it experiences total resistance forces of 200 000 N.

Find the power produced by the train, giving your answer in kilowatts. *(6 marks)*

6 A block, of mass 5 kg, is attached to one end of a length of elastic string. The other end of the string is fixed to a vertical wall. The block is placed on a horizontal surface.

The elastic string has natural length 1.2 m and modulus of elasticity 180 N. The block is pulled so that it is 2 m from the wall and is then released from rest. Whilst taut, the string remains horizontal. It may be assumed that, after the string becomes slack, it does not interfere with the movement of the block.



(a) Calculate the elastic potential energy when the block is 2 m from the wall. *(2 marks)*

(b) If the horizontal surface is smooth, find the speed of the block when it hits the wall. *(3 marks)*

(c) The surface is in fact rough and the coefficient of friction between the block and the surface is μ.

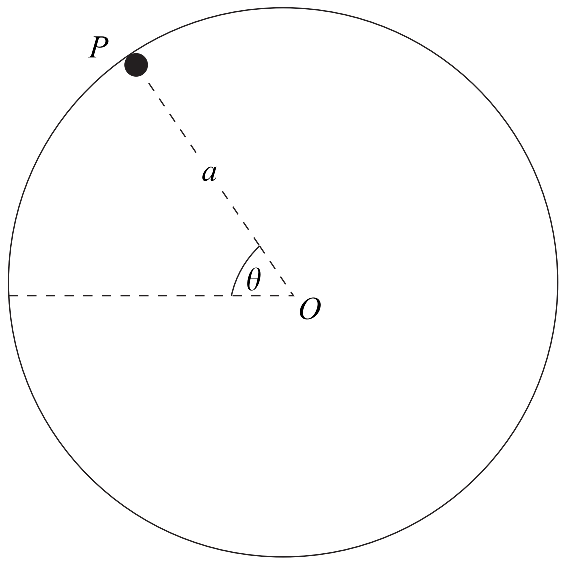
Find μ if the block comes to rest just as it reaches the wall. *(7 marks)*

Turn over for the next question

7 In crazy golf, a golf ball is hit so that it starts to move in a vertical circle on the inside of a smooth cylinder.

Model the golf ball as a particle, P, of mass m. The circular path of the golf ball has radius a and centre O. At time t, the angle between OP and the horizontal is θ, as shown in the diagram.

The golf ball has speed u at the lowest point of its circular path.



(a) Show that, while the golf ball is in contact with the cylinder, the reaction of the cylinder on the golf ball is

$$\frac{mu^2}{a} - 3mg\sin\theta - 2mg$$ *(6 marks)*

(b) Given that $u = \sqrt{3ag}$, the golf ball will not complete a vertical circle inside the cylinder. Find the angle which OP makes with the horizontal when the golf ball leaves the surface of the cylinder. *(4 marks)*

8 A stone, of mass m, is moving in a straight line along smooth horizontal ground.

At time t, the stone has speed v. As the stone moves, it experiences a total resistance force of magnitude $\lambda m v^{\frac{3}{2}}$, where λ is a constant. No other horizontal force acts on the stone.

(a) Show that

$$\frac{\mathrm{d}v}{\mathrm{d}t} = -\lambda v^{\frac{3}{2}}$$

(2 marks)

(b) The initial speed of the stone is $9\,\mathrm{m\,s^{-1}}$.

Show that

$$v = \frac{36}{(2 + 3\lambda t)^2}$$

(7 marks)

(c) Find, in terms of λ, the time taken for the speed of the stone to drop to $4\,\mathrm{m\,s^{-1}}$.

(3 marks)

END OF QUESTIONS

There are no questions printed on this page