SUMMARY SHEET – MECHANICS 1

Momentum

The main ideas are

	AQA	Edx	MEI	OCR
Momentum	M1	M1	M2	M1
Conservation law	M1	M1	M2	M1

Before the exam you should know:

- What momentum is, in particular that it is a vector.
- How to calculate the momentum of objects.
- About conservation of momentum and understand the calculations involved.

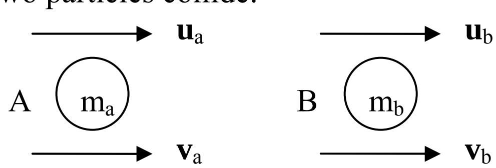
Momentum

If an object of mass m has velocity $\mathbf{v}$, then the momentum of the object is its mass x velocity,
i.e. Momentum = $m\mathbf{v}$
Momentum is a vector quantity because it has both a magnitude and direction associated with it. Its units are kg ms^{-1} or Newton seconds Ns.

Conservation law

For a system of interacting particles, the total momentum of a system remains constant when there is no resultant external force acting.

If two particles collide:



Where:

m_a = mass of particle A
m_b = mass of particle B
$\mathbf{u}_a$ = velocity of particle A before collision
$\mathbf{u}_b$ = velocity of particle B before collision
$\mathbf{v}_a$ = velocity of particle A after collision
$\mathbf{v}_b$ = velocity of particle B after collision

Then, the principle of conservation of momentum is: $m_a\mathbf{u}_a + m_b\mathbf{u}_b = m_a\mathbf{v}_a + m_b\mathbf{v}_b$

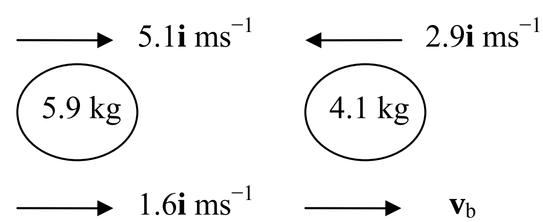
i.e. Total momentum before collision
= Total momentum after collision

It is key to read these questions carefully noting the directions of the velocities of the particles both before and after a collision. Remember two particles could coalesce and move as one particle.

Example *(Momentum)*
A biker and their motorbike have a combined mass of 320 kg and are travelling along a straight horizontal road at 22 ms^{-1}. A cyclist and their cycle, which have a combined mass of 80 kg, travel in the opposite direction at 10 ms^{-1}. What is the momentum of the biker and what is the momentum of the cyclist?

Defining the direction of the biker as positive, then:
momentum of biker = $320 \times 22 = 7040$ kg ms^{-1}
momentum of cyclist = $80 \times (-10) = -800$ kg ms^{-1}

Example *(Conservation law)*
A particle, of mass 5.9 kg, travelling in a straight line at $5.1\mathbf{i}$ ms^{-1} collides with another particle, of mass 4.1 kg, travelling in the same straight line, but in the opposite direction, with a velocity of $2.9\mathbf{i}$ ms^{-1}. Given that after the collision the first particle continues to move in the same direction at $1.5\mathbf{i}$ ms^{-1}, what velocity does the second particle move with after the collision?



$$m_a\mathbf{u}_a + m_b\mathbf{u}_b = m_a\mathbf{v}_a + m_b\mathbf{v}_b$$
$$5.9 \times 5.1\mathbf{i} + 4.1 \times (-2.9\mathbf{i}) = 2.9 \times 1.6\mathbf{i} + 4.1 \times \mathbf{v}_b$$
$$30.09\mathbf{i} - 11.89\mathbf{i} - 4.64\mathbf{i} = 4.1 \times \mathbf{v}_b$$
$$13.56\mathbf{i} = 4.1\,\mathbf{v}_b$$
$$\mathbf{v}_b = 3.31\mathbf{i}\ ms^{-1}$$

REVISION SHEET – MECHANICS 1

MOTION GRAPHS

The main ideas are

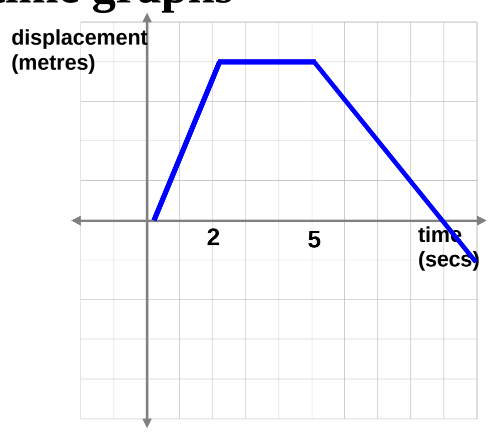
	AQA	Edx	MEI	OCR
Displacement-time graphs	M1	M1	M1	M1
Distance-time graphs	M1	M1	M1	M1
Velocity-time graphs	M1	M1	M1	M1
Interpreting the graphs	M1	M1	M1	M1
Using differentiation and integration	M2	M2	M1	M1

Before the exam you should know

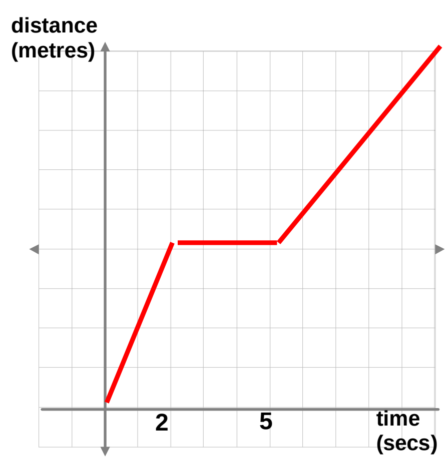
- The difference between a distance-time graph and a displacement-time graph, and the difference between a speed-time graph and a velocity-time graph.
- The gradient of a displacement time graph gives the velocity and the gradient of a distance time graph gives the speed.
- The displacement from the starting point is given by the area under the velocity-time graph (where area below the *x*-axis is given a negative sign)
- In the case of a particle moving with a variable acceleration, know how to use differentiation and integration to calculate acceleration, velocity and displacement via the gradients and areas described in 2 and 3 above.

Displacement-time graphs and distance-time graphs

A displacement time graph plots the displacement of the object from some fixed origin against time.
For example, the journey described in the graph to the right can be described as follows,
Phase 1: The object moves away from the origin (at a constant speed) for 2 seconds
Phase 2: It remains motionless for 3 seconds.
Phase 3: It starts moving back towards the origin at a constant speed returning to the origin after about 4 seconds and then continues in the same direction another second.

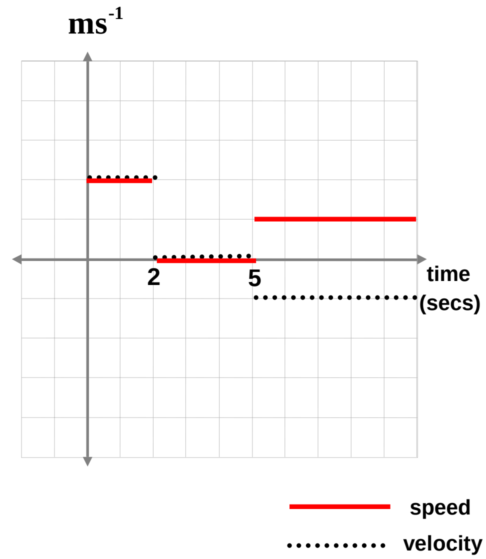


Here is a distance-time graph of the same journey. Notice that the gradient of the distance-time graph is never negative (as time increases it is impossible for the total distance you have travelled to decrease). The gradient is positive whenever the object is moving and it is zero whenever the object is stationary. In fact the gradient of the distance-time graph is the modulus of the gradient of the displacement-time graph at all times. Think about this!



Velocity-time graphs and speed-time graphs

A velocity time graph plots the velocity of the object against time, whereas a speed-time graph plots speed against time. Remember an objects speed is just the modulus of its velocity.
For example for the journey we looked at in the displacement-time graph over the page the velocity-time graphs is drawn to the right as a dotted line and the speed time graph is drawn in as a solid line.
Phase 1: Object moving in a direction (which has been designated the "positive direction" at a constant speed of $2\,\text{ms}^{-1}$.
Phase 2: Stationary, zero velocity.
Phase 3: Moving in the opposite direction to phase 1(the negative direction) at a constant speed of $1\ \text{ms}^{-1}$, and therefore with a velocity of $-1\ \text{ms}^{-1}$.



Interpreting the features of a graph.

Key Points

The *gradient* of a velocity time graph gives the *acceleration*.
The *area underneath* (taking area below the *x*-axis to have a negative sign) a velocity time graph gives the *displacement*.

The *gradient* of a displacement-time graph gives the *velocity*
The *gradient* of a distance-time graph gives the *speed*.

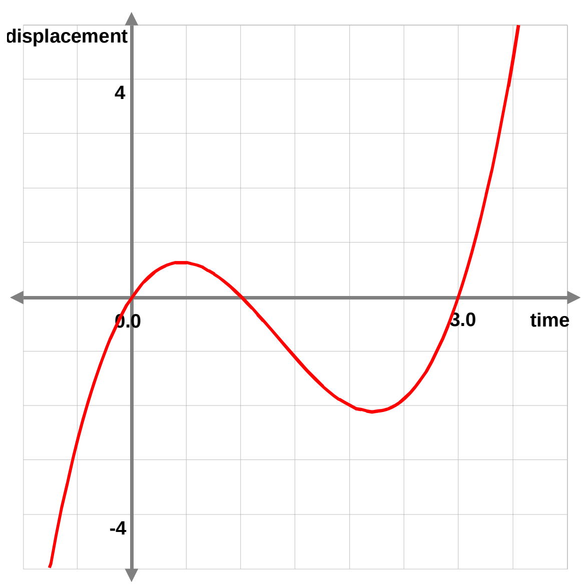
In the case of non-constant acceleration the gradients and areas can be calculated using differentiation and integration.

Here is an example involving a particle moving with a non-constant acceleration. (In fact the acceleration at time *t* is 6*t*, can you see why? Hint – differentiate the expression for the displacement twice.)
Example. *A particle moves so that its displacement, x, from a fixed origin at time t where* $0 \le t \le 3$ *is given by* $x = t(t-1)(t-3)$.

i) *Sketch a displacement-time graph of this journey.*
ii) *What is the velocity of the particle when t = 1 and when t = 2?*

Solution.

i)



ii) To calculate the velocity at the times given firstly differentiate the displacement with respect to time.
$x = t(t-1)(t-3)$ so the easiest way to do this is probably to multiply out the brackets so that there is a polynomial in *t* to differentiate.
Hence, $x = t(t-1)(t-3) = t^3 - 4t^2 + 3t$.

And so $\dfrac{dx}{dt} = 3t^2 - 8t + 3$.

When $t = 1$: $\dfrac{dx}{dt} = 3 - 8 + 3 = -2\text{ms}^{-1}$

and when $t = 2$:
$\dfrac{dx}{dt} = (3\times 4) - (8\times 2) + 3 = -1\text{ms}^{-1}$.

REVISION SHEET – MECHANICS 1

NEWTON'S LAWS APPLIED ALONG A LINE

The main ideas are

	AQA	Edx	MEI	OCR
Motion in a horizontal plane	M1	M1	M1	M1
Motion in a vertical plane	M1	M1	M1	M1
Pulleys	M1	M1	M1	M1
Connected bodies	M1	M1	M1	M1

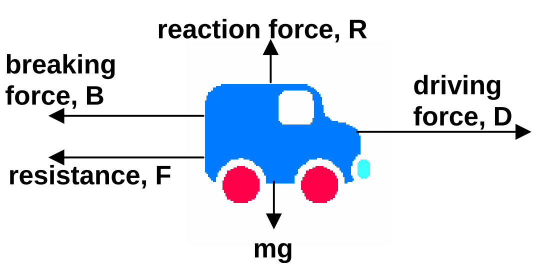
Before the exam you should know:

1. You should know Newton's second law, that the change in motion is proportional to the force, or as an equation
$$F = ma$$
where F is the resultant force, m is the mass of the object and a is its acceleration.
2. Newton's first law – that every particle continues in a state of rest or uniform motion in a straight line unless acted on by a resultant external force.
3. Newton's third law, that when one object exerts a force on another there is always a reaction, which is equal and opposite in direction, to the acting force.
4. The five SUVAT equations, what each letter stands for and when to use them. They are:
$$v = u + at, \qquad s = \left(\frac{u+v}{2}\right) \times t,$$
$$s = ut + \frac{1}{2}at^2,\ v^2 = u^2 + 2as\ ,\ s = vt - \frac{1}{2}at^2$$

Type of Problem

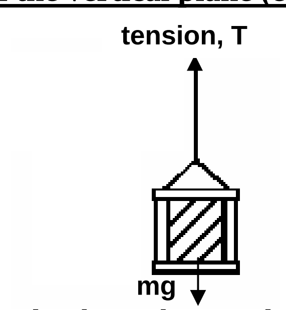
The type of problem that will be looked at on this sheet is largely concerned with objects which are moving in a horizontal or vertical straight line. Note that in many of these problems the resistive force could be taken to represent friction. However, friction is discussed in the revision sheet 'vectors and Newton's laws in 2D' (and is not required for the MEI M1 specification)

<u>Moving in the horizontal plane (e.g a car, a train, a ship)</u>



When the vehicle is moving along a horizontal road, the reaction force and the weight force (labeled *mg* in the diagram) are equal and opposite i.e. **R = mg** and as they are perpendicular to the horizontal motion they do not contribute (unless Friction is involved, which is related to R). So the resultant force acts in the horizontal direction and equals (taking right as positive): **D – B – F**

<u>Moving in the vertical plane (e.g balls thrown vertically in the air, crates hanging from ropes, lifts)</u>

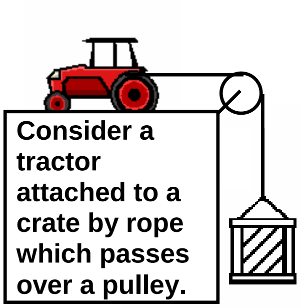


The only forces acting on the crate are in the vertical direction. Hence the resultant force acts in the vertical direction. If the tension is greater than *mg* the crate will accelerate upwards (or decelerate if the crate is already moving down). If the tension equals *mg* the crate will maintain its current velocity (or remain stationary). If the tension is less than mg the crate will accelerate downwards (or decelerate if the crate is already moving upwards).

We also need to know how to deal with situations of this type in which objects are connected. It is possible to have an object moving in a straight line in the horizontal plane connected to an object moving in a straight line in the vertical plane (via a pulley). We need to know how to deal with this too.

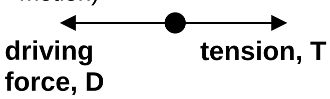
The common theme is that Newton's second law will always be used, which when written as an equation, reads: resultant force $=$ mass $\times$ acceleration .

Pulleys The tension in a rope passing over a pulley is constant throughout the length of the rope (assuming no friction in the pulley, which is always the case in Mechanics 1).

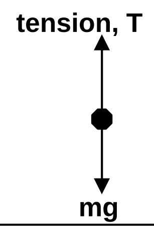


Consider a tractor attached to a crate by rope which passes over a pulley.

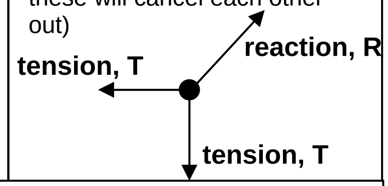
Forces acting on tractor
(ignoring vertical forces which are perpendicular to the motion and hence do not contribute to the horizontal motion)



Forces acting on crate



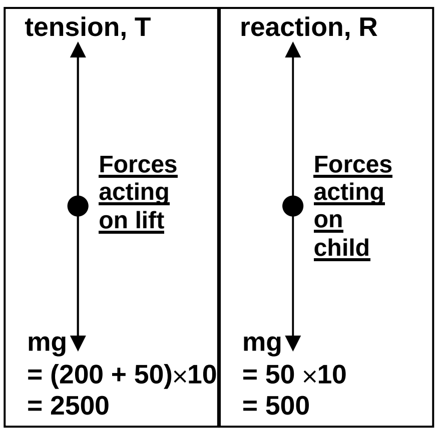
Forces acting on pulley (of negligible mass)
(assuming the pulley is fixed, these will cancel each other out)



***Key Point:* The value of T is the same in all three of the above force diagrams.**

Typical Standard Problem: A child of mass 50kg is in a lift of mass 200kg. Calculate the tension in the lift cable and the reaction force (of the lift floor on the child) when a) the lift is stationary, b) the lift going up and accelerating at $5\,\text{ms}^{-2}$, c) the lift is going down and decelerating at $5\,\text{ms}^{-2}$. (let $g = 10\,\text{ms}^{-2}$).

Solution:



(a) When the lift is stationary the acceleration is zero, therefore we have (taking up to be positive), considering the lift:

$$T - mg = T - 2500 = \text{resultant force} = ma = m \times 0 = 0 \Rightarrow T = 2500\text{N}$$

And by considering the child:

$$R - mg = R - 500 = \text{resultant force} = ma = m \times 0 = 0 \Rightarrow R = 500\text{N}$$

(b) When the lift is accelerating upwards at $5\,\text{ms}^{-2}$, we have (taking up to be positive), considering the lift:

$$T - mg = T - 2500 = \text{resultant force} = ma = 2500 \times 5 = 12500$$

$$\Rightarrow T = 12500 + 2500 = 15000\text{N}$$

And by considering the child:

$$R - mg = R - 500 = \text{resultant force} = ma = 500 \times 5 = 2500$$

$$\Rightarrow T = 500 + 250 = 750N$$

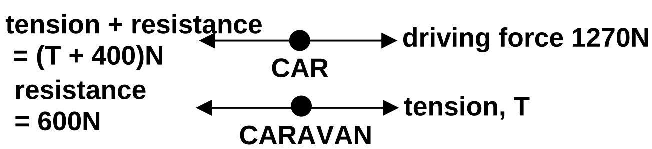
(c) When the lift is going down and decelerating at $5\,\text{ms}^{-2}$, as the acceleration is in the opposite direction to the motion, the acceleration is upwards. Thus, we have an upwards acceleration of $5\,\text{ms}^{-2}$ and so our answers are exactly as in (b).

Connected Bodies Example:

A car, mass 800kg is pulling a caravan, mass 1000kg along a straight, horizontal road. The caravan is connected to the car with a light, rigid tow bar. The car is exerting a driving force of 1270N. The resistances to forward motion are 400N on the car and 600N on the caravan. These resistances remain constant. Calculate the acceleration of the car and caravan.

Solution: There are two possible approaches:

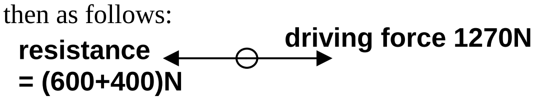
Approach 1: Consider forces on the car and the caravan separately.



Considering the car $1270 - T - 400 = 800a$, and by considering the caravan $T - 600 = 1000a$. By adding these equations together we get

$$270 = 1800a \Rightarrow a = \frac{270}{1800} = 0.15\text{ms}^{-2}$$

Approach 2: Treat the system as a whole. The forces are then as follows:



This gives

$$1270 - 1000 = 1800a$$

$$\Rightarrow a = \frac{270}{1800} = 0.15\text{ms}^{-2}$$

Notice the equation we arrive at immediately here is exactly the same as the one obtained by adding the two equations in approach 1, where the "internal" tension forces cancel. To calculate the tension in the tow bar, we could use this value of a in either of the equations in approach 1.

REVISION SHEET – MECHANICS 1

CONSTANT ACCELERATION & "SUVAT" EQUATIONS

The main ideas are

	AQA	Edx	MEI	OCR
Introduction to the variables	M1	M1	M1	M1
Using the variables	M1	M1	M1	M1

Before the exam you should know:

1. The five SUVAT equations, what each letter stands for and when to use them. They are:

$$v = u + at,$$

$$s = \left(\frac{u+v}{2}\right) \times t,$$

$$v^2 = u^2 + 2as$$

$$s = ut + \frac{1}{2}at^2,$$

$$s = vt - \frac{1}{2}at^2$$

2. You should know the units of all the quantities in the SUVAT equations.

Problems involving bodies or systems acted upon by constant forces often begin by calculating the acceleration using Newton's second law; once the acceleration has been found, they become *suvat* problems.

The variables which appear in the SUVAT equations are:

u = initial velocity
v = velocity after t seconds
a = acceleration
t = time
s = displacement (from the initial displacement) at time t.

Important things to remember:

- ***The SUVAT equations can only be used for objects moving under a constant acceleration.*** This occurs whenever a constant force is applied to a body for a period of time.
- ***If, over the course of a journey, the acceleration changes from one constant rate to another constant rate (e.g* 5ms^{-2} *for 2 seconds followed by* -3ms^{-2} *for 5 seconds) the SUVAT equations must be applied to each leg of the journey separately***. The final velocity for the first leg will be the initial velocity for the second leg.
- ***Be careful with units.*** Make sure that your units are consistent with one another. e.g. if acceleration is given in ms^{-2} it might be wise to give all displacements in m, all times in seconds and all velocities in ms^{-1}.
- ***Get your signs right***. In these problems the particle is always moving in a straight line. From the starting position you should decide which way along the line you are going to specify as the positive direction and which way the negative. From then on be consistent with your choice. Remember it is possible for a particle to have a positive velocity and a negative acceleration.
- ***Select the equation you use appropriately.*** Work out which variables you know and which variable you need to do this.

Try to have a clear picture in your mind of what is going on.

Here are some examples of this. In them we've specified right as positive and left as negative.

Example 1. If $u = -3\text{ms}^{-1}$ and $v = 6\text{ms}^{-1}$ when $t = 3$ seconds we expect the particle to have a positive acceleration

Example 2. If $u = 6\text{ms}^{-1}$ and $a = -3\text{ms}^{-1}$ the particle begins by moving to the right. Its acceleration is in the opposite direction to this movement and so it's actually a deceleration. After two seconds its velocity is zero (why?). From then on it moves to the left accelerating in that direction.

Example 3. If $u = 4\text{ms}^{-1}$ with $a = -6\text{ms}^{-2}$ then plugging s = 3m into the equation $s = ut + \frac{at^2}{2}$ gives

$3 = 4t - 3t^2$ or $3t^2 - 4t - 3 = (3t-1)(t-3) = 0$. Which means that $t = \frac{1}{3}$ or t = 3. This is because there are two occasions when the particle is 3m to the right of the starting point. The first time when the particle has positive velocity, the second time after the particle has slowed to a stop and began moving in the opposite direction due to the leftwards acceleration.

Standard Questions

1. *A car decelerates from* 24ms^{-1} *to rest in* 5s. *Assuming the deceleration is constant, calculate how for the car travels in this time.*

Solution

We know $u = 24\text{ms}^{-1}$, $v = 0\text{ms}^{-1}$, t = 5s. We want s.

The equation to use is therefore $s = \left(\frac{u+v}{2}\right) \times t$.

This gives $s = \frac{24}{2} \times 5 = 60\text{m}$.

2. *A ball is thrown vertically in the air with a speed of* 20ms^{-1}. *It accelerates (downwards) at a rate of* 9.8ms^{-2}. *How long does it take to hit the ground?*

Solution

In this we have $u = 20\text{ms}^{-1}$. (I've designated up as being positive here.) We have $a = -9.8\text{m/s}^2$. We would like to know at what time t, s = 0. It looks as though $s = ut + 0.5at^2$ is the right choice of equation. We have

$0 = 20t - 4.9t^2 = t(20 - 4.9t)$. The values of t for which this is true are t = 0 and

$t = \frac{20}{4.9} = 4.08$ to two decimal places. Clearly t = 0 is when the ball is thrown so the time we require is 4.08 seconds.

Harder Questions

In early examples using the SUVAT equations you are given three of the quantities u, v, a, t and s and are asked to calculate one of the other two. It's easy to do this by selecting the appropriate equation. A more difficult question is one where given enough information simultaneous equations involving two of the variables can be set-up. The following is an example of this:

A ball is dropped from a building and falls with acceleration 10ms^{-2}. *The distance between floors is constant. The ball takes 0.5 secs to fall from floor 8 to floor 7 and only 0.3 secs to fall from floor 7 to floor 6. What is the distance between the floors?*

Solution

The trick is to see that we have information about two journeys both starting at floor 8. One is the journey from floor 8 to floor 7 which takes 0.5s the other is twice as long, the journey from floor 8 to floor 6 which takes 0.5 + 0.3 = 0.8s If we let u be the velocity at floor 8. Then we have using $s = ut + 0.5at^2$,

Journey 1:

$s = u \times 0.5 + 0.5 \times 10 \times 0.5^2 = 0.5u + 1.25$

Journey 2:

$2s = u \times 0.8 + 0.5 \times 10 \times 0.8^2 = 0.5u + 3.2$

This gives simultaneous equations

$s - 0.5u = 1.25$ and $2s - 0.5u = 3.2$ which can be solved to discover s. (i.e. s = 1.95 m)

REVISION SHEET – MECHANICS 1

VARIABLE ACCELERATION USING DIFFERENTIATION AND INTEGRATION

The main ideas are

	AQA	Edx	MEI	OCR
Differentiation	M2	M2	M1	M1
Integration	M2	M2	M1	M1
Differentiation in 2 dimensions	M2	M2	M1	M1
Integration in 2 dimensions	M2	M2	M1	M1

There are two main ideas in this topic. It is:

- **Using differentiation and integration to obtain expressions for the displacement, velocity and acceleration from one another. You should be able to do this in one, two or three dimensions.**
- **Obtaining values of associated quantities such as speed and distance travelled.**

Before the exam you should know:

- **Velocity is the rate of change of displacement.** Therefore to obtain an expression for a particle's velocity at time t you should differentiate the expression for its displacement.
- **Acceleration is the rate of change of velocity.** Therefore to obtain an expression for a particle's acceleration at time t you should differentiate the expression for its velocity.
- Reversing the two ideas above, **a particle's velocity can be obtained by integrating the expression for its acceleration** and **a particle's displacement can be obtained by integrating the expression for its velocity**. In both cases this will introduce a constant of integration whose value can be found if the particle's displacement or velocity is known at some particular time.
- The above facts apply to both:
  1. particles travelling in one dimension. In this case each of the displacement, velocity and acceleration is a (scalar valued) function of time, all of which can be differentiated and integrated in the usual way.
  2. particles travelling in two and three dimensions, when the displacement, velocity and acceleration are all vectors with components dependent on t (time). We differentiate and integrate such expressions in the usual way, dealing with each component separately. There are several examples of this on this sheet.
- You should be comfortable with both column vector and **i**, **j**, **k** notation for vectors.

Using differentiation - a particle travelling in a straight line.

Example*: An object moves in a straight line, so that its displacement relative to some fixed origin at time t is given by* $s = t^3 - 5t^2 + 4$.

1. *Find expressions for its velocity and acceleration at time t.*
2. *Calculate the velocity and acceleration of the object when t = 0 and when t = 1.*
3. *What is the displacement of the object when its velocity is zero?*

Solution.

1. $s = t^3 - 5t^2 + 4$ so that $v = \frac{ds}{dt} = 3t^2 - 10t$ and $a = \frac{dv}{dt} = 6t - 10$.
2. When $t = 0, v = 3\times 0 - 10\times 0 = 0\text{ms}^{-1}$ and $a = 6\times 0 - 10 = -10\text{ms}^{-2}$ and when $t = 1, v = 3\times 1 - 10\times 1 = -7\text{ms}^{-1}$ and $a = 6\times 1 - 10 = -4\text{ms}^{-2}$.
3. The velocity of the object at time t is $3t^2 - 10t = t(3t - 10)$. This is zero when t = 0 or when $t = \frac{10}{3}$. The displacement of the particle when t = 0 is $s = 4\text{m}$ and then displacement of the particle when $t = \frac{10}{3}$ is

Using integration – a particle travelling in a straight line.

Example*: An object is moving in a straight line with acceleration at time t given by* $a = 10 - 6t$.
Given that when t = 1, s = 0 and v = –5, where s is the object's displacement and v is the object's velocity, find an expression for v and s in terms of t.
Hence find out the displacement of the particle when it first comes to rest.

Solution

$v = \int a \ \mathrm{d}t = \int (10-6t)\mathrm{d}t = 10t - 3t^2 + c$. But when t = 1, $v = 5 = 10 - 3 + c \Rightarrow c = -2$. So $v = 10t - 3t^2 - 2$.

$s = \int v \ \mathrm{d}t = \int (10t - 3t^2 - 2)\ \mathrm{d}t = 5t^2 - t^3 - 2t + c$. But when t = 1, $s = 0 = 5 - 1 - 2 + c \Rightarrow c = -2$. So $s = 5t^2 - t^3 - 2t - 2$

Using differentiation – an example in two dimensions using column vector notation.

Example*: A girl throws a ball and, t seconds after she releases it, its position in metres relative to the point where she is standing is modelled by*

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 15t \\ 2 + 16t - 5t^2 \end{pmatrix}$$

where the directions are horizontal and vertical.

1. *Find expressions for the velocity and acceleration of the ball at time t.*
2. *The vertical component of the velocity is zero when the ball is at its highest point. Find the time taken for the ball to reach this point.*
3. *What is the speed of the ball when it hits the ground.*

Solution

1. The velocity is obtained by differentiating (with respect to t) the components in the vector giving the ball's position. This gives $v = \begin{pmatrix} 15 \\ 16 - 10t \end{pmatrix}$. The acceleration is obtained by differentiating (with respect to t) the components in the vector giving the ball's velocity. This gives $a = \begin{pmatrix} 0 \\ -10 \end{pmatrix}$.
2. The vertical component of the velocity is 16 – 10t. This is zero when $t = \frac{16}{10} = \frac{8}{5} = 1.6$ seconds.
3. The ball hits the ground when the vertical component of the balls position is zero. In other words when $2 + 16t - 5t^2 = 0$. Rearranging this as $5t^2 - 16t - 2 = 0$ and then using the formula for the solutions of a quadratic we see that the solutions of this are $t = -0.12$ and $t = 3.3$ (to 2 s.f). Clearly the value we require is t = 3.3. The velocity of the ball when t = 3.3 is $\begin{pmatrix} 15 \\ -17 \end{pmatrix}$ and so the speed is $\sqrt{15^2 + (-17)^2} = 22.67\ \mathrm{ms}^{-1}$.

Using Integration and Newtons 2nd Law an example in 2 dimensions with i, j notation.

Example*: A particle of mass 0.5 kg is acted on by a force, in Newtons, of* $\mathbf{F} = t^2\mathbf{i} + 2t\mathbf{j}$. *The particle is initially at rest and t is measured in seconds.*

1. *Find the acceleration of the particle at time t.*
2. *Find the velocity of the particle at time t.*

Solution

Newton's second law, $\mathbf{F} = m\mathbf{a}$ gives that $\mathbf{F} = t^2\mathbf{i} + 2t\mathbf{j} = 0.5\mathbf{a}$ so that $\mathbf{a} = 2t^2\mathbf{i} + 4t\mathbf{j}$.

We have that $\mathbf{v} = \int \mathbf{a}\ \mathrm{d}t = \left(\frac{2t^3}{3} + c\right)\mathbf{i} + \left(2t^2 + d\right)\mathbf{j}$ where c and d are the so-called "constants of integration".

We are told that the particle is at rest when t = 0 and so $c = d = 0$. This gives $\mathbf{v} = \frac{2t^3}{3}\mathbf{i} + 2t^2\mathbf{j}$.

REVISION SHEET – MECHANICS 1

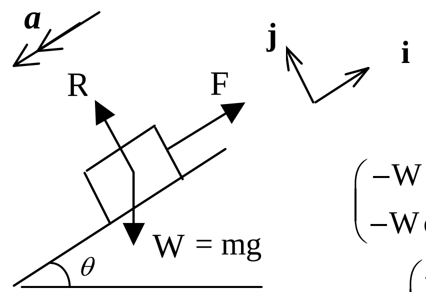
VECTORS AND NEWTON'S LAWS IN 2 DIMENSIONS

The main ideas are

	AQA	Edx	MEI	OCR
Resolving forces into components	M1	M1	M1	M1
Motion on a slope	M1	M1	M1	M1
Motion on a slope (including friction)	M1	M1	M2	M1

Before the exam you should know:

1. You must be confident with the use of vectors.
2. You must be able to draw clear diagrams showing forces.
3. You must be confident at resolving forces into components, both horizontally and vertically, or parallel and perpendicular to a slope.



$$\begin{pmatrix} -W\sin\theta \\ -W\cos\theta \end{pmatrix} + \begin{pmatrix} F \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ R \end{pmatrix} = m\begin{pmatrix} -a \\ 0 \end{pmatrix}$$

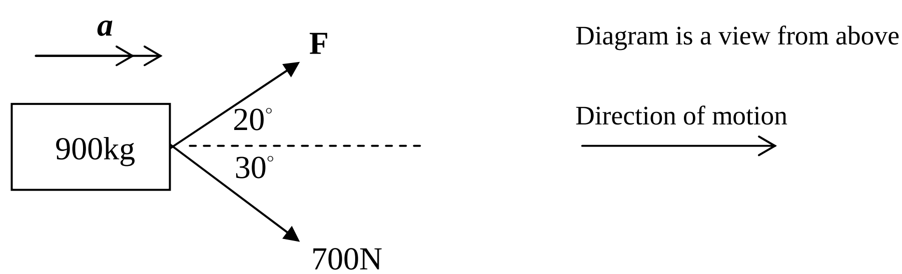
4. You must be confident at using Newton's second law, **F** = m**a**

Example

Two people pull on separate light inextensible ropes, which are attached to the front of a stationary car of mass 900kg. The first pulls the rope at an angle of 30° to the right of the direction of motion and the tension in the rope is 700N. The second pulls at an angle of 20° to the left of the line of motion.

(a) With what force is the second person pushing?
(b) What is the acceleration of the car?
(c) Assuming the forces remain constant, how fast will the car be moving after 3 seconds?

Solution Begin with a clear diagram:



(a) There can be no force perpendicular to the direction of motion so, resolving perpendicular to the direction of motion: $700\sin 30^\circ - F\sin 20^\circ = 0 \Rightarrow F = 1020$N (3s.f.)

(b) Resolving in the direction of motion:
From Newton's second law, $700\cos 30^\circ + 1023\cos 20^\circ = 900a \Rightarrow a = 1.74\text{ms}^{-2}$

(c) $u = 0,\ a = 1.742,\ t = 3,\ v = ?$
$v = u + at \Rightarrow v = 5.23\text{ms}^{-1}$

The previous example was solved by resolving parallel and perpendicular to the direction of motion separately. In the next example vector equations are used. Both methods are equally good. You should be happy with both.

Example

A block of mass 10kg is at rest on a plane which is inclined at 20° to the horizontal. A light, inelastic string is attached to the block, passes over a smooth pulley and supports a mass m which is hanging freely. The part of the string between the block and the pulley is parallel to the line of greatest slope of the plane. A friction force of 20N opposes the motion of the block.

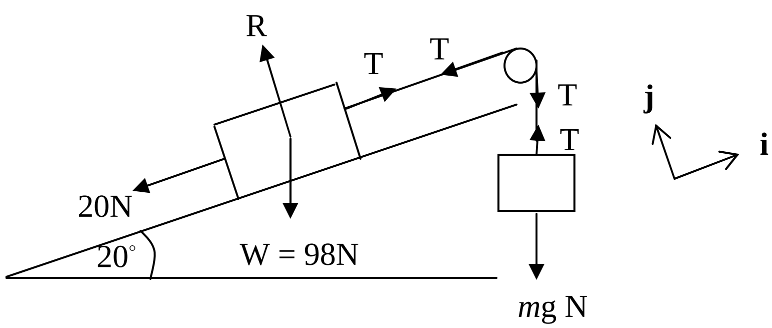
(a) Draw a diagram and mark on all the forces on the block and the hanging mass, including the tension in the string.

(b) Calculate the value of *m* when the block slides up the plane at a constant speed and find the tension in the string and the normal reaction between the block and the plane.

(c) Calculate the acceleration of the system when *m* = 6 kg and find the tension in the string in this case.

Solution

(a)



(b) When the block is moving with constant velocity, the forces on the block must be in equilibrium. Using Newton's second law:

$$\begin{pmatrix}-20\\0\end{pmatrix}+\begin{pmatrix}0\\R\end{pmatrix}+\begin{pmatrix}-98\sin 20^\circ\\-98\cos 20^\circ\end{pmatrix}+\begin{pmatrix}T\\0\end{pmatrix}=10\begin{pmatrix}0\\0\end{pmatrix}\quad\Rightarrow$$

T = 53.5 N (3 s.f.) (from the **i** component)
R = 92.1 N (3 s.f.) (from the **j** component)

Considering the forces on the hanging mass, $T = mg \Rightarrow m = \frac{53.52}{9.8} = 5.46$ N

(c) As the block and the mass are connected bodies, they both experience the same acceleration. Let this acceleration be *a*.
Using Newton's second law:
Considering the 10 kg block, resolving parallel to the slope: $T - 20 - 98\sin 20^\circ = 10a$ [1]
Considering the 6 kg hanging mass: $6g - T = 6a$ [2]

Solving [1] and [2] simultaneously gives: $a = 0.330\ \text{ms}^{-2}$ (3 s.f.), $T = 56.8$ N (3 s.f.)