

## **General Certificate of Education**

# **Mathematics 6360**

MM2B Mechanics 2B

# **Mark Scheme**

2009 examination - January series

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

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### Key to mark scheme and abbreviations used in marking

| M       | mark is for method   |
|---------|--|
| m or dM | mark is dependent on one or more M marks and is for method         |
| A       | mark is dependent on M or m marks and is for accuracy              |
| В       | mark is independent of M or m marks and is for method and accuracy |
| Е       | mark is for explanation  |

| √or ft or F | follow through from previous   |     |                            |
|-------------|--------------------------------|-----|----------------------------|
|             | incorrect result               | MC  | mis-copy                   |
| CAO         | correct answer only            | MR  | mis-read                   |
| CSO         | correct solution only          | RA  | required accuracy          |
| AWFW        | anything which falls within    | FW  | further work               |
| AWRT        | anything which rounds to       | ISW | ignore subsequent work     |
| ACF         | any correct form               | FIW | from incorrect work        |
| AG          | answer given                   | BOD | given benefit of doubt     |
| SC          | special case                   | WR  | work replaced by candidate |
| OE          | or equivalent                  | FB  | formulae book              |
| A2,1        | 2 or 1 (or 0) accuracy marks   | NOS | not on scheme              |
| –x EE       | deduct x marks for each error  | G   | graph                      |
| NMS         | no method shown                | c   | candidate                  |
| PI          | possibly implied               | sf  | significant figure(s)      |
| SCA         | substantially correct approach | dp  | decimal place(s)           |

#### No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

## MM2B

| Q      | Solution   | Marks | Total    | Comments                            |
|--------|--|-------|----------|-------------------------------------|
| 1      | $r = \int v  \mathrm{d}t$  | M1    |          |                                     |
|        | $= t^4 + 4\cos 2t + 5t \ (+c)$   | A1    |          |                                     |
|        | When $t = 0$ , $r = 0 \implies c = -4$   | M1    |          | Finding c correctly                 |
|        | $\therefore r = t^4 + 4\cos 2t + 5t - 4$   | A1ft  | 4        |                                     |
|        | Total  |       | 4        |                                     |
| 2(a)   | Initial KE = $\frac{1}{2}mv^2$   |       |          |                                     |
|        | $=\frac{1}{2}\times 6\times 12^2$  | M1    |          | Allow one of $m$ and $v$ incorrect  |
|        | = 432 J  | A1    | 2        |                                     |
| (b)(i) | When it hits the ground, conservation of energy gives KE = Initial KE + loss in PE |       |          |                                     |
|        | $= 432 + 6 \times g \times 4$<br>= 667.2   | M1    |          | Need $6 \times g \times 4$ or 235.2 |
|        | = 667  J (3sf)   | A1    | 2        | AG                                  |
| (ii)   | $667.2 = \frac{1}{2} \times 6 \times v^2$  | M1A1  |          |                                     |
|        | Speed is 14.9 m s <sup>-1</sup>  | A1    | 3        |                                     |
| (iii)  | Stone is a particle  | B1    |          | Not g constant                      |
|        | No air resistance Total  | B1    | <u>2</u> | No other forces acting              |
|        | 1 otal   |       | 9        |                                     |

## MM2B (cont)

| MM2B (con<br>Q | Solution   | Marks      | Total | Comments   |
|----------------|--|------------|-------|--|
|                |  |            | Total | Comments   |
| 3(a)           | $\mathbf{v} = \frac{\mathrm{d}\mathbf{r}}{\mathrm{d}t}$  | M1         |       |  |
|                | $\mathbf{v} = (e^{\frac{1}{2}t} - 8)\mathbf{i} + (2t - 6)\mathbf{j}$   | A1         |       | i terms  |
|                | $\mathbf{v} = (\mathbf{e}^2 - 8)\mathbf{i} + (2t - 6)\mathbf{j}$   | A1         | 3     | <b>j</b> terms   |
|                |  |            |       |  |
| (b)(i)         | When $t = 3$ , $\mathbf{v} = -3.52\mathbf{i}$  | B1         |       | Accept $(e^{\frac{3}{2}} - 8)i$  |
| , , , , ,      | Speed is 3.52 m s <sup>-1</sup>  | B1         | 2     | 3.5 does not give 2 <sup>nd</sup> B mark   |
|                | •  |            |       |  |
| (ii)           | West   | B1         | 1     |  |
|                | 1  |            |       |  |
| (c)            | $\mathbf{a} = \frac{1}{2} e^{\frac{1}{2}t} \mathbf{i} + 2\mathbf{j}$   | M1A1       |       |  |
|                |  |            |       |  |
|                | When $t = 3$ , $\mathbf{a} = \frac{1}{2}e^{\frac{3}{2}}\mathbf{i} + 2\mathbf{j}$ or $2.24\mathbf{i} + 2\mathbf{j}$ | A1         | 3     |  |
|                | 2  |            |       |  |
| (d)            | Using $\mathbf{F} = m\mathbf{a}$ :   | M1         |       | Accept $\mathbf{F} = 7\mathbf{a}$  |
|                |  |            |       |  |
|                | $\mathbf{F} = 7(\frac{1}{2}e^{\frac{3}{2}}\mathbf{i} + 2\mathbf{j})$   |            |       |  |
|                | ∴ Magnitude of force is  |            |       |  |
|                | $-(1^{\frac{3}{2}})^2$   | M1         |       |  |
|                | $7\left((\frac{1}{2}e^{\frac{3}{2}})^2+2^2\right)^{\frac{1}{2}}$   | M1         |       |  |
|                | $\mathbf{F} = 21.025$  |            |       |  |
|                | $\mathbf{F} = 21.0$  | A1         | 3     | Accept 21  |
| 4(a)           | Total  |            | 12    |  |
| <b>4(a)</b>    | Taking moments about AD:   |            |       | M1 for moments and 1 term on left  |
|                | $8 \times 10 + 2 \times 15 = 10\overline{x}$   | M1A1       |       | correct and 1 term on right  |
|                | $\overline{x} = \frac{110}{10}$  |            |       | , and the second |
|                |  | . 1        | 2     |  |
|                | = 11 cm  | A1         | 3     |  |
| (b)            | 5 cm   | B1         | 1     |  |
|                |  |            |       |  |
| (c)            | $(\tan \theta) = \frac{1}{5} \text{ ie } \frac{(a)-10}{(b)}$   | M1         |       | From areas; $\frac{1.4}{5} \Rightarrow \theta = 15.6$ or 15.7  |
| (6)            |  |            |       | $\frac{110111 \text{ areas, } \frac{1}{5}}{5} \rightarrow 0 - 13.0 \text{ or } 13.7$   |
|                | = 0.2  | A1ft       |       |  |
|                | Angle is $ton^{-1}(0.2)$   | M1         |       |  |
|                | Angle is $\tan^{-1}(0.2)$<br>= 11.3°   | M1<br>A1ft | 4     |  |
|                | - 11.3   | 13111      | 7     |  |
| (d)            | Centre of mass is at middle of lamina  | E1         | 1     |  |
|                | Total  |            | 9     |  |

## MM2B (cont)

| Q Q        | Solution  | Marks      | Total | Comments                                      |
|------------|---|------------|-------|---|
| 5(a)       | 40 revolutions per minute   |            |       |   |
|            | $=80\pi$ radians per minute   | B1         |       | or $\frac{2}{3}$ rev per second               |
|            | $=\frac{4\pi}{3}$ radians per second  | B1         | 2     | AG  |
| <b>(b)</b> | Resolve vertically:<br>$T \cos 30 = 6g$<br>T = 67.9  N                                    | M1A1<br>A1 | 3     | M1 1 term each side, 1 correct AG             |
| (c)        | Resolve horizontally:<br>$T \sin 30 = m\omega^2 r$  | M1<br>A1   |       | M1 1 term each side, 1 correct A1 $T \sin 30$ |
|            | $67.9 \sin 30 = 6 \times r \times \left(\frac{4\pi}{3}\right)^2$                          | A1         |       | A1 RHS  |
|            | r = 0.322  m  | A1         | 4     | Condone 0.323 (using $\pi$ as 3.14)           |
|            | Total   |            | 9     |   |
| 6(a)       | At maximum speed,<br>tractive force = resistance force<br>Using power = force × velocity: | M1         |       |   |
|            | $800\ 000 = F \times 40$  | M1         |       |   |
|            | $F = 20\ 000\ N$  | A1         | 3     |   |
| (b)        | Using force × distance = work done = change in energy:                                    |            |       |   |
|            | 1 2 2   | M1         |       | M1 $Fs = \text{change of KE}$                 |
|            | $20\ 000\ s = \frac{1}{2} \times 60\ 000 \times (40^2 - 36^2)$                            | A1         |       | A1 2 of 3 terms correct                       |
|            | <del>-</del>  | A1         |       | A1 all 3 terms correct                        |
|            | Distance = 456 m  | A1         | 4     |   |
|            | Total   |            | 7     |   |
| 7(a)       | $\frac{1}{2}mv^2 = \frac{1}{2}m \times 8^2 - mg2$   | M1<br>A1   |       | M1 3 terms, 2 KE and 1 PE                     |
|            | $v^2 = 64 - 39.2$<br>= 24.8   |            |       |   |
|            | v = 4.98  | A1         | 3     | Accept $\sqrt{24.8}$                          |
| (b)        | Using $F = ma$ radially:  |            |       | M1 2  |
|            | $R = mg\cos 60 + \frac{mv^2}{r}$  | M1         |       | M1 3 correct terms (not necessarily           |
|            | $\overline{r}$  | A1<br>B1   |       | correct signs)                                |
|            | $=6g\cos 60 + \frac{6 \times 24.8}{4}$  | DI         |       | B1 for 60°                                    |
|            | = 66.6 N  | A1         | 4     |   |
|            | Total   |            | 7     |   |

## MM2B (cont)

| Q Q        | Solution   | Marks | Total | Comments  |
|------------|--|-------|-------|---|
| 8(a)       | Using $F = ma$ :   |       |       |   |
|            | $-0.08v^2 = 0.05a$   | B1    |       |   |
|            |  |       | 2     | A.C. condens sign amon in first D1                                    |
|            | $\therefore \frac{\mathrm{d}v}{\mathrm{d}t} = -1.6v^2$                             | B1    | 2     | AG; condone sign error in first B1                                    |
|            |  |       |       |   |
| <b>(b)</b> | $\int \frac{dv}{2} = -1.6 \int dt$   | M1    |       |   |
|            | $\int \frac{\mathrm{d}v}{v^2} = -1.6 \int \mathrm{d}t$ $-\frac{1}{v} = -1.6t (+c)$ |       |       | 1 1   |
|            | $-\frac{1}{11} = -1.6t (+c)$   | A1    |       | Condone $-\frac{1}{v} = -1.6t + c \Rightarrow \frac{1}{v} = 1.6t + c$ |
|            |  |       |       | V   |
|            | When $t = 0$ , $v = 3 \Rightarrow c = -\frac{1}{3}$                                | M1    |       |   |
|            |  |       |       |   |
|            | $\frac{1}{v} = \frac{1}{3} + 1.6t$ *   | A1    |       |   |
|            | $\frac{1}{v} = \frac{1}{3} + \frac{8}{5}t$   |       |       |   |
|            |  |       |       |   |
|            | $\frac{1}{v} = \frac{5+24t}{15}$   |       |       |   |
|            |  |       |       |   |
|            | $v = \frac{15}{5 + 24t}$   | A1    | 5     | AG; all working lines correct from *                                  |
|            | Total  |       | 7     |   |
| 9(a)       | When acceleration is zero,   |       | •     |   |
| . ,        | tension = gravitational force  |       |       |   |
|            | $\frac{784x}{16} = 80g$  | M1    |       | Both terms correct  |
|            |  |       |       |   |
|            | x = 16, $x + 16 = 32$ m  | A1    |       | A1 for $x=16$   |
|            | Length of cord is 32 m   | A1    | 3     |   |
| (b)(i)     | When bungee jumper comes to rest,  |       |       |   |
| (2)(1)     |  | 3.61  |       |   |
|            | $EPE = \frac{784 \times x^2}{2 \times 16}$   | M1    |       |   |
|            | $=\frac{49x^2}{2}$   |       |       |   |
|            | 2  |       |       |   |
|            | Change in PE = $80 \times g \times (16 + x)$                                       | M1    |       | Or $80 \times g \times 65 - (80g[16+x])$                              |
|            |  |       |       | (or 80g(49-x))  |
|            | $49x^{2}$  |       |       |   |
|            | $\frac{49x^2}{2} = 80 \times 9.8 \times (16 + x)$                                  | A1    |       |   |
|            | $x^2 = 32x + 512$  |       |       |   |
|            | $x^2 - 32x - 512 = 0$  | A1    | 4     | AG  |
|            |  |       |       |   |
| (ii)       | $x = \frac{32 \pm \sqrt{32^2 + 2048}}{2}$  | M1    |       |   |
| (11)       | <i>L</i>   |       |       |   |
|            | x = 43.7128  | A1    |       |   |
|            | Distance below point of jump is  |       |       |   |
|            | 43.7 + 16 = 59.7  m<br>Distance between jumper and ground is                       |       |       |   |
|            | 65 – 59.7  | M1    |       |   |
|            | = 5.29  m  | A1    | 4     | Accept 5.287, 5.3   |
|            | Total  |       | 11    |   |
|            | TOTAL  |       | 75    |   |