General Certificate of Education
January 2008
Advanced Level Examination

ASSESSMENT and QUALIFICATIONS ALLIANCE

MATHEMATICS **MM2B**
Unit Mechanics 2B

Tuesday 15 January 2008 9.00 am to 10.30 am

For this paper you must have:
- an 8-page answer book
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MM2B.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Take $g = 9.8\,\text{m}\,\text{s}^{-2}$, unless stated otherwise.

Information
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.
- Unit Mechanics 2B has a **written paper only**.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 A ball is thrown vertically upwards from ground level with an initial speed of $15\,\text{m}\,\text{s}^{-1}$. The ball has a mass of 0.6 kg. Assume that the only force acting on the ball after it is thrown is its weight.

(a) Calculate the initial kinetic energy of the ball. *(2 marks)*

(b) By using conservation of energy, find the maximum height above ground level reached by the ball. *(3 marks)*

(c) By using conservation of energy, find the kinetic energy and the speed of the ball when it is at a height of 3 m above ground level. *(4 marks)*

(d) State one modelling assumption which has been made. *(1 mark)*

2 A particle moves in a straight line and at time t it has velocity v, where

$$v = 3t^2 - 2\sin 3t + 6$$

(a) (i) Find an expression for the acceleration of the particle at time t. *(2 marks)*

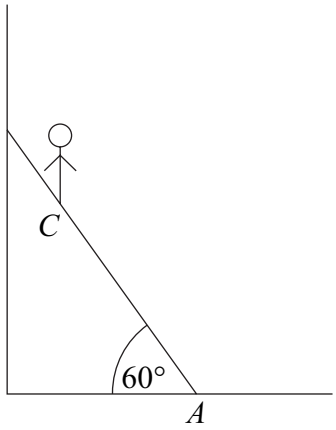
(ii) When $t = \frac{\pi}{3}$, show that the acceleration of the particle is $2\pi + 6$. *(2 marks)*

(b) When $t = 0$, the particle is at the origin.

Find an expression for the displacement of the particle from the origin at time t. *(4 marks)*

3 A uniform ladder of length 4 metres and mass 20 kg rests in equilibrium with its foot, A, on a rough horizontal floor and its top leaning against a smooth vertical wall. The vertical plane containing the ladder is perpendicular to the wall and the angle between the ladder and the floor is 60°.

A man of mass 80 kg is standing at point C on the ladder. With the man in this position, the ladder is on the point of slipping. The coefficient of friction between the ladder and the floor is 0.4. The man may be modelled as a particle at C.



(a) Draw a diagram to show the forces acting on the ladder. *(2 marks)*

(b) Show that the magnitude of the frictional force between the ladder and the ground is 392 N. *(3 marks)*

(c) Find the distance AC. *(6 marks)*

4 A particle moves in a horizontal plane under the action of a single force, **F** newtons. The unit vectors **i** and **j** are directed east and north respectively. At time t seconds, the position vector, **r** metres, of the particle is given by

$$\mathbf{r} = (t^3 - 3t^2 + 4)\mathbf{i} + (4t + t^2)\mathbf{j}$$

(a) Find an expression for the velocity of the particle at time t. *(2 marks)*

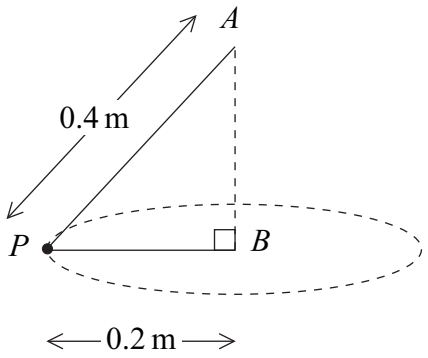
(b) The mass of the particle is 3 kg.

(i) Find an expression for **F** at time t. *(3 marks)*

(ii) Find the magnitude of **F** when $t = 3$. *(2 marks)*

(c) Find the value of t when **F** acts due north. *(2 marks)*

5 Two light inextensible strings, of lengths 0.4 m and 0.2 m, each have one end attached to a particle, P, of mass 4 kg. The other ends of the strings are attached to the points A and B respectively. The point A is vertically above the point B. The particle moves in a horizontal circle, centre B and radius 0.2 m, at a speed of $2\,\text{m}\,\text{s}^{-1}$. The particle and strings are shown in the diagram.



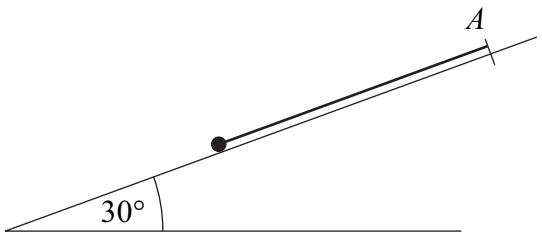
(a) Calculate the magnitude of the acceleration of the particle. *(2 marks)*

(b) Show that the tension in string PA is 45.3 N, correct to three significant figures. *(4 marks)*

(c) Find the tension in string PB. *(3 marks)*

6 A light elastic string has one end attached to a point A fixed on a smooth plane inclined at 30° to the horizontal. The other end of the string is attached to a particle of mass 6 kg. The elastic string has natural length 4 metres and modulus of elasticity 300 newtons.

The particle is pulled down the plane in the direction of the line of greatest slope through A. The particle is released from rest when it is 5.5 metres from A.

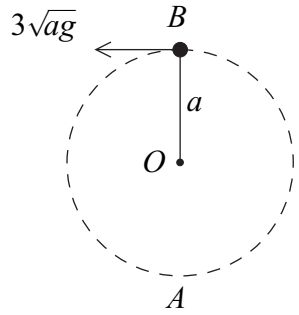


(a) Calculate the elastic potential energy of the string when the particle is 5.5 metres from the point A. *(2 marks)*

(b) Show that the speed of the particle when the string becomes slack is $3.66\,\text{m}\,\text{s}^{-1}$, correct to three significant figures. *(5 marks)*

(c) Show that the particle will **not** reach point A in the subsequent motion. *(3 marks)*

7 A light inextensible string, of length a, has one end attached to a fixed point O. A particle, of mass m, is attached to the other end. The particle is moving in a vertical circle, centre O. When the particle is at B, vertically above O, the string is taut and the particle is moving with speed $3\sqrt{ag}$.



(a) Find, in terms of g and a, the speed of the particle at the lowest point, A, of its path. *(4 marks)*

(b) Find, in terms of g and m, the tension in the string when the particle is at A. *(4 marks)*

8 A car of mass 600 kg is driven along a straight horizontal road. The resistance to motion of the car is kv^2 newtons, where $v\,\mathrm{m\,s^{-1}}$ is the velocity of the car at time t seconds and k is a constant.

(a) When the engine of the car has power 8 kW, show that the equation of motion of the car is

$$600\frac{\mathrm{d}v}{\mathrm{d}t} - \frac{8000}{v} + kv^2 = 0 \qquad \textit{(4 marks)}$$

(b) When the velocity of the car is $20\,\mathrm{m\,s^{-1}}$, the engine is turned off.

(i) Show that the equation of motion of the car now becomes

$$600\frac{\mathrm{d}v}{\mathrm{d}t} = -kv^2 \qquad \textit{(1 mark)}$$

(ii) Find, in terms of k, the time taken for the velocity of the car to drop to $10\,\mathrm{m\,s^{-1}}$. *(5 marks)*

END OF QUESTIONS

There are no questions printed on this page

There are no questions printed on this page

There are no questions printed on this page