General Certificate of Education
June 2007
Advanced Level Examination

ASSESSMENT and QUALIFICATIONS ALLIANCE

MATHEMATICS **MM2B**
Unit Mechanics 2B

Thursday 7 June 2007 9.00 am to 10.30 am

For this paper you must have:
- an 8-page answer book
- the **blue** AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MM2B.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Take $g = 9.8\,\text{m}\,\text{s}^{-2}$, unless stated otherwise.

Information
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.
- Unit Mechanics 2B has a **written paper only**.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 A hot air balloon moves vertically upwards with a constant velocity. When the balloon is at a height of 30 metres above ground level, a box of mass 5 kg is released from the balloon.

After the box is released, it initially moves vertically upwards with speed $10\,\mathrm{m\,s^{-1}}$.

(a) Find the initial kinetic energy of the box. *(2 marks)*

(b) Show that the kinetic energy of the box when it hits the ground is 1720 J. *(3 marks)*

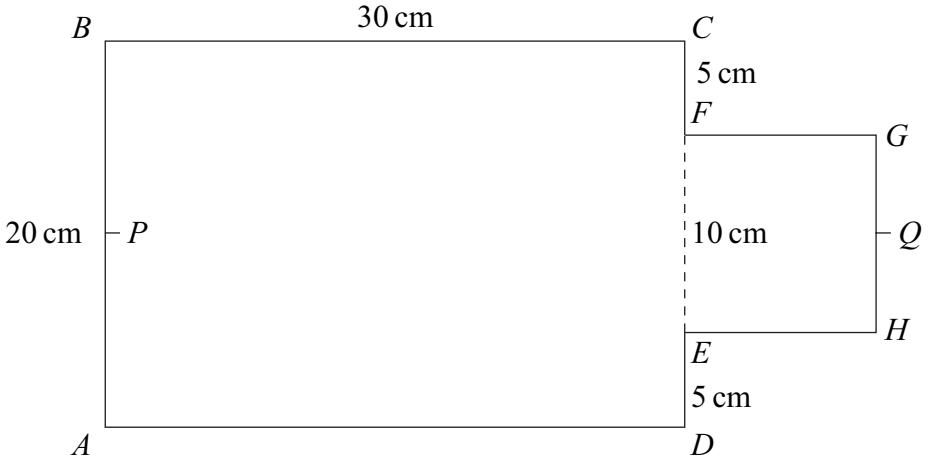
(c) Hence find the speed of the box when it hits the ground. *(3 marks)*

(d) State **two** modelling assumptions which you have made. *(2 marks)*

2 A uniform lamina is in the shape of a rectangle $ABCD$ and a square $EFGH$, as shown in the diagram.

The length AB is 20 cm, the length BC is 30 cm, the length DE is 5 cm and the length EF is 10 cm.

The point P is the midpoint of AB and the point Q is the midpoint of HG.



(a) Explain why the centre of mass of the lamina lies on PQ. *(1 mark)*

(b) Find the distance of the centre of mass of the lamina from AB. *(4 marks)*

(c) The lamina is freely suspended from A.

Find, to the nearest degree, the angle between AD and the vertical when the lamina is in equilibrium. *(4 marks)*

3 A particle has mass 800 kg. A single force of $(2400\,\mathbf{i} - 4800t\,\mathbf{j})$ newtons acts on the particle at time t seconds. No other forces act on the particle.

(a) Find the acceleration of the particle at time t. *(2 marks)*

(b) At time $t = 0$, the velocity of the particle is $(6\mathbf{i} + 30\mathbf{j})\,\mathrm{m\,s^{-1}}$. The velocity of the particle at time t is $\mathbf{v}\,\mathrm{m\,s^{-1}}$.

Show that

$$\mathbf{v} = (6 + 3t)\,\mathbf{i} + (30 - 3t^2)\,\mathbf{j}$$ *(4 marks)*

(c) Initially, the particle is at the point with position vector $(2\mathbf{i} + 5\mathbf{j})$ m.

Find the position vector, $\mathbf{r}$ metres, of the particle at time t. *(5 marks)*

4 A uniform plank is 10 m long and has mass 15 kg. It is placed on horizontal ground at the edge of a vertical river bank, so that 2 m of the plank is projecting over the edge, as shown in the diagram below.

(a) A woman of mass 50 kg stands on the part of the plank which projects over the river.

Find the greatest distance from the river bank at which she can safely stand. *(3 marks)*

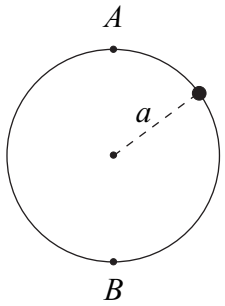
(b) The woman wishes to stand safely at the end of the plank which projects over the river.

Find the minimum mass which she should place on the other end of the plank so that she can do this. *(4 marks)*

(c) State how you have used the fact that the plank is uniform in your solution. *(1 mark)*

(d) State one other modelling assumption which you have made. *(1 mark)*

5 A bead of mass m moves on a smooth circular ring of radius a which is fixed in a vertical plane, as shown in the diagram. Its speed at A, the highest point of its path, is v and its speed at B, the lowest point of its path, is $7v$.



(a) Show that $v = \sqrt{\frac{ag}{12}}$. *(5 marks)*

(b) Find the reaction of the ring on the bead, in terms of m and g, when the bead is at A. *(4 marks)*

6 An elastic string has one end attached to a point O, fixed on a horizontal table. The other end of the string is attached to a particle of mass 5 kilograms. The elastic string has natural length 2 metres and modulus of elasticity 200 newtons. The particle is pulled so that it is 2.5 metres from the point O and it is then released from rest on the table.

(a) Calculate the elastic potential energy when the particle is 2.5 m from the point O. *(2 marks)*

(b) If the table is smooth, show that the speed of the particle when the string becomes slack is $\sqrt{5}$ m s^{-1}. *(3 marks)*

(c) The table is, in fact, rough and the coefficient of friction between the particle and the table is 0.4 .

Find the speed of the particle when the string becomes slack. *(7 marks)*

7 A stone of mass m is moving along the smooth horizontal floor of a tank which is filled with a viscous liquid. At time t, the stone has speed v. As the stone moves, it experiences a resistance force of magnitude λmv, where λ is a constant.

(a) Show that

$$\frac{\mathrm{d}v}{\mathrm{d}t} = -\lambda v \qquad \textit{(2 marks)}$$

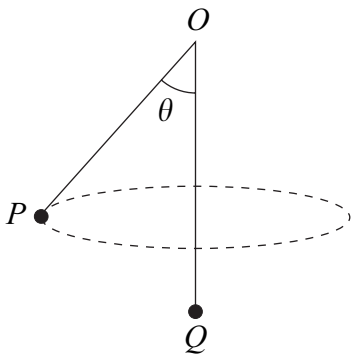
(b) The initial speed of the stone is U.

Show that

$$v = U\mathrm{e}^{-\lambda t} \qquad \textit{(4 marks)}$$

8 A particle, P, of mass 3 kg is attached to one end of a light inextensible string. The string passes through a smooth fixed ring, O, and a second particle, Q, of mass 5 kg is attached to the other end of the string. The particle Q hangs at rest vertically below the ring and the particle P moves with speed $4\,\mathrm{m\,s^{-1}}$ in a horizontal circle, as shown in the diagram.

The angle between OP and the vertical is θ.



(a) Explain why the tension in the string is 49 N. *(2 marks)*

(b) Find θ. *(3 marks)*

(c) Find the radius of the horizontal circle. *(4 marks)*

END OF QUESTIONS

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