

- 1** The map of a large area of open land is marked in 1 km squares and a point near the middle of the area is defined to be the origin. The vectors $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$ are in the directions east and north.

At time t hours the position vectors of two hikers, Ashok and Kumar, are given by:

$$\text{Ashok} \quad \mathbf{r}_A = \begin{pmatrix} -2 \\ 0 \end{pmatrix} + \begin{pmatrix} 8 \\ 1 \end{pmatrix} t,$$

$$\text{Kumar} \quad \mathbf{r}_K = \begin{pmatrix} 7t \\ 10 - 4t \end{pmatrix}.$$

(i) Prove that the two hikers meet and give the coordinates of the point where this happens. **[4]**

(ii) Compare the speeds of the two hikers. **[3]**

- 2 A box of emergency supplies is dropped to victims of a natural disaster from a stationary helicopter at a height of 1000 metres. The initial velocity of the box is zero.

At time t s after being dropped, the acceleration, $a \text{ m s}^{-2}$, of the box in the vertically downwards direction is modelled by

$$a = 10 - t \quad \text{for } 0 \leq t \leq 10,$$

$$a = 0 \quad \text{for } t > 10.$$

- (i) Find an expression for the velocity, $v \text{ m s}^{-1}$, of the box in the vertically downwards direction in terms of t for $0 \leq t \leq 10$.

Show that for $t > 10$, $v = 50$. [4]

- (ii) Draw a sketch graph of v against t for $0 \leq t \leq 20$. [3]

- (iii) Show that the height, h m, of the box above the ground at time t s is given, for $0 \leq t \leq 10$, by

$$h = 1000 - 5t^2 + \frac{1}{6}t^3.$$

Find the height of the box when $t = 10$. [4]

- (iv) Find the value of t when the box hits the ground. [2]

- (v) Some of the supplies in the box are damaged when the box hits the ground. So measures are considered to reduce the speed with which the box hits the ground the next time one is dropped. Two different proposals are made. Carry out suitable calculations and then comment on each of them.

(A) The box should be dropped from a height of 500 m instead of 1000 m. [2]

(B) The box should be fitted with a parachute so that its acceleration is given by

$$a = 10 - 2t \quad \text{for } 0 \leq t \leq 5,$$

$$a = 0 \quad \text{for } t > 5. \quad \text{[3]}$$

- 3 In this question the origin is a point on the ground. The directions of the unit vectors $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$, $\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ are east, north and vertically upwards.



Alesha does a sky-dive on a day when there is no wind. The dive starts when she steps out of a moving helicopter. The dive ends when she lands gently on the ground.

- During the dive Alesha can reduce the magnitude of her acceleration in the vertical direction by spreading her arms and increasing air resistance.
- During the dive she can use a power unit strapped to her back to give herself an acceleration in a horizontal direction.
- Alesha's mass, including her equipment, is 100 kg.
- Initially, her position vector is $\begin{pmatrix} -75 \\ 90 \\ 750 \end{pmatrix}$ m and her velocity is $\begin{pmatrix} -5 \\ 0 \\ -10 \end{pmatrix}$ m s⁻¹.

- (i) Calculate Alesha's initial speed, and the initial angle between her motion and the downward vertical. [4]

At a certain time during the dive, forces of $\begin{pmatrix} 0 \\ 0 \\ -980 \end{pmatrix}$ N, $\begin{pmatrix} 0 \\ 0 \\ 880 \end{pmatrix}$ N and $\begin{pmatrix} 50 \\ -20 \\ 0 \end{pmatrix}$ N are acting on Alesha.

- (ii) Suggest how these forces could arise. [3]
- (iii) Find Alesha's acceleration at this time, giving your answer in vector form, and show that, correct to 3 significant figures, its magnitude is 1.14 m s⁻². [3]

One suggested model for Alesha's motion is that the forces on her are constant throughout the dive from when she leaves the helicopter until she reaches the ground.

- (iv) Find expressions for her velocity and position vector at time t seconds after the start of the dive according to this model. Verify that when $t = 30$ she is at the origin. [6]
- (v) Explain why consideration of Alesha's landing velocity shows this model to be unrealistic. [2]

4 A particle moves along a straight line through an origin O. Initially the particle is at O.

At time t s, its displacement from O is x m and its velocity, v m s^{-1} , is given by

$$v = 24 - 18t + 3t^2.$$

(i) Find the times, T_1 s and T_2 s (where $T_2 > T_1$), at which the particle is stationary. [2]

(ii) Find an expression for x at time t s.

Show that when $t = T_1$, $x = 20$ and find the value of x when $t = T_2$. [4]

5 In this question, positions are given relative to a fixed origin, O. The x -direction is east and the y -direction north; distances are measured in kilometres.

Two boats, the *Rosemary* and the *Sage*, are having a race between two points A and B.

The position vector of the *Rosemary* at time t hours after the start is given by

$$\mathbf{r} = \begin{pmatrix} 3 \\ 2 \end{pmatrix} + \begin{pmatrix} 6 \\ 8 \end{pmatrix} t, \text{ where } 0 \leq t \leq 2.$$

The *Rosemary* is at point A when $t = 0$, and at point B when $t = 2$.

(i) Find the distance AB. [3]

(ii) Show that the *Rosemary* travels at constant velocity. [1]

The position vector of the *Sage* is given by

$$\mathbf{r} = \begin{pmatrix} 3(2t + 1) \\ 2(2t^2 + 1) \end{pmatrix}.$$

(iii) Plot the points A and B.

Draw the paths of the two boats for $0 \leq t \leq 2$. [3]

(iv) What can you say about the result of the race? [1]

(v) Find the speed of the *Sage* when $t = 2$. Find also the direction in which it is travelling, giving your answer as a compass bearing, to the nearest degree. [6]

(vi) Find the displacement of the *Rosemary* from the *Sage* at time t and hence calculate the greatest distance between the boats during the race. [4]