General Certificate of Education
June 2008
Advanced Level Examination

MATHEMATICS **MM2B**
Unit Mechanics 2B

Friday 6 June 2008 1.30 pm to 3.00 pm

For this paper you must have:
- an 8-page answer book
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MM2B.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Take $g = 9.8\ \text{m s}^{-2}$, unless stated otherwise.

Information
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.
- Unit Mechanics 2B has a **written paper only**.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 A particle moves in a straight line and at time t seconds has velocity $v\,\mathrm{m\,s^{-1}}$, where

$$v = 6t^2 + 4t - 7, \quad t \geqslant 0$$

(a) Find an expression for the acceleration of the particle at time t. *(2 marks)*

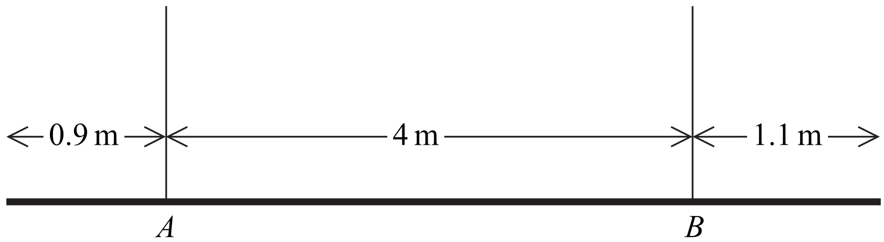
(b) The mass of the particle is 3 kg.

Find the resultant force on the particle when $t = 4$. *(2 marks)*

(c) When $t = 0$, the displacement of the particle from the origin is 5 metres.

Find an expression for the displacement of the particle from the origin at time t. *(4 marks)*

2 A uniform plank, of length 6 metres, has mass 40 kg. The plank is held in equilibrium in a horizontal position by two vertical ropes attached to the plank at A and B, as shown in the diagram.



(a) Draw a diagram to show the forces acting on the plank. *(1 mark)*

(b) Show that the tension in the rope attached to the plank at B is $21g$ N. *(3 marks)*

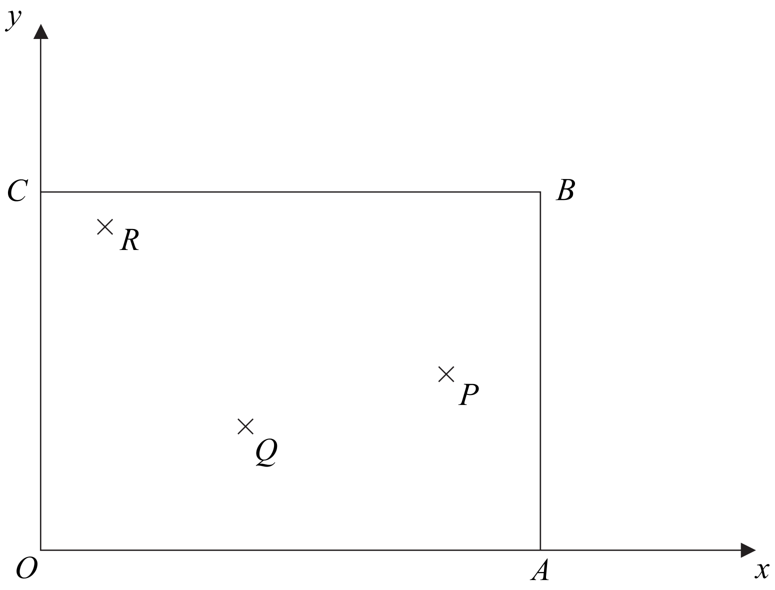
(c) Find the tension in the rope that is attached to the plank at A. *(2 marks)*

(d) State where in your solution you have used the fact that the plank is uniform. *(1 mark)*

3 Three particles are attached to a light rectangular lamina $OABC$, which is fixed in a horizontal plane.

Take OA and OC as the x- and y-axes, as shown.

Particle P has mass 1 kg and is attached at the point $(25, 10)$.
Particle Q has mass 4 kg and is attached at the point $(12, 7)$.
Particle R has mass 5 kg and is attached at the point $(4, 18)$.



Find the coordinates of the centre of mass of the three particles. *(4 marks)*

4 A van, of mass 1500 kg, has a maximum speed of $50\,\text{m s}^{-1}$ on a straight horizontal road. When the van travels at a speed of $v\,\text{m s}^{-1}$, it experiences a resistance force of magnitude $40v$ newtons.

(a) Show that the maximum power of the van is 100 000 watts. *(2 marks)*

(b) The van is travelling along a straight horizontal road.

Find the maximum possible acceleration of the van when its speed is $25\,\text{m s}^{-1}$. *(3 marks)*

(c) The van starts to climb a hill which is inclined at 6° to the horizontal. Find the maximum possible constant speed of the van as it travels in a straight line up the hill. *(6 marks)*

5 A particle moves on a horizontal plane in which the unit vectors **i** and **j** are directed east and north respectively.

At time t seconds, the particle's position vector, **r** metres, is given by

$$\mathbf{r} = 8\left(\cos\tfrac{1}{4}t\right)\mathbf{i} - 8\left(\sin\tfrac{1}{4}t\right)\mathbf{j}$$

(a) Find an expression for the velocity of the particle at time t. *(2 marks)*

(b) Show that the speed of the particle is a constant. *(3 marks)*

(c) Prove that the particle is moving in a circle. *(2 marks)*

(d) Find the angular speed of the particle. *(2 marks)*

(e) Find an expression for the acceleration of the particle at time t. *(2 marks)*

(f) State the magnitude of the acceleration of the particle. *(1 mark)*

6 A car, of mass m, is moving along a straight smooth horizontal road. At time t, the car has speed v. As the car moves, it experiences a resistance force of magnitude $0.05mv$. No other horizontal force acts on the car.

(a) Show that

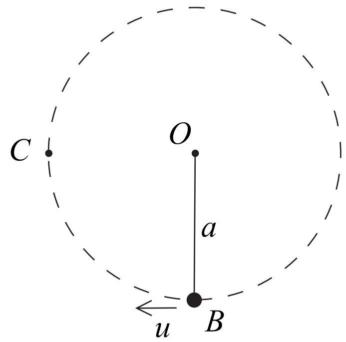
$$\frac{\mathrm{d}v}{\mathrm{d}t} = -0.05v$$ *(1 mark)*

(b) When $t = 0$, the speed of the car is $20\,\mathrm{m\,s^{-1}}$.

Show that $v = 20\mathrm{e}^{-0.05t}$. *(4 marks)*

(c) Find the time taken for the speed of the car to reduce to $10\,\mathrm{m\,s^{-1}}$. *(3 marks)*

7 A small bead, of mass m, is suspended from a fixed point O by a light inextensible string, of length a. The bead is then set into circular motion with the string taut at B, where B is vertically below O, with a horizontal speed u.



(a) Given that the string does not become slack, show that the least value of u required for the bead to make complete revolutions about O is $\sqrt{5ag}$. *(5 marks)*

(b) In the case where $u = \sqrt{5ag}$, find, in terms of g and m, the tension in the string when the bead is at the point C, which is at the same horizontal level as O, as shown in the diagram. *(3 marks)*

(c) State one modelling assumption that you have made in your solution. *(1 mark)*

Turn over for the next question

8 (a) Hooke's law states that the tension in a stretched string of natural length l and modulus of elasticity λ is $\dfrac{\lambda x}{l}$ when its extension is x.

Using this formula, prove that the work done in stretching a string from an unstretched position to a position in which its extension is e is $\dfrac{\lambda e^2}{2l}$. *(3 marks)*

(b) A particle, of mass 5 kg, is attached to one end of a light elastic string of natural length 0.6 metres and modulus of elasticity 150 N. The other end of the string is fixed to a point O.

(i) Find the extension of the elastic string when the particle hangs in equilibrium directly below O. *(2 marks)*

(ii) The particle is pulled down and held at the point P, which is 0.9 metres vertically below O.

Show that the elastic potential energy of the string when the particle is in this position is 11.25 J. *(2 marks)*

(iii) The particle is released from rest at the point P. In the subsequent motion, the particle has speed $v\,\mathrm{m\,s^{-1}}$ when it is x metres **above *P***.

Show that, while the string is taut,

$$v^2 = 10.4x - 50x^2$$ *(7 marks)*

(iv) Find the value of x when the particle comes to rest for the first time after being released, given that the string is still taut. *(2 marks)*

END OF QUESTIONS

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