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General Certificate of Education

Mathematics 6360

MM2B Mechanics 2B

Mark Scheme

2010 examination - January series

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

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Key to mark scheme and abbreviations used in marking

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation

√ or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	or equivalent	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
–*x* EE	deduct *x* marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

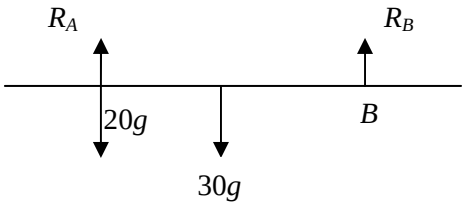
Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MM2B

Q	Solution	Marks	Total	Comments
1	Work done = $Fs\cos\theta$	M1		Accept $Fs\sin\theta$ for M1
	$= 40 \times 5 \times \cos 30$	A1		
	= 173 J	A1	3	
	Total		**3**	
2	$\bar{X} = \frac{3\times15+1\times7+6\times8+10\times12}{3+1+6+10}$	M1A1		M1 for at least 3 multiplication & addition
	$= \frac{220}{20}$ or 11	A1		
	$\bar{Y} = \frac{3\times6+1\times14+6\times7+10\times9}{20}$	M1A1		
	$= \frac{164}{20}$ or 8.2	A1	6	SC 4 (10, 7.4) [omit lamina] ie: B2, B2
	$\therefore$ Centre of mass is at (11, 8.2)			
	Total		**6**	
3(a)		B2	2	B1 for four forces B2 for two different reactions and 30g and 20g marked
(b)	Taking moments about A:			
	$3.2\times30g = R_B \times 5$	M1B1		B1 for 3.2
	$R_B = 19.2g$	A1	3	AG
(c)	Resolve vertically: $R_A + R_B = 50g$	M1		Can be awarded in (b)
	$R_A = 30.8g$ or 302 N	A1	2	
(d)	Gravitational force acts through mid-point of the rod	E1	1	
	Total		**8**	

MM2B (cont)

Q	Solution	Marks	Total	Comments
4(a)	$\mathbf{r} = \int \mathbf{v}\,dt$	M1		M1 for at least one term correct
	$= \left(t^4 - 6t^2 + 3t\right)\mathbf{i} + 5t\mathbf{j} + 4t^2\mathbf{k} + \mathbf{c}$	A1m1		m1 for + **c**
	When $t = 0$, $\mathbf{r} = -5\mathbf{i} + 6\mathbf{k}$ $\therefore \mathbf{c} = -5\mathbf{i} + 6\mathbf{k}$			
	$\therefore \mathbf{r} = \left(t^4 - 6t^2 + 3t - 5\right)\mathbf{i} + 5t\mathbf{j} + \left(6 + 4t^2\right)\mathbf{k}$	A1	4	
(b)	$\mathbf{a} = \left(12t^2 - 12\right)\mathbf{i} + 8\mathbf{k}$	M1A1	2	M1 for either component
(c)	Magnitude is $\left\{\left(12t^2 - 12\right)^2 + 64\right\}^{\frac{1}{2}}$	M1 A1F	2	
(d)	Magnitude is a minimum when $12t^2 - 12$ is zero	M1		M1 for correct differentiation of correct expression in (c)
	ie when $t = 1$	A1	2	
(e)	Minimum acceleration is 8			
	Using F = *ma*,	M1		*a* could be a vector
	$F = 7 \times 8 = 56$	A1	2	CAO
	Total		**12**	

MM2B (cont)

Q	Solution	Marks	Total	Comments
5(a)	Using $F = ma$,			
	$-0.2mv^{\frac{1}{2}} = m\dfrac{\mathrm{d}v}{\mathrm{d}t}$			
	$\therefore \dfrac{\mathrm{d}v}{\mathrm{d}t} = -0.2v^{\frac{1}{2}}$	B1	1	AG Must see equ'n containing m
(b)	$\displaystyle\int \frac{\mathrm{d}v}{v^{\frac{1}{2}}} = -\int 0.2\ \mathrm{d}t$	M1		
	$2v^{\frac{1}{2}} = -0.2t + c$	A1m1		m1 for + c
	When $t=0$, $v=16 \therefore C=8$	A1		
	$2v^{\frac{1}{2}} = -0.2t + 8$			
	$v = (4 - 0.1t)^2$	A1	5	AG
(c)	When $v=1, 1=(4-0.1t)^2$	M1		
	$4 - 0.1t = \pm 1$			
	$t=30$ or 50	A1		[if use $2v^{\frac{1}{2}} = 8 - 0.2t$ no need to see 50]
	$t=30$	A1	3	$t \neq 50$ as ball stops when $t = 40$
(d)	Integrating $v=(4-0.1t)^2$:			
	$v = 16 - 0.8t + 0.01t^2$			
	$x = 16t - 0.4t^2 + \dfrac{0.01}{3}t^3 + d$	M1		M1 for first 3 terms or $-\dfrac{10}{3}(4-01t)^3$
	When $t=0$, $x=0 \Rightarrow d=0$	A1		
	$x = 16t - 0.4t^2 + \dfrac{0.01}{3}t^3$			
	When speed is $1\,\mathrm{m\,s^{-1}}$, $t = 30$			
	$x = 480 - 360 + 90$	m1		dep on M1 above
	$= 210$	A1	4	[No 'd', 3 marks only]
	Total		**13**	

MM2B (cont)

Q	Solution	Marks	Total	Comments
6(a)	$r = 1.2\sin\theta$	B1	1	1.2 $\cos\theta$ 0 marks
(b)	Resolve horiz: $T\sin\theta = m\omega^2 r$	M1A1		$T\cos\theta = m\omega^2 r$ etc M1 (+ second M1)
	$T\sin\theta = 4\times 5^2 \times 1.2\sin\theta$			
	$T = 120$	A1		
	Resolve vert: $T\cos\theta = 4g$	M1A1		
	$\cos\theta = 0.32666$			M1 for $\tan\theta = \dfrac{30\sin\theta}{g}$
	$\theta = 70.9°$ or 1.24^c	A1	6	
	Total		**7**	
7(a)	Using conservation of energy:			
	$\frac{1}{2}mu^2 = \frac{1}{2}mv^2 - mgh$	M1A1		M1 for 3 terms, 2 KE and PE or 4 terms , 2 KE and 2 PE
	$\frac{1}{2}mu^2 = \frac{1}{2}mv^2 - mga(1-\cos\theta)$	M1A1		M1A1 for finding h
	$v^2 = u^2 + 2ga(1-\cos\theta)$			
	$v = \left(u^2 + 2ga[1-\cos\theta]\right)^{\frac{1}{2}}$	A1	5	AG
(b)	Using $F = ma$ radially,			
	$mg\cos\theta - \mathrm{N} = \dfrac{mv^2}{a}$	M1A1		M1 Correct 3 terms A1 Correct signs $(-\mathrm{N}$ or $+\mathrm{N})$
	Particle leaves surface of hemisphere when N = 0	B1		
	$mg\cos\theta = \dfrac{m}{a}\left(u^2 + 2ga[1-\cos\theta]\right)$	M1		
	$\cos\theta = \dfrac{u^2}{ga} + 2 - 2\cos\theta$			
	$\cos\theta = \dfrac{1}{3}\left(\dfrac{u^2}{ga} + 2\right)$	A1	5	
	Total		**10**	

MM2B (cont)

Q	Solution	Marks	Total	Comments
8(a)	When $x \geq 22$, KE is $\frac{1}{2}\times 49\times v^2$			
	EPE is $\frac{1078(x-22)^2}{2\times 22}$	M1A1		M1 for any $\frac{1078p^2}{2\times 22}$
	Change in PE is $49 \times g \times x$			
	Conservation of energy:			
	$\frac{1}{2}\times 49\times v^2 + \frac{1078(x-22)^2}{2\times 22} = 49\times g\times x$	M1A1 A1		M1 3 terms (KE, PE, EPE) A1 2 terms correct A1 all 3 terms correct
	$\frac{49}{2}v^2 + \frac{49}{2}(x-22)^2 = 49gx$			
	$v^2 + (x-22)^2 = 19.6x$			SC3 $\frac{49}{2}v^2 + \frac{49}{2}e^2 = 49g(e+22)$ [could use x for e]
	$5v^2 = 318x - 5x^2 - 2420$	A1	6	AG
(b)	If x is not greater than 22, cord is not stretched	B1	1	
(c)	At maximum value of x, $v = 0$	M1		
	$\therefore 5x^2 - 318x + 2420 = 0$			
	$x = \frac{318 \pm \sqrt{318^2 - 4\times 5\times 2420}}{2\times 5}$	m1		dep on M1 above
	x = 54.76.. or 8.84..	A1		A1 for either solution
	= 54.8	E1	4	Needs to give a reason for deletion of second root. Both roots must be positive: one above 22, one below 22
(d)(i)	When speed is a maximum, $a = 0$			**or**
	tension = gravitational force	M1		$\frac{d(5v^2)}{dx} = 318 - 10x$
	$\frac{1078(x-22)}{22} = 49g$	A1		= 0 at maximum speed $\Rightarrow 318 - 10x = 0$
	$x - 22 = 9.8$			
	x = 31.8	A1	3	AG
(ii)	From part (a), $v^2 = 19.6\times 31.8 - 9.8^2$	M1		
	v = 22.96	A1	2	
	Maximum speed is $23.0\,\text{m s}^{-1}$			
	Total		**16**	
	TOTAL		**75**	