General Certificate of Education
January 2009
Advanced Level Examination

MATHEMATICS MM2B
Unit Mechanics 2B

Thursday 29 January 2009 9.00 am to 10.30 am

For this paper you must have:
- an 8-page answer book
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MM2B.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Take $g = 9.8\,\text{m}\,\text{s}^{-2}$, unless stated otherwise.

Information
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 A particle moves along a straight line. At time t, it has velocity v, where

$$v = 4t^3 - 8\sin 2t + 5$$

When $t = 0$, the particle is at the origin.

Find an expression for the displacement of the particle from the origin at time t. *(4 marks)*

2 A stone, of mass 6 kg, is thrown vertically upwards with a speed of $12\,\text{m s}^{-1}$ from a point at a height of 4 metres above ground level.

(a) Calculate the initial kinetic energy of the stone. *(2 marks)*

(b) (i) Show that the kinetic energy of the stone when it hits the ground is 667 J, correct to three significant figures. *(2 marks)*

(ii) Hence find the speed of the stone when it hits the ground. *(3 marks)*

(iii) State two modelling assumptions that you have made. *(2 marks)*

3 A particle moves on a horizontal plane, in which the unit vectors **i** and **j** are directed east and north respectively.

At time t seconds, the position vector of the particle is **r** metres, where

$$\mathbf{r} = \left(2e^{\frac{1}{2}t} - 8t + 5\right)\mathbf{i} + (t^2 - 6t)\mathbf{j}$$

(a) Find an expression for the velocity of the particle at time t. *(3 marks)*

(b) (i) Find the speed of the particle when $t = 3$. *(2 marks)*

(ii) State the direction in which the particle is travelling when $t = 3$. *(1 mark)*

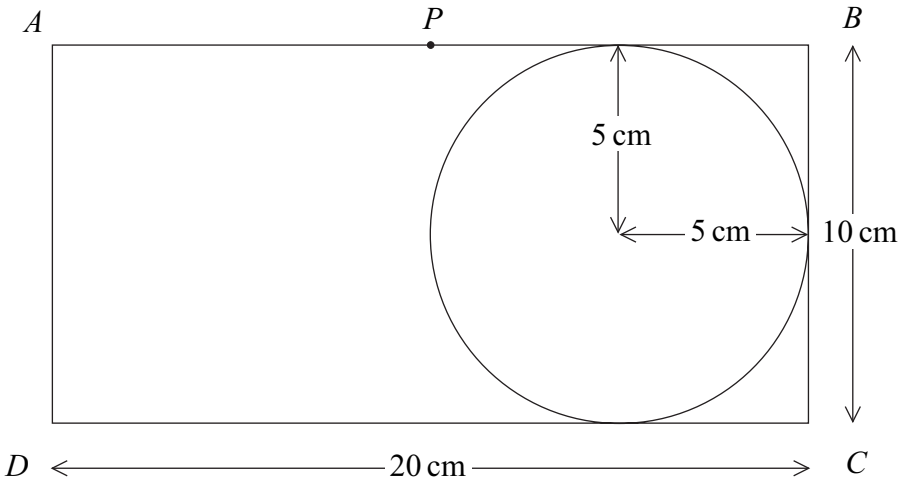
(c) Find the acceleration of the particle when $t = 3$. *(3 marks)*

(d) The mass of the particle is 7 kg.

Find the magnitude of the resultant force on the particle when $t = 3$. *(3 marks)*

4 A uniform rectangular lamina $ABCD$ has a mass of 8 kg. The side AB has length 20 cm, the side BC has length 10 cm, and P is the mid-point of AB.

A uniform circular lamina, of mass 2 kg and radius 5 cm, is fixed to the rectangular lamina to form a sign. The centre of the circular lamina is 5 cm from each of AB and BC, as shown in the diagram.



(a) Find the distance of the centre of mass of the sign from AD. *(3 marks)*

(b) Write down the distance of the centre of mass of the sign from AB. *(1 mark)*

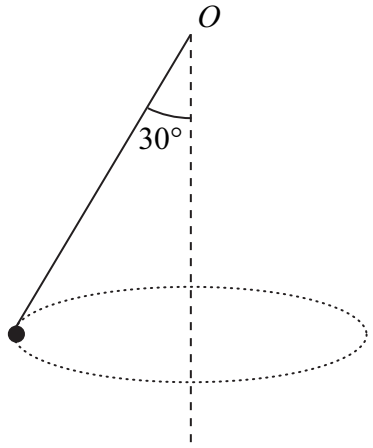
(c) The sign is freely suspended from P.

Find the angle between AD and the vertical when the sign is in equilibrium. *(4 marks)*

(d) Explain how you have used the fact that each lamina is uniform in your solution to this question. *(1 mark)*

Turn over for the next question

5 A particle, of mass 6 kg, is attached to one end of a light inextensible string. The other end of the string is attached to the fixed point O. The particle is set in motion, so that it moves in a horizontal circle at constant speed, with the string at an angle of 30° to the vertical. The centre of this circle is vertically below O.



The particle moves in a horizontal circle with an angular speed of 40 revolutions per minute.

(a) Show that the angular speed of the particle is $\frac{4\pi}{3}$ radians per second. *(2 marks)*

(b) Show that the tension in the string is 67.9 N, correct to three significant figures. *(3 marks)*

(c) Find the radius of the horizontal circle. *(4 marks)*

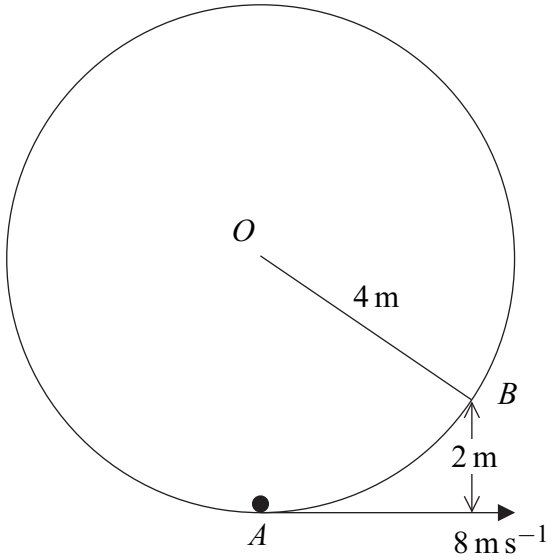
6 A train, of mass 60 tonnes, travels on a straight horizontal track. It has a maximum speed of $40\,\text{m}\,\text{s}^{-1}$ when its engine is working at 800 kW.

(a) Find the magnitude of the resistance force acting on the train when the train is travelling at its maximum speed. *(3 marks)*

(b) When the train is travelling at $40\,\text{m}\,\text{s}^{-1}$, the power is turned off. Assume that the resistance force is constant and is equal to that found in part (a). Also assume that this resistance force is the only horizontal force acting on the train.

Use an energy method to find how far the train travels when slowing from $40\,\text{m}\,\text{s}^{-1}$ to $36\,\text{m}\,\text{s}^{-1}$. *(4 marks)*

7 A hollow cylinder, of internal radius 4 m, is fixed so that its axis is horizontal. The point O is on this axis. A particle, of mass 6 kg, is set in motion so that it moves on the smooth **inner** surface of the cylinder in a vertical circle about O. Its speed at the point A, which is vertically below O, is $8\,\mathrm{m\,s^{-1}}$.



When the particle is at the point B, at a height of 2 m above A, find:

(a) its speed; *(3 marks)*

(b) the normal reaction between the cylinder and the particle. *(4 marks)*

8 A stone, of mass 0.05 kg, is moving along the smooth horizontal floor of a tank, which is filled with oil. At time t, the stone has speed v. As the stone moves, it experiences a resistance force of magnitude $0.08v^2$.

(a) Show that

$$\frac{\mathrm{d}v}{\mathrm{d}t} = -1.6v^2$$ *(2 marks)*

(b) The initial speed of the stone is $3\,\mathrm{m\,s^{-1}}$.

Show that

$$v = \frac{15}{5 + 24t}$$ *(5 marks)*

Turn over ▶

9 A bungee jumper, of mass 80 kg, is attached to one end of a light elastic cord, of natural length 16 metres and modulus of elasticity 784 N. The other end of the cord is attached to a horizontal platform, which is at a height of 65 metres above the ground.

The bungee jumper steps off the platform at the point where the cord is attached and falls vertically. The bungee jumper can be modelled as a particle. Hooke's law can be assumed to apply throughout the motion and air resistance can be assumed to be negligible.

(a) Find the length of the cord when the acceleration of the bungee jumper is zero. *(3 marks)*

(b) The cord extends by x metres beyond its natural length before the bungee jumper first comes to rest.

(i) Show that $x^2 - 32x - 512 = 0$. *(4 marks)*

(ii) Find the distance above the ground at which the bungee jumper first comes to rest. *(4 marks)*

END OF QUESTIONS

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