1	(i)		Vertical motion: initial speed $40\sin\alpha$ $h = (40\sin\alpha)t - \frac{1}{2}gt^2$	B1		
			$h = 0 \Rightarrow t = 0$ or $\dfrac{2\times 40\times\sin\alpha}{g}$	M1	Correct expression for *h* must be seen. Condone omission of the case $t = 0$	
			$\Rightarrow T = \dfrac{80\sin\alpha}{g}$	E1	Perfect argument (but still condone omission of $t = 0$)	
			Alternative Vertical motion: initial speed $40\sin\alpha$ $v = 40\sin\alpha - gt$	(B1)		
			When $v = 0$, $t = \dfrac{T}{2}$	(M1)	Correct expression for *v* must be seen	
			$\Rightarrow T = \dfrac{80\sin\alpha}{g}$	(E1)	Perfect argument	
			Horizontal motion: initial speed $40\cos\alpha$	B1		
			$R = 40\cos\alpha \times T$	M1	There must be evidence of intention to use *T*	
			$\Rightarrow R = \dfrac{3200\sin\alpha\cos\alpha}{g}$	E1	Perfect argument	
				[6]		
	(ii)		$\alpha = 30°$: $T = \dfrac{80\sin 30°}{9.8} \simeq 4.08$ $\Rightarrow R = \dfrac{3200\times\sin 30°\times\cos 30°}{9.8} = 141.4$	B1	Both answers required for the mark. Evidence of substitution required	
			$\alpha = 45°$: $T = 5.77$	B1		

			$\alpha = 45°$: $R = 163.3$	B1 **[3]**	Accept 3 significant figures	
	(iii)		The standard model is not accurate; 125 is much less than 141.4	B1 **[1]**	The comment must be based on the figures given in the question	
	(iv)		Horizontal motion: $s = ut + \frac{1}{2}at^2$ $x = 40\cos 30° \times t - \frac{1}{2} \times 2 \times t^2$ $x = 40t\cos 30° - t^2$ Flight time = 4.08 s $R = 40 \times \cos 30° \times 4.08 - \frac{1}{2} \times 2 \times 4.08^2$ $R = 124.7$ This is close to the experimental result of 125 m	M1 A1 M1 E1 **[4]**	Use of correct formula A comparison with 125 m is required	

	(v)		When $\alpha = 45°$, $T = 5.77$ $R = 40 \times \cos 45° \times 5.77 - \frac{1}{2} \times 2 \times 5.77^2$ $R = 129.9$ 129.9 m is not very close to 135 m so the model is not very accurate for this angle.	M1 A1 B1 **[3]**	Use of correct formula, with substitution for α and T. FT their T from (ii) but not 4 SC1 for substituting for T but using 30^o for α Comparison of their 129.9 with 135 If 4.08 used for T and answer 98.8 obtained for R allow FT for this mark Allow argument that to get to 135m takes 6.07 s which is greater than 5.77 s	
	(vi)		Allow for resistance in the vertical direction as well	B1 **[1]**	Any sensible comment, but do not award a mark for "Allow for air resistance" without mention of the vertical direction.	

2		mark	notes
(i) (A)	$x = Ut\cos 68.5°$	B1 1	
(i) (B)	$y = Ut\sin 68.5° - 4.9\times t^2$	M1 A1 2	Allow 'u' = U. Allow $s \leftrightarrow c$. Allow g as g, ±9.8, ±9.81, ±10. Allow +2. Accept not 'shown'. Do not allow +2. Allow e.g + 0.5 × (- 9.8) × t^2 instead of – 4.9t^2. Accept g not evaluated
(ii)	**either** At D, $y = 0$ so $U\sin 68.5°T - 4.9\times T^2 = 0$ $\Rightarrow T\left(U\sin 68.5° - 4.9T\right) = 0$ so T = 0 (at C) or $T = \dfrac{U\sin 68.5°}{4.9}$ (at D) **or** Use (i)(A) and put x = 10 with $t = T$ to get $UT\cos 68.5° = 10$	 M1 M1 E1 M1 M1 E1 B1 4	 Equating correct y to 0 or **their** y to correct value. Attempting to factorise (or solve). Allow ÷ T without comment. Properly shown. Accept no ref to T = 0. Accept T = 0 given as well without comment. Find time to top Double time to the top
(iii)	Eliminating T from the results in (ii) gives $U\cos 68.5°\times\dfrac{U\sin 68.5°}{4.9} = 10$ so U = 11.98729… so 12.0 (3 s. f.)	 M1 M1 E1 3	 Substituting, using correct expressions or **their** expressions from (ii). Attempt to solve for U^2 or U. Some evidence seen. e.g. $142.8025.. < U^2 < 145.2025...$ with clear statement, or 11.9… seen with clear statement or 11.98… seen. Accept 11.98… seen for full marks.
(iv)	**continued**		

(iv)	Require $Ut\sin 68.5° - 4.9t^2 = -2$ Solving $4.9t^2 - Ut\sin 68.5° - 2 = 0$ $t = -\ 0.1670594541\ldots\ ,\ 2.4431591\ldots$ (Using 12: $-\ 0.1669052502..\ ,\ 2.445478886..$) Require $U\cos 68.5° \times 2.44\ldots. -\ 10$ $= 0.7336\ldots.$ so 0.734 m (3 s. f.) (Using 12 consistently, 0.7552… so 0.755 (3 s. f.)) 5	M1 M1 A1 M1 A1	Equating correct y to $-\ 2$ or **their** y to correct value. Allow use of U, 11.987… or 12. Allow implicit '= 0' Dep on 1st M1. Attempt to solve a 3 term quadratic to find at least the +ve root. Allow if two correct roots seen WW. Accept only + ve root given Alternative method of e.g. finding time to highest point and then time to the ground. M1 all times attempted, at least one by a sound method. M1 both methods sound and complete. A1. Dep on first M1. Allow **their** expression for x. Allow '$-\ 10$' omitted. cao. Accept $0.73 \le x \le 0.76$
(v)	Eliminate t from (i) (B) using $t = \dfrac{x}{U\cos 68.5°}$ from (i)(A) so $y = x\tan 68.5° - \dfrac{4.9x^2}{U^2(\cos 68.5°)^2}$ We require $y = 0$ when $x = 10$ so $U = 11.98729\ldots$ so 12.0 (3 s. f.) 4	 M1 E1 M1 E1	 May be implied. FT their (i). Clearly shown. Must see attempt to solve. Or use $x = 10.73\ldots$ when $y = -\ 2$. Must see evidence of fresh calculation or statement that they have now got the same expression for evaluation.
		19	

3 (i)	$y(0) = 1$	B1		1
	Either			
(ii)	$\frac{1}{2}(20+5)-5=7.5$	M1	Use of symmetry e.g. use of $\frac{1}{2}(20+5)$	
		A1	12.5 o.e. seen	
		A1	7.5 cao	
	or	**M1**	Att pt at y' and to solve $y' = 0$	
		A1	$k(15 - 2x)$ where $k = 1$ or $\frac{1}{100}$	
		A1	7.5 cao, seen as final answer	
	$y(7.5)=\frac{1}{100}(100+15\times7.5-7.5^2)$	M1	FT **their** 7.5	
	$= \frac{25}{16}$ (1.5625) so 1.5625 m	E1	A	
			[SC2 only showing 1.5625 leads to $x = 7.5$]	5

(iii)	$4.9t^2 = \frac{25}{16}$ (1.5625)	M1	Use of $s = ut + 0.5at^2$ with $u = 0$. Condone use of $\pm 10,\ \pm 9.8,\ \pm 9.81$. If sequence of *suvat* used, complete method required.	
		A1	In any method only error accepted is sign error	
	$t^2 = 0.31887\ldots$ so $t = \pm 0.56469\ldots$ Hence 0.565 s (3 s. f.)	E1	AG. Condone no reference to –ve value. www. 0.565 must be justified as answer to 3 s. f.	3
(iv)	$\dot{x} = \frac{12.5}{0.56469...} = 22.1359...$	M1	or 25 / (2×0.56469..)	
		B1	Use of 12.5 or equivalent	
	so 22.1 m s $^{-1}$ (3 s. f.))	E1	22.1 must be justified as answer to 3 s. f. Don't penalise if penalty already given in (iii).	
	Either			
	Time is $\frac{20}{12.5} \times 0.56469...$ s	M1		
	so 0.904 s (3 s. f.)	A1	cao Accept 0.91 (2 s. f.)	
	or			
	Time is $\frac{20}{22.1359...}$ s	M1		
	= 0.903507… so 0.904 s (3 s. f.)	A1	cao Accept 0.91 (2 s. f.)	
	or			
	(iii) $+ \frac{7.5}{\textbf{their}\ \dot{x}}$	M1		
	so 0.904 s (3 s. f.)	A1	cao Accept 0.91 (2 s. f.)	5
(v)	$v = \sqrt{\dot{x}^2 + \dot{y}^2}$	M1	Must have attempts at both components	
	$\dot{y}^2 = 0^2 + 2 \times 9.8 \times \frac{25}{16}$ or $\dot{y} = 0 + 9.8 \times 0.5646...$	M1	Or equiv. $u = 0$. Condone use of $\pm 10,\ \pm 9.8,\ \pm 9.81$.	
			Accept wrong *s* (or *t* in alternative method)	
	= $\frac{245}{8}$ (30.625) $\quad \dot{y} = \pm 5.539...$	A1	Or equivalent. May be implied. Could come from (iii) $v^2 = u^2 + 2as$ used there. Award marks again.	
	so $v = \sqrt{490 + 30.625}$ = 22.8172… m s $^{-1}$ so 22.8 m s $^{-1}$ (3 s. f.)	A1	cao. www	4
				18

4		Mark	Comment	Sub
	either			
	We need $3.675 = 9.8t - 4.9t^2$	*M1	Equating given expression or **their** attempt at y to ± 3.675 . If **they** attempt y, allow sign errors, g = 9.81 etc. and u = 35.	
	Solving $4t^2 - 8t + 3 = 0$	M1*	Dependent. Any method of solution of a 3 term quadratic.	
	gives t = 0.5 or t = 1.5	A1	cao. Accept only the larger root given	
		F1	Both roots shown and larger chosen provided both +ve. Dependent on 1st M1. [Award M1 M1 A1 for 1.5 seen WW]	
	or			
		M1	Complete method for total time from motion in separate parts. Allow sign errors, g = 9.81 etc. Allow u = 35 initially only.	
	Time to greatest height			
	$0 = 35 \times 0.28 - 9.8t$ so t = 1	A1	Time for 1st part	
	Time to drop is 0.5	A1	Time for 2nd part	
	total is 1.5 s	A1	cao	
	then			
	Horiz distance is $35 \times 0.96t$	B1	Use of $x = u\cos\alpha t$. May be implied.	
	So distance is $35 \times 0.96 \times 1.5 = 50.4$ m	F1	FT **their** quoted t provided it is positive.	
				6
		6		

5		Mark	Comment	Sub
	Method 1			
	$\uparrow\ v_A = 29.4 - 9.8T \qquad \downarrow\ v_B = 9.8T$	M1	Either attempted. Allow sign errors and g = 9.81 etc	
		A1	Both correct	
	For same speed $29.4 - 9.8T = 9.8T$	M1	Attempt to equate. Accept sign errors and T = 1.5 substituted in both.	
	so T = 1.5	E1	If 2 subs there must be a statement about equality	
	and V = 14.7	F1	FT T or V, whichever is found second	
	$H = 29.4\times1.5 - 0.5\times9.8\times1.5^2$	M1	Sum of the distance travelled by each attempted	
	$+\ 0.\ \times9.8\times1.5^2$			
	$= 44.1$	A1	cao	
	Method 2			
	$V^2 = 29.4^2 - 2\times9.8\times x = 2\times9.8\times(H - x)$	M1	Attempts at V^2 for each particle equated. Allow sign errors, 9.81 etc Allow h_1, h_2 without $h_1 = H - h_2$	
		B1	Both correct. Require $h_1 = H - h_2$ but not an equation.	
	$29.4^2 = 19.6H$ so H = 44.1	A1	cao	
	Relative velocity is 29.4 so	M1	Any method that leads to T or V	
	$T = \frac{44.1}{29.4}$	E1		
	Using $v = u + at$	M1	Any method leading to the other variable	
	$V = 0 + 9.8\times1.5\ = 14.7$	F1		
			Other approaches possible. If 'clever' ways seen, reward according to weighting above.	7
		7		