1 Fig. 7 shows the trajectory of an object which is projected from a point O on horizontal ground. Its initial velocity is $40\,\text{m}\,\text{s}^{-1}$ at an angle of α to the horizontal.

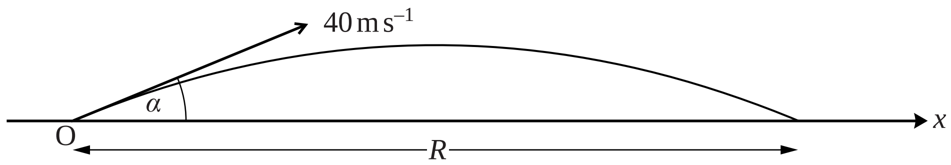

Fig. 7

(i) Show that, according to the standard projectile model in which air resistance is neglected, the flight time, T s, and the range, R m, are given by

$$T = \frac{80\sin\alpha}{g} \quad \text{and} \quad R = \frac{3200\sin\alpha\cos\alpha}{g}.$$ **[6]**

A company is designing a new type of ball and wants to model its flight.

(ii) Initially the company uses the standard projectile model.

Use this model to show that when $\alpha = 30°$ and the initial speed is $40\,\text{m}\,\text{s}^{-1}$, T is approximately 4.08 and R is approximately 141.4.

Find the values of T and R when $\alpha = 45°$. **[3]**

The company tests the ball using a machine that projects it from ground level across horizontal ground. The speed of projection is set at $40\,\text{m}\,\text{s}^{-1}$.

When the angle of projection is set at 30°, the range is found to be 125 m.

(iii) Comment briefly on the accuracy of the standard projectile model in this situation. **[1]**

The company refines the model by assuming that the ball has a constant deceleration of $2\,\text{m}\,\text{s}^{-2}$ in the horizontal direction.

In this new model, the resistance to the vertical motion is still neglected and so the flight time is still 4.08 s when the angle of projection is 30°.

(iv) Using the new model, with $\alpha = 30°$, show that the horizontal displacement from the point of projection, x m at time t s, is given by

$$x = 40t\cos 30° - t^2.$$

Find the range and hence show that this new model is reasonably accurate in this case. **[4]**

The company then sets the angle of projection to 45° while retaining a projection speed of $40\,\text{m}\,\text{s}^{-1}$. With this setting the range of the ball is found to be 135 m.

(v) Investigate whether the new model is also accurate for this angle of projection. **[3]**

(vi) Make one suggestion as to how the model could be further refined. **[1]**

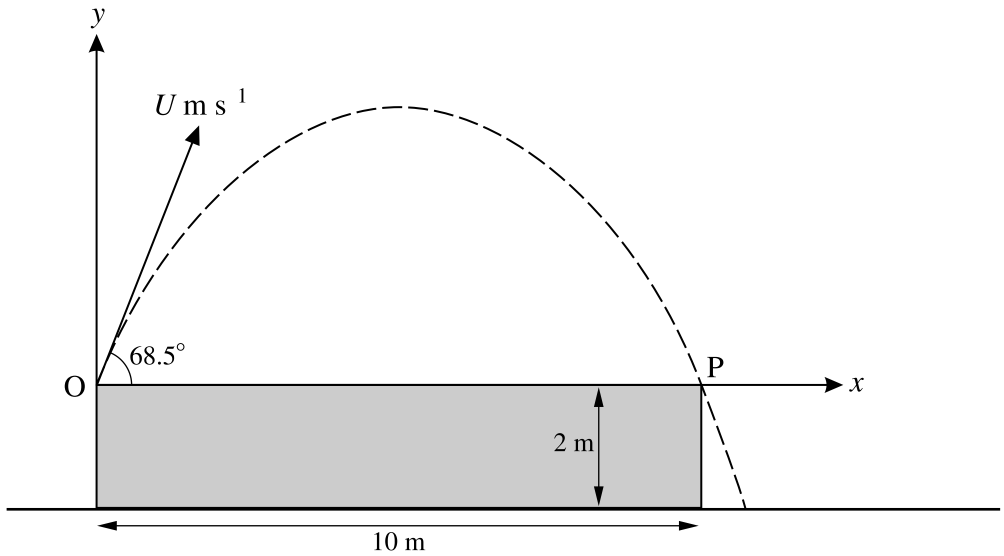

Fig. 7

Fig. 7 shows a platform 10 m long and 2 m high standing on horizontal ground. A small ball projected from the surface of the platform at one end, O, just misses the other end, P. The ball is projected at 68.5° to the horizontal with a speed of $U\,\text{m s}^{-1}$. Air resistance may be neglected.

At time t seconds after projection, the horizontal and vertical displacements of the ball from O are x m and y m.

(i) Obtain expressions, in terms of U and t, for

(A) x,

(B) y. **[3]**

(ii) The ball takes T s to travel from O to P.

Show that $T = \dfrac{U \sin 68.5°}{4.9}$ and write down a second equation connecting U and T. **[4]**

(iii) Hence show that $U = 12.0$ (correct to three significant figures). **[3]**

(iv) Calculate the horizontal distance of the ball from the platform when the ball lands on the ground. **[5]**

(v) Use the expressions you found in part **(i)** to show that the cartesian equation of the trajectory of the ball in terms of U is

$$y = x \tan 68.5° - \frac{4.9x^2}{U^2(\cos 68.5°)^2}.$$

Use this equation to show again that $U = 12.0$ (correct to three significant figures). **[4]**

3

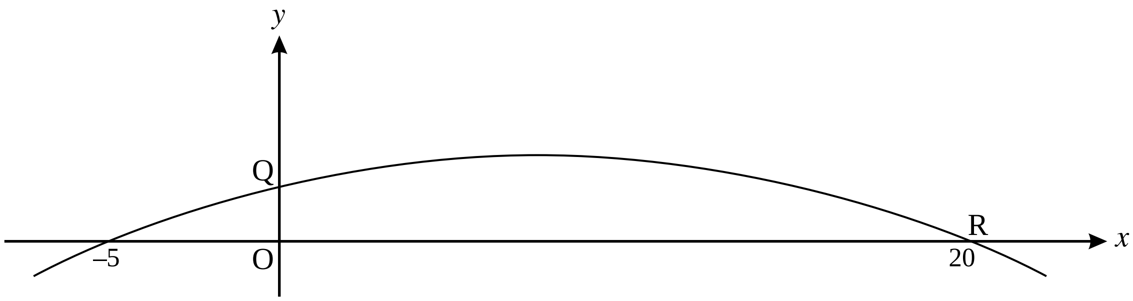

Fig. 7

Fig. 7 shows the graph of $y = \frac{1}{100}(100 + 15x - x^2)$.

For $0 \leqslant x \leqslant 20$, this graph shows the trajectory of a small stone projected from the point Q where y m is the height of the stone above horizontal ground and x m is the horizontal displacement of the stone from O. The stone hits the ground at the point R.

(i) Write down the height of Q above the ground. **[1]**

(ii) Find the horizontal distance from O of the highest point of the trajectory and show that this point is 1.5625 m above the ground. **[5]**

(iii) Show that the time taken for the stone to fall from its highest point to the ground is 0.565 seconds, correct to 3 significant figures. **[3]**

(iv) Show that the horizontal component of the velocity of the stone is 22.1 m s^{-1}, correct to 3 significant figures. Deduce the time of flight from Q to R. **[5]**

(v) Calculate the speed at which the stone hits the ground. **[4]**

4 Sandy is throwing a stone at a plum tree. The stone is thrown from a point O at a speed of 35 m s^{-1} at an angle of α to the horizontal, where $\cos\alpha = 0.96$. You are *given* that, t seconds after being thrown, the stone is $(9.8t - 4.9t^2)$ m higher than O.

When descending, the stone hits a plum which is 3.675 m higher than O. Air resistance should be neglected.

Calculate the horizontal distance of the plum from O. **[6]**

5 Small stones A and B are initially in the positions shown in Fig. 6 with B a height H m directly above A.

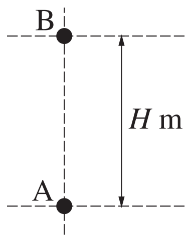

Fig. 6

At the instant when B is released from rest, A is projected vertically upwards with a speed of 29.4 m s^{-1}. Air resistance may be neglected.

The stones collide T seconds after they begin to move. At this instant they have the same speed, $V\text{ m s}^{-1}$, and A is still rising.

By considering when the speed of A upwards is the same as the speed of B downwards, or otherwise, show that $T = 1.5$ and find the values of V and H. **[7]**