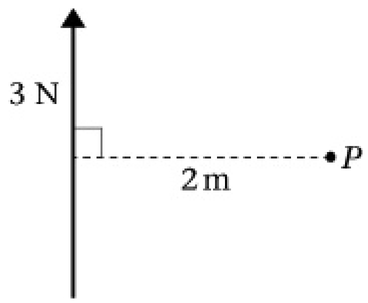
Solutionbank M1
Edexcel AS and A Level Modular Mathematics

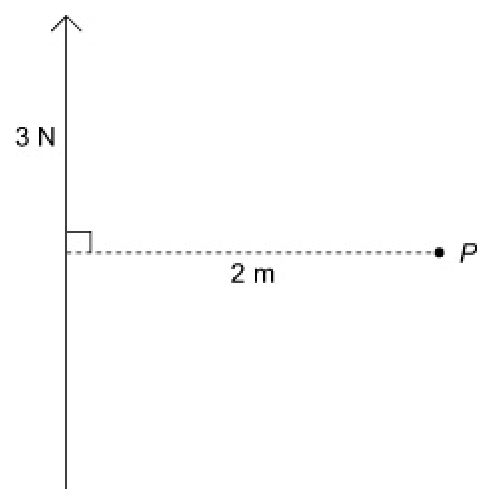
Moments
Exercise A, Question 1

Question:

Calculate the moment about P of each of these forces acting on a lamina.



Solution:



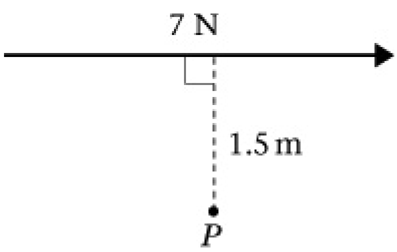
Moment $= 3 \times 2 = 6$ Nm clockwise

Solutionbank M1
Edexcel AS and A Level Modular Mathematics

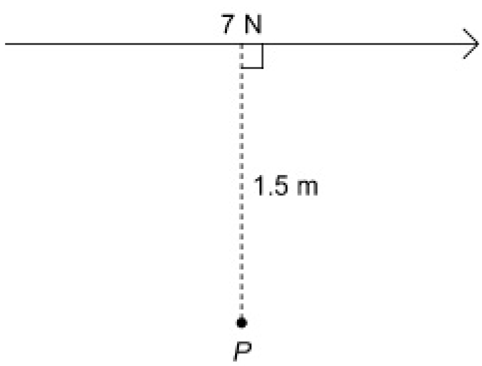
Moments
Exercise A, Question 2

Question:

Calculate the moment about P of each of these forces acting on a lamina.



Solution:



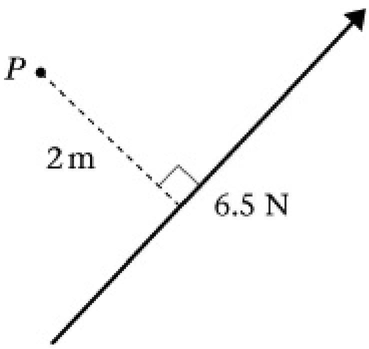
Moment $= 7 \times 1.5 = 10.5$ Nm clockwise

Solutionbank M1
Edexcel AS and A Level Modular Mathematics

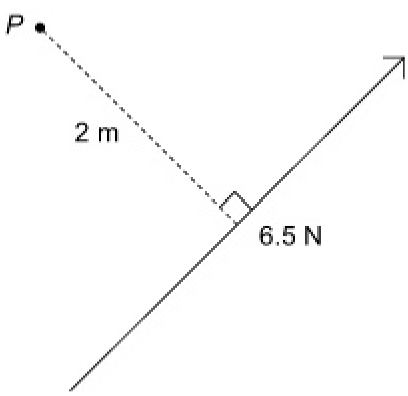
Moments
Exercise A, Question 3

Question:

Calculate the moment about *P* of each of these forces acting on a lamina.



Solution:



Moment $= 2 \times 6.5 = 13$ Nm anticlockwise

Solutionbank M1
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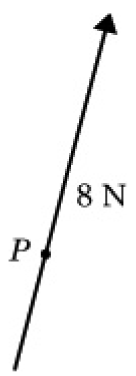
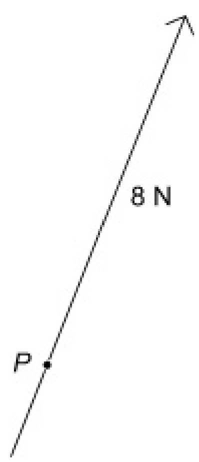
Moments
Exercise A, Question 4

Question:

Calculate the moment about *P* of each of these forces acting on a lamina.



Solution:



Line of action passes through *P* so the distance is zero.
Moment = 0 Nm
(No turning effect)

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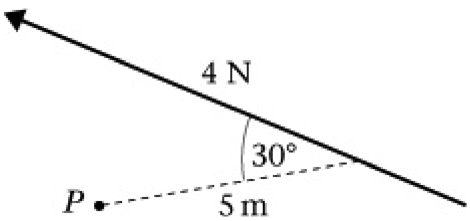
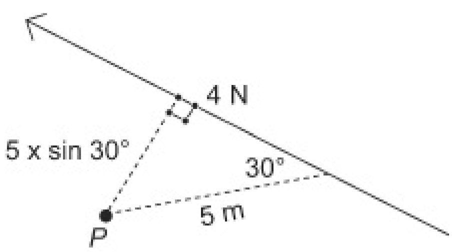
Moments
Exercise A, Question 5

Question:

Calculate the moment about P of each of these forces acting on a lamina.



Solution:



$\boxed{\text{Draw in the right angled triangle.}}$

Perpendicular distance = $5 \times \sin 30°$
Moment = $4 \times 5 \sin 30°$
= 10 Nm anticlockwise

Solutionbank M1

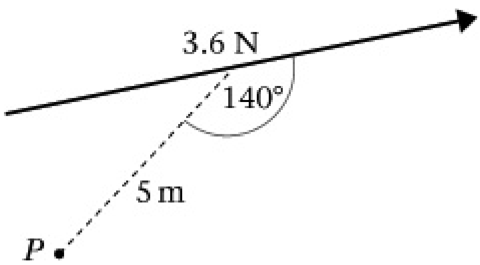
Edexcel AS and A Level Modular Mathematics

Moments

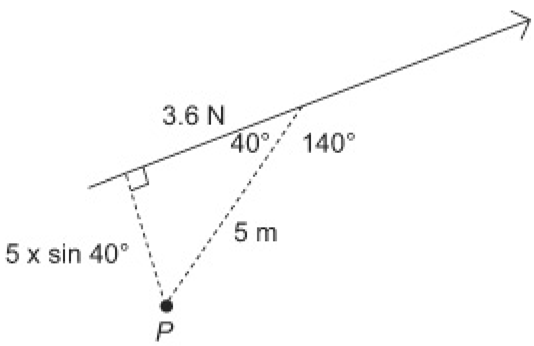
Exercise A, Question 6

Question:

Calculate the moment about P of each of these forces acting on a lamina.



Solution:



Draw in the right angled triangle.

The angle inside the triangle is $180° - 140°$
$= 40°$,
so the distance $= 5 \times \sin 40°$
Moment $= 3.6 \times 5 \sin 40°$
≈ 11.6 Nm clockwise

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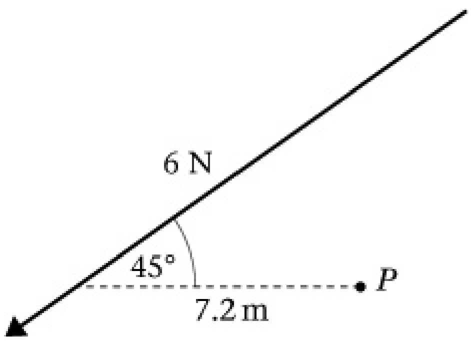
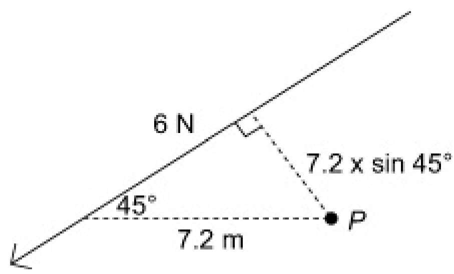
Moments
Exercise A, Question 7

Question:

Calculate the moment about P of each of these forces acting on a lamina.



Solution:



Distance $= 7.2 \times \sin 45°$
Moment $= 6 \times 7.2 \sin 45°$
≈ 30.5 Nm anticlockwise

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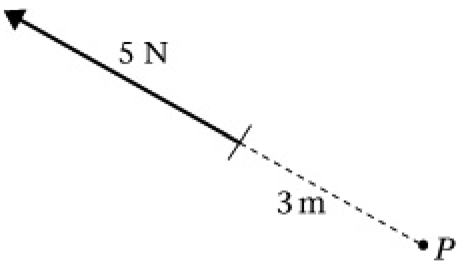
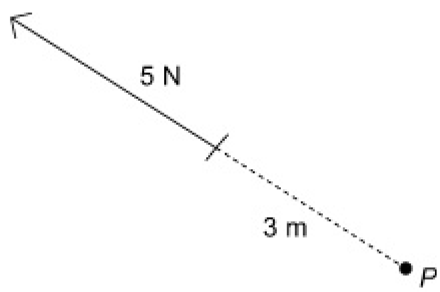
Moments
Exercise A, Question 8

Question:

Calculate the moment about P of each of these forces acting on a lamina.



Solution:



The line of action of the force acts through P, so
moment = 0 Nm

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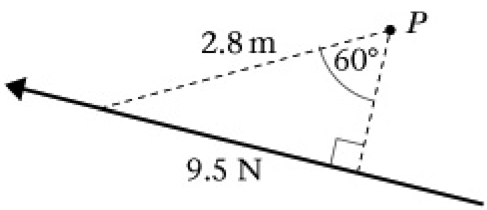
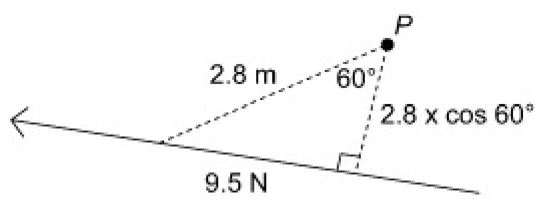
Moments
Exercise A, Question 9

Question:

Calculate the moment about P of each of these forces acting on a lamina.



Solution:



Distance $= 2.8 \times \cos 60°$
Moment $= 9.5 \times 2.8 \cos 60°$
$= 13.3$ Nm clockwise

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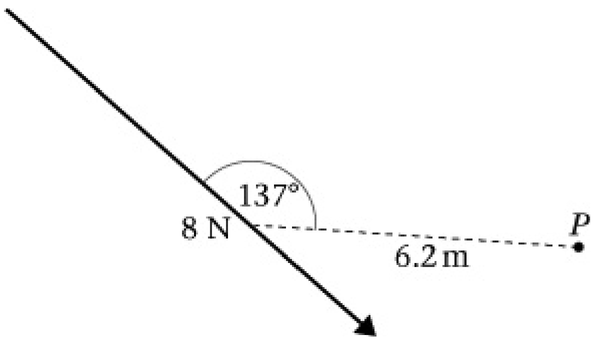
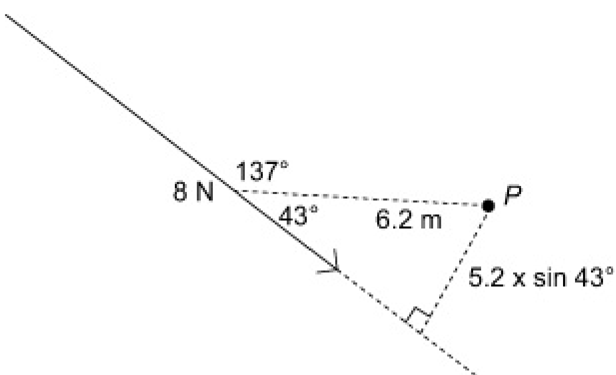
Moments
Exercise A, Question 10

Question:

Calculate the moment about *P* of each of these forces acting on a lamina.



Solution:



Draw in the right angled triangle.

The angle inside the triangle $= 180° - 137°$
$= 43°$
Distance $= 6.2 \times \sin 43°$
Moment $= 8 \times 6.2 \sin 43°$
≈ 33.8 Nm anticlockwise

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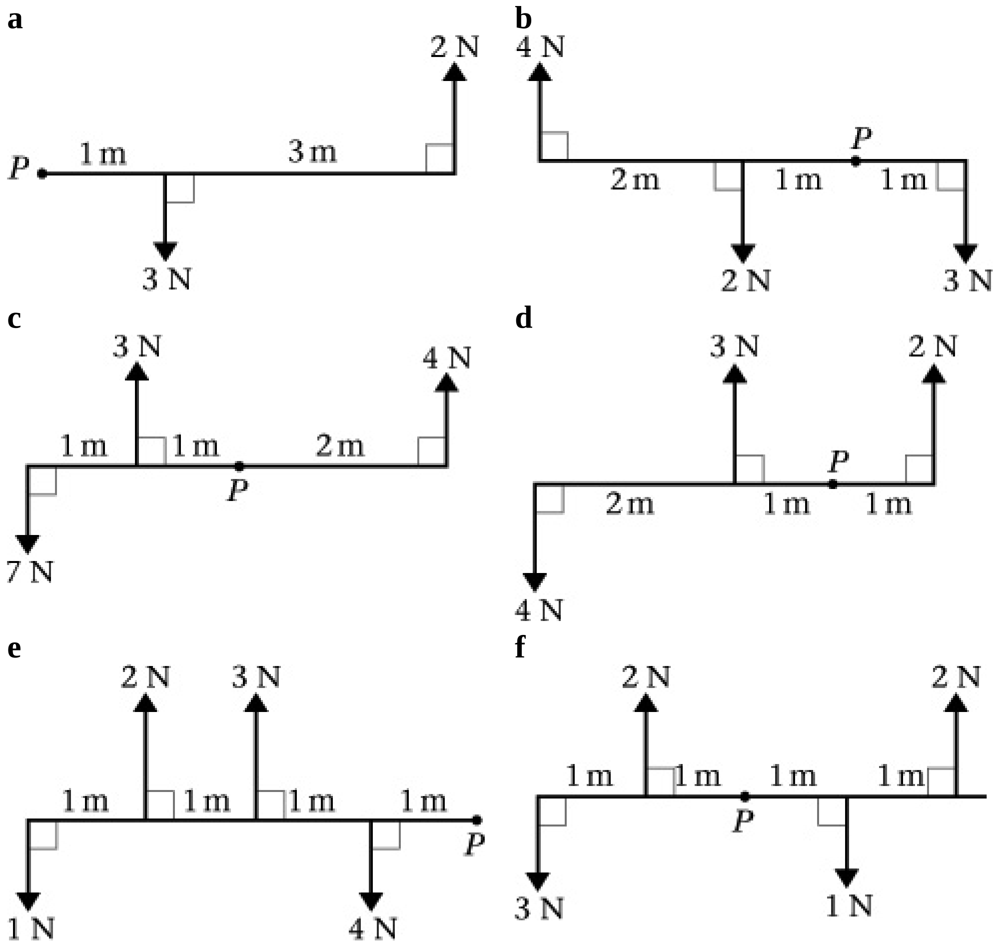
Moments
Exercise B, Question 1

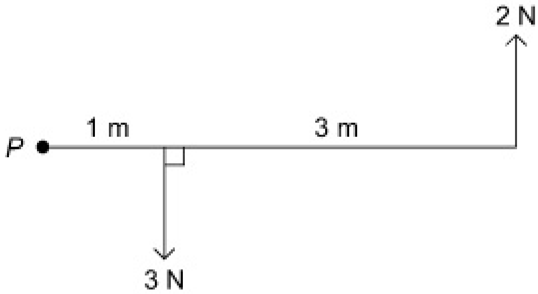
Question:

These diagrams show sets of forces acting on a light rod. For each rod, calculate the sum of the moments about *P*.



Solution:

a



↻ (Total of moments for forces acting clockwise about P): $3 \times 1 = 3$ Nm

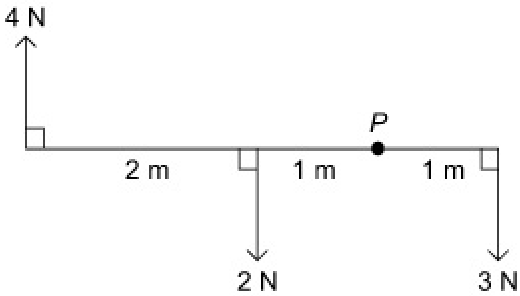
↺ (Total of moments for forces acting anticlockwise about P):

$(1 + 3) \times 2 = 8$ Nm

Sum of moments $= 8 - 3$

$= 5$ Nm anticlockwise

b



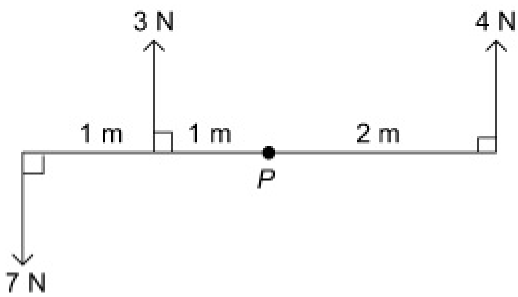
$\circlearrowright\ 4 \times (2+1) + 3 \times 1 = 15\ \text{Nm}$

$\circlearrowleft\ 2 \times 1 = 2\ \text{Nm}$

Sum of moments $= 15 - 2$

$= 13\ \text{Nm}$ clockwise

c



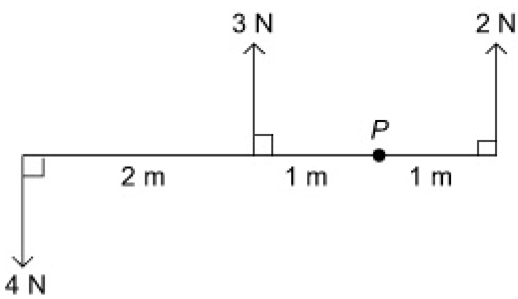
$\circlearrowright\ 3 \times 1 = 3\ \text{Nm}$

$\circlearrowleft\ 7 \times (1+1) + 4 \times 2 = 22\ \text{Nm}$

Sum of moments $= 22 - 3$

$= 19\ \text{Nm}$ anticlockwise

d



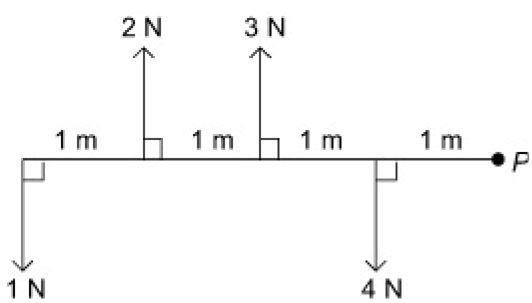
$\circlearrowright\ 3 \times 1 = 3\ \text{Nm}$

$\circlearrowleft\ 2 \times 1 + 4 \times (2+1) = 14\ \text{Nm}$

Sum of moments $= 14 - 3$

$= 11\ \text{Nm}$ anticlockwise

e



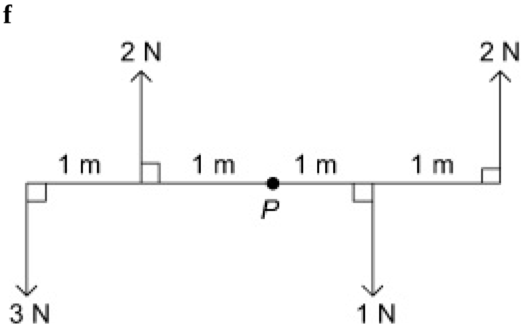
$\circlearrowright\ 3 \times (1+1) + 2 \times (1+1+1) = 12\ \text{Nm}$

$\circlearrowleft\ 4 \times 1 + 1 \times (1+1+1+1) = 8\ \text{Nm}$

Sum of moments $= 12 - 8$

$= 4\ \text{Nm}$ clockwise

f



$\circlearrowright\ 1 \times 1 + 2 \times 1 = 3\ \text{Nm}$

$\circlearrowleft\ 2 \times (1+1) + 3 \times (1+1) = 10\ \text{Nm}$

Sum of moments $= 10 - 3$

$= 7\ \text{Nm}$ anticlockwise

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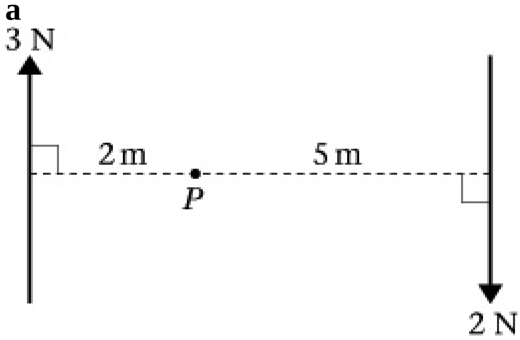
Edexcel AS and A Level Modular Mathematics

Moments
Exercise B, Question 2

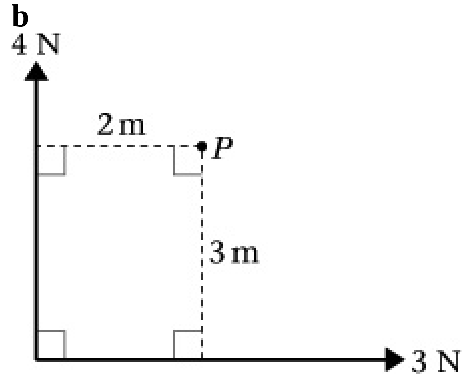
Question:

These diagrams show forces acting on a lamina. In each case, find the sum of the moments of the set of forces about *P*.

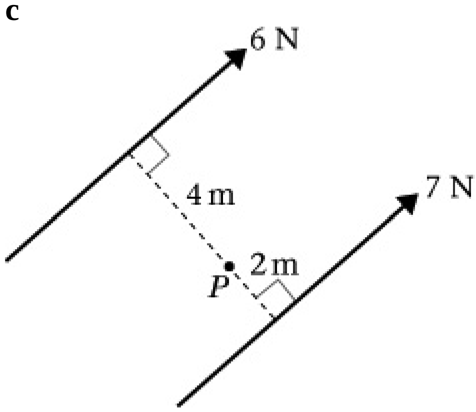
a



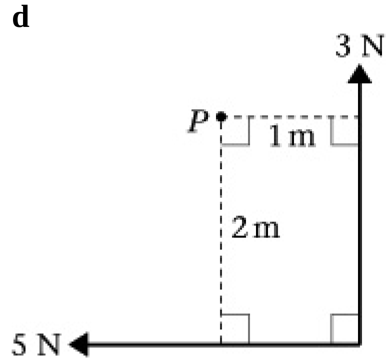
b



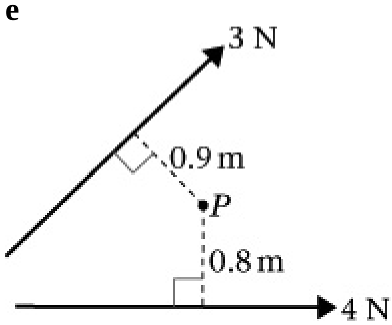
c



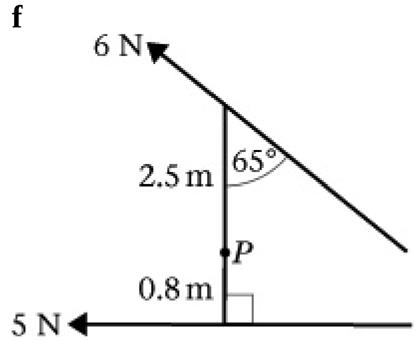
d



e

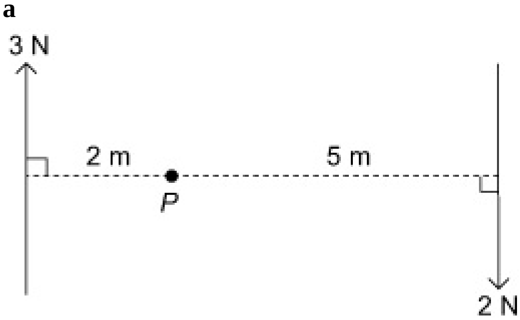


f



Solution:

a

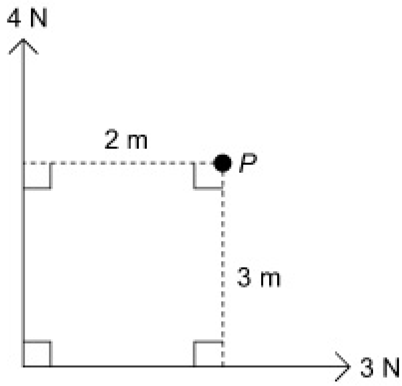


↻ $3 \times 2 + 2 \times 5 = 16$ Nm

↺ None

Sum of moments $= 16$ Nm clockwise

b



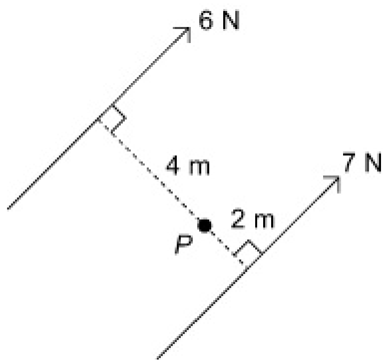
↻ $4 \times 2 = 8$ Nm

↺ $3 \times 3 = 9$ Nm

Sum of moments $= 9 - 8$

$= 1$ Nm anticlockwise

c



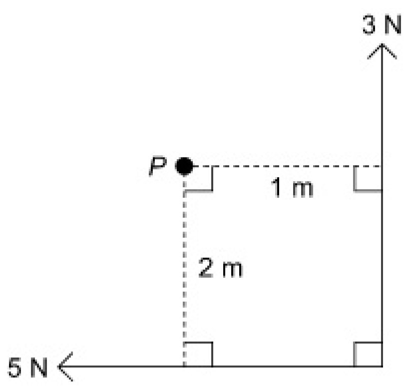
↻ $6 \times 4 = 24$ Nm

↺ $7 \times 2 = 14$ Nm

Sum of moments $= 24 - 14$

$= 10$ Nm clockwise

d



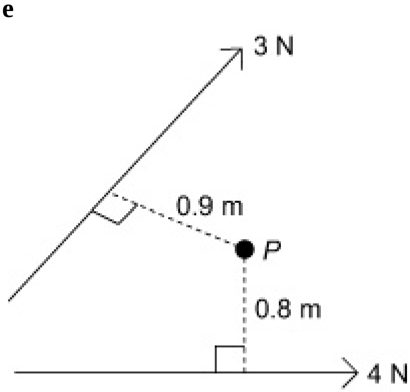
↻ $5 \times 2 = 10$ Nm

↺ $3 \times 1 = 3$ Nm

Sum of moments $= 10 - 3$

$= 7$ Nm clockwise

e



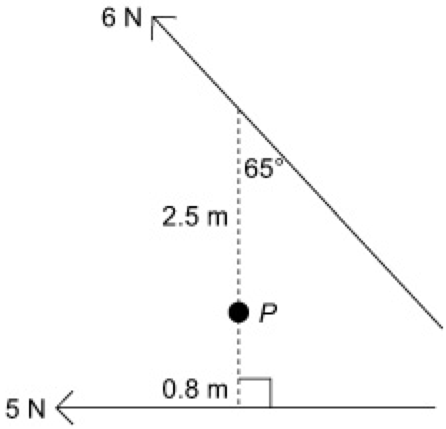
↻ $3 \times 0.9 = 2.7$ Nm

↺ $4 \times 0.8 = 3.2$ Nm

Sum of moments $= 3.2 - 2.7$

$= 0.5$ Nm anticlockwise

f



↻ 5×0.8	$= 4$ Nm
↺ $6 \times (2.5 \times \sin 65°)$	$= 13.594\ldots$ Nm
or $(6 \times \sin 65°) \times 2.5$	
or $(6 \times \cos 25°) \times 2.5$	
Sum of moments	$= 13.594.\ldots - 4$
	≈ 9.59 Nm anticlockwise

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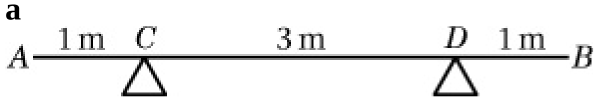
Moments
Exercise C, Question 1

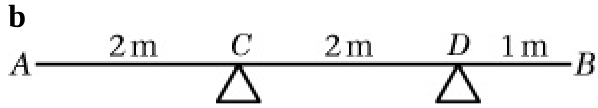
Question:

AB is a uniform rod of length 5 m and weight 20 N. In these diagrams *AB* is resting in a horizontal position on supports at *C* and *D*. In each case, find the magnitudes of the reactions at *C* and *D*.

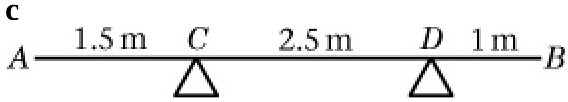
a



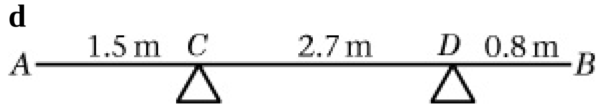
b



c

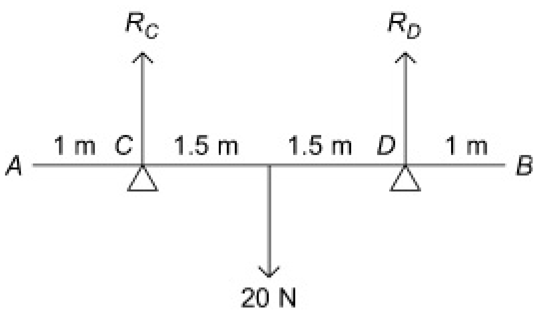


d



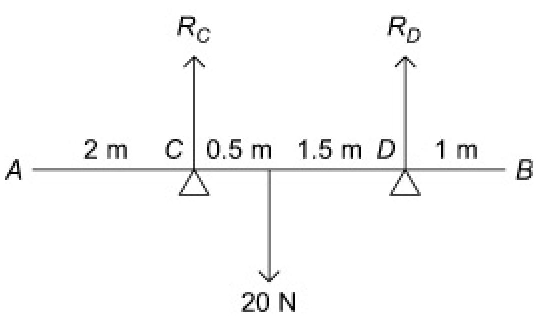
Solution:

a



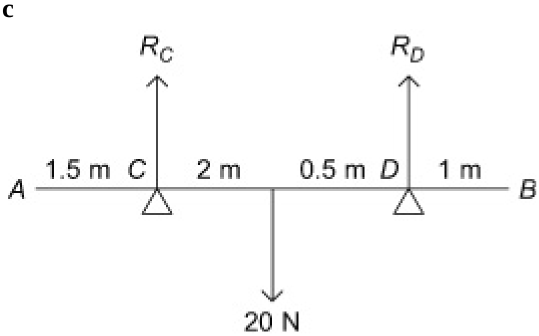
R(↑)
$R_C + R_D = 20$
Taking moments about *C*:
$3 \times R_D = 20 \times 1.5 = 30$
$\Rightarrow R_D = 10$ N and $R_C = 10$ N

b



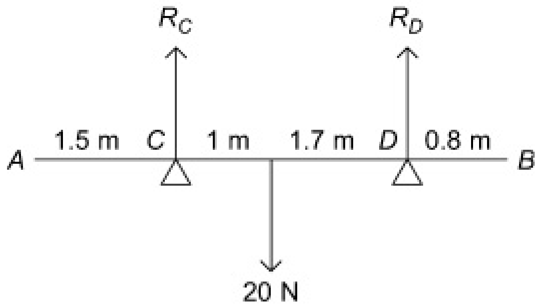
R(↑)
$R_C + R_D = 20$
Taking moments about *C*:
$R_D \times 2 = 20 \times 0.5 = 10$
$\Rightarrow R_D = 5$ N and $R_C = 15$ N

c



R(↑)
$R_C + R_D = 20$
Taking moments about *C*:
$R_D \times 2.5 = 20 \times 1 = 20$
$\Rightarrow R_D = \frac{20}{2.5} = 8$ N and $R_C = 12$ N

d



R(↑)

$R_C + R_D = 20$

Taking moments about C:

$2.7 \times R_D = 20 \times 1 = 20$

$\Rightarrow R_D = \frac{20}{2.7} \approx 7.4$ N and $R_C \approx 12.6$ N

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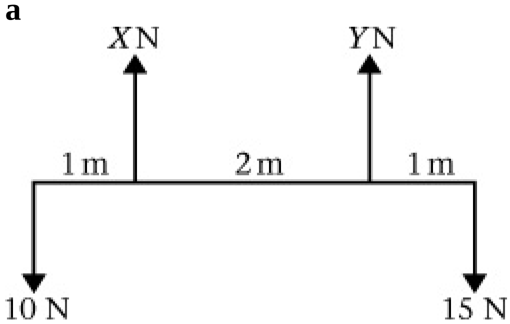
Moments
Exercise C, Question 2

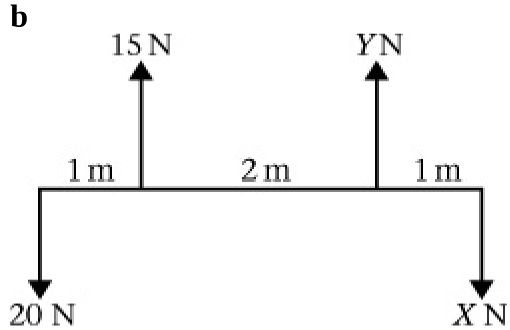
Question:

Each of these diagrams shows a light rod in equilibrium in a horizontal position under the action of a set of forces.

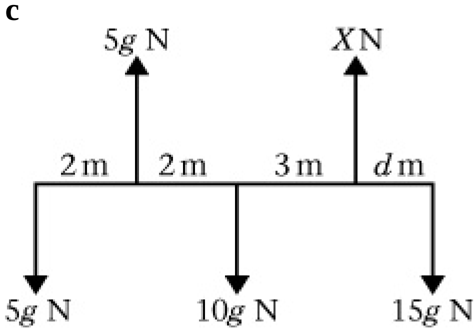
a



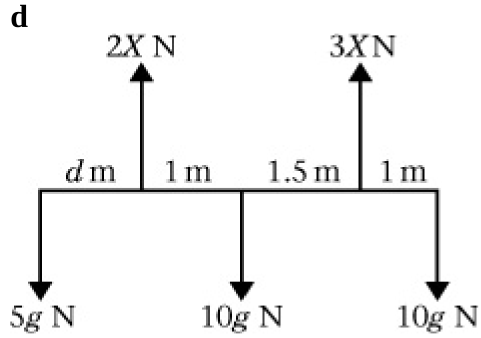
b



c

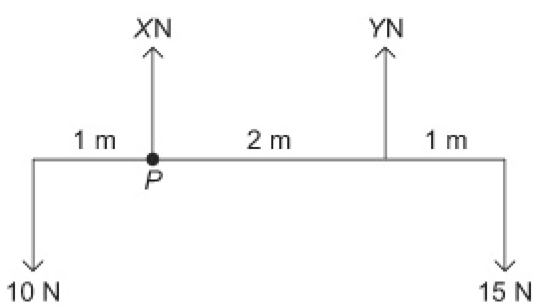


d



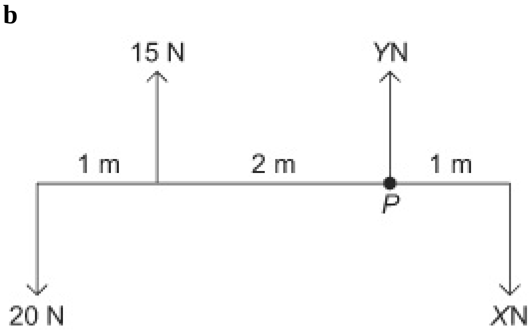
Solution:

a



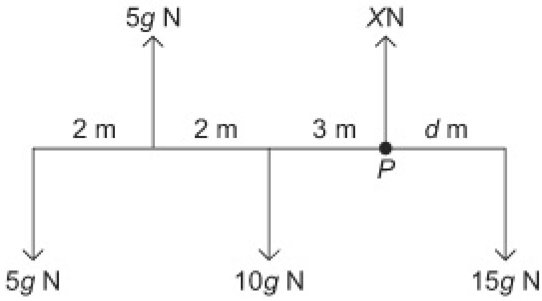
R(↑)
$X + Y = 10 + 15 = 25$
Taking moments about P:
$15 \times (2 + 1) = 10 \times 1 + Y \times 2$
$\Rightarrow 45 = 10 + 2Y, \quad 2Y = 35, Y = 17.5$
$\Rightarrow X = 7.5$ and $Y = 17.5$

b



R(↑)
$15 + Y = 20 + X, \; Y - X = 5$
Taking moments about P:
$20 \times (2 + 1) = 15 \times 2 + X \times 1$
$\Rightarrow 60 = 30 + X, \quad X = 30$
$\Rightarrow X = 30$ and $Y = 35$

c



R(↑)

$5g + X = 5g + 10g + 15g = 30g$

$\Rightarrow X = 25g = 245$

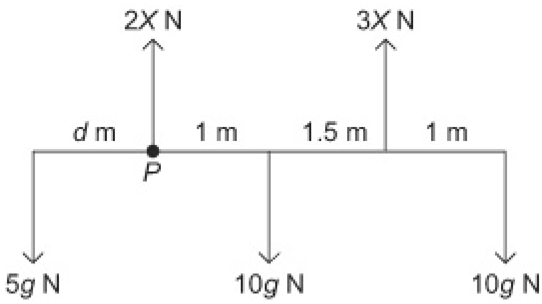
Taking moments about P:

$15g \times d + 5g \times (2 + 3) = 10g \times 3 + 5g \times (2 + 2 + 3)$

$\Rightarrow 15g\ d + 25g = 30g + 35g$

$\Rightarrow 15d = 40$, $d = 2\frac{2}{3}$

d



R(↑)

$2X + 3X = 5g + 10g + 10g = 25g$

$\Rightarrow X = 5g = 49$

Taking moments about P:

$5g \times d + 15g \times (1 + 1.5) = 10g \times 1 + 10g \times (1 + 1.5 + 1)$

$\Rightarrow 5d + 15 \times 2.5 = 10 + 35$, $5d = 7.5$,

$\Rightarrow d = 1.5$

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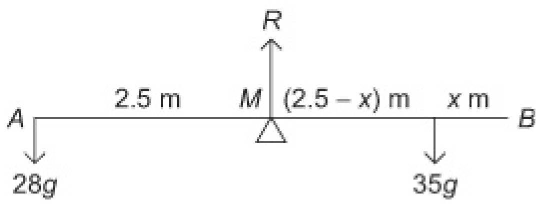
Moments
Exercise C, Question 3

Question:

Jack and Jill are playing on a see-saw made from a uniform plank AB of length 5 m pivoted at M, the mid-point of AB. Jack has mass 35 kg and Jill has mass 28 kg. Jill sits at A. Where must Jack sit for the plank to be in equilibrium when horizontal?

Solution:



Suppose that Jack sits x m from B.
Taking moments about the pivot (M):
$28g \times 2.5 = 35g \times (2.5 - x)$
$\Rightarrow 28 \times 2.5 = 35 (2.5 - x)$
$5 (2.5 - x) = 4 \times 2.5 = 10$
$2.5 - x = 2 , \Rightarrow x = 0.5$
Jack sits 0.5 m from B

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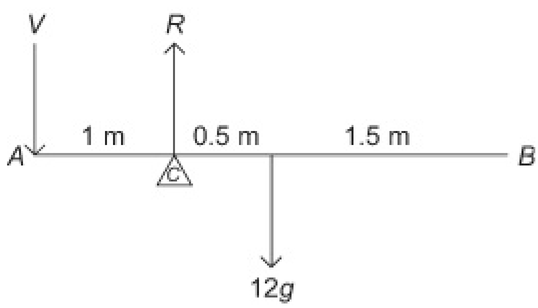
Moments
Exercise C, Question 4

Question:

A uniform rod AB of length 3 m and mass 12 kg is pivoted at C, where $AC = 1$ m. Calculate the vertical force that must be applied at A to maintain equilibrium with the rod horizontal.

Solution:



Suppose that the force required is V N acting vertically downwards at A.
Taking moments about the pivot (C):

$$V \times 1 = 0.5 \times 12g$$

$$\Rightarrow V = 6g = 59 \text{ N (2 s.f.)}$$

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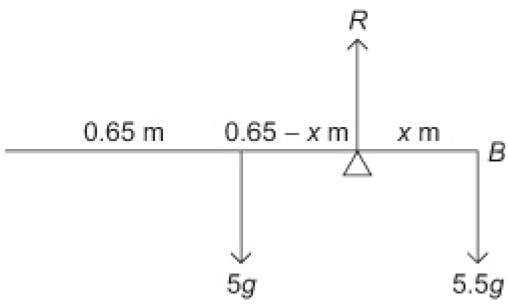
Moments
Exercise C, Question 5

Question:

A broom consists of a broomstick of length 130 cm and mass 5 kg and a broomhead of mass 5.5 kg attached at one end. By modelling the broomstick as a rod and the broomhead as a particle, find where a support should be placed so that the broom will balance horizontally.

Solution:



Let the support be x m from the broomhead.
Taking moments about the support:

$$5.5g \times x = 5g \times (0.65 - x)$$
$$5.5x = 5 \times 0.65 - 5x$$
$$10.5x = 3.25$$
$$x \approx 0.31$$

The support should be 31 cm from the broomhead.

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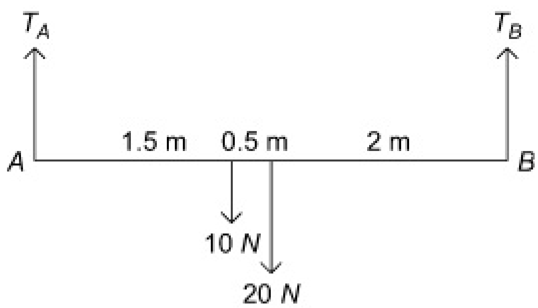
Moments
Exercise C, Question 6

Question:

A uniform rod AB of length 4 m and weight 20 N is suspended horizontally by two vertical strings attached at A and at B. A particle of weight 10 N is attached to the rod at point C, where $AC = 1.5$ m. Find the magnitudes of the tensions in the two strings.

Solution:



Let the tensions in the two strings be T_A and T_B respectively.

R(↑)

$T_A + T_B = 10 + 20 = 30$

Taking moments about point A:

$10 \times 1.5 + 20 \times (1.5 + 0.5) = 4 \times T_B$

$\Rightarrow 4T_B = 15 + 40 = 55$, $T_B = 13.75$ N

and $T_A = 16.25$ N

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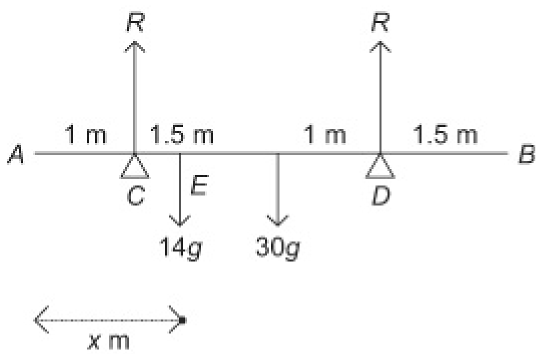
Moments
Exercise C, Question 7

Question:

A uniform plank AB of length 5 m and mass 30 kg is resting horizontally on supports at C and D, where $AC = 1$ m and $AD = 3.5$ m. When a particle of mass 14 kg is attached to the rod at point E the magnitude of the reaction at C is equal to the magnitude of the reaction at D. Find the distance AE.

Solution:



R(↑)

$R + R = 30g + 14g = 44g \Rightarrow R = 22g$

Let distance $AE = x$ m

Taking moments about A:

$R \times 1 + R \times 3.5 = 30g \times 2.5 + 14g \times x$

$\Rightarrow\ 4.5 \times 22g = 75g + 14gx$

$99 = 75 + 14x$

$\Rightarrow\ 24 = 14x\ ,\ x \approx 1.71$

$AE \approx 1.71$ m

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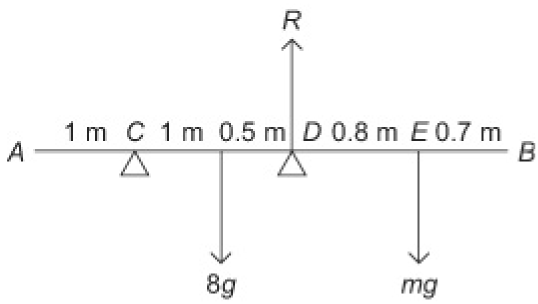
Moments
Exercise C, Question 8

Question:

A uniform rod AB has length 4 m and mass 8 kg. It is resting in a horizontal position on supports at points C and D were $AC = 1\,\text{m}$ and $AD = 2.5\,\text{m}$. A particle of mass m kg is placed at point E where $AE = 3.3\,\text{m}$. Given that rod is about to tilt about D, calculate the value of m.

Solution:



If the rod is about to turn about D then the reaction at C is zero.
Taking moments about point D:

$$8g \times 0.5 = m g \times 0.8$$

$$\Rightarrow m = 5$$

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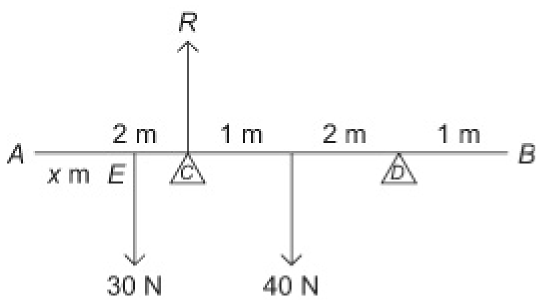
Moments
Exercise C, Question 9

Question:

A uniform bar AB of length 6 m and weight 40 N is resting in a horizontal position on supports at points C and D where $AC = 2\,\text{m}$ and $AD = 5\,\text{m}$. When a particle of weight 30 N is attached to the bar at point E the bar is on the point of tilting about C. Calculate the distance AE.

Solution:



If the bar is about to tilt about C then the reaction at D is zero.
Let the distance $AE = x$ m
Taking moments about C:

$40 \times 1 = 30 \times (2 - x)$, $40 = 60 - 30x$

$\Rightarrow 30x = 20$, $x = \frac{2}{3}$

The distance $AE = \frac{2}{3}$ m

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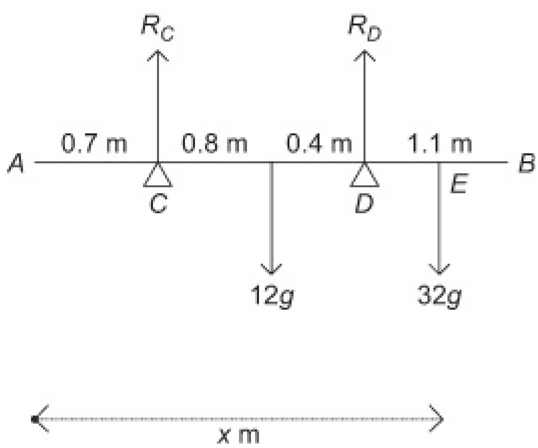
Moments
Exercise C, Question 10

Question:

A plank AB of mass 12 kg and length 3 m is in equilibrium in a horizontal position resting on supports at C and D where $AC = 0.7$ m and $DB = 1.1$ m. A boy of mass 32 kg stands on the plank at point E. The plank is about to tilt about D. By modelling the plank as a uniform rod and the boy as a particle, calculate the distance AE.

Solution:



Let the distance AE be x m.
If the plank is about to tilt about D then $R_C = 0$.
Taking moments about D:

$12g \times 0.4 = 32g \times (x - 1.9)$

$12 \times 0.4 = 32x - 32 \times 1.9$

$32x = 4.8 + 60.8 = 65.6$

$\Rightarrow x = 65.6 \div 32 = 2.05$ m

E is 2.05 m from A

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Moments
Exercise C, Question 11

Question:

A uniform rod AB has length 5 m and weight 20 N. The rod is resting on supports at points C and D where $AC = 2$ m and $BD = 1$ m.

a Find the magnitudes of the reactions at C and D.

A particle of weight 12 N is placed on the rod at point A.

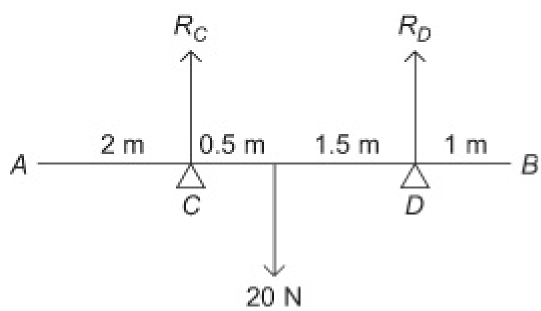
b Show that this causes the rod to tilt about C.

A second particle of weight 12 N is placed on the rod at E to hold it in equilibrium.

c How far must E be from A?

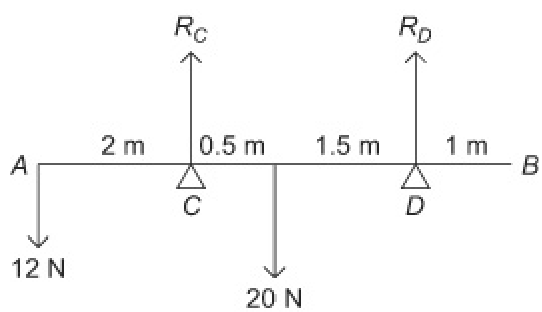
Solution:

a



R(↑)
$R_C + R_D = 20$
Taking moments about C:
$20 \times 0.5 = R_D \times 2$, $R_D = 5$ N
$R_C = 15$ N

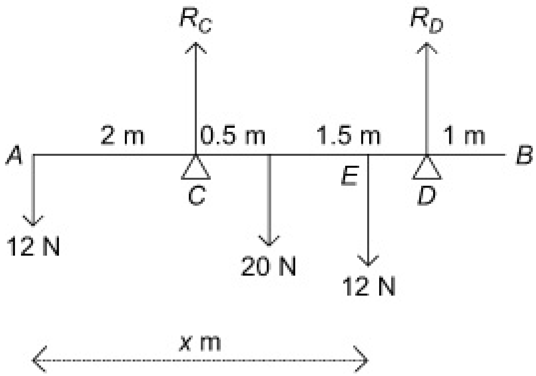
b Adding the weight of 12 N:



Taking moments about C:
$20 \times 0.5 = 12 \times 2 + R_D \times 2$
$10 = 24 + 2R_D$
$\Rightarrow R_D$ is negative, which is impossible, therefore there is an anticlockwise moment about C – the rod will tilt.

c Adding the second particle:

Let the distance AE be x m.
If the system is just about to tilt about C, taking moments about C and $R_D = 0$



$$12 \times (x - 2) + 20 \times 0.5 = 12 \times 2$$
$$12x - 24 + 10 = 24$$
$$12x = 38$$
$$\Rightarrow x = 38 \div 12 \approx 3.17$$

The second particle needs to be 3.17 m from *A* to prevent tilting.

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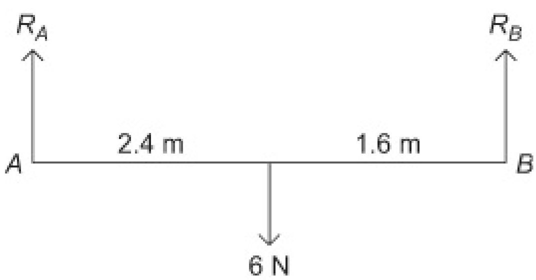
Moments
Exercise D, Question 1

Question:

A non-uniform rod AB of length 4 m and weight 6 N rests horizontally on two supports at A and B. Given that the centre of mass of the rod is 2.4 m from the end A, find the reactions at the two supports.

Solution:



R(↑)

$6 = R_A + R_B$

Taking moments about A:

$6 \times 2.4 = 4 \times R_B$

$\Rightarrow R_B = 3.6$ N

$R_A = 2.4$ N

The reactions at A and B are 2.4 N and 3.6 N respectively.

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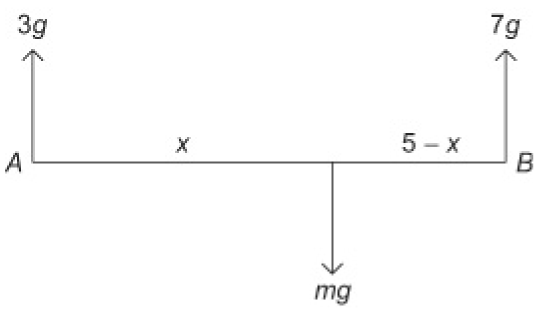
Moments
Exercise D, Question 2

Question:

A non-uniform bar AB of length 5 m is supported horizontally on supports at A and B. The reactions at these supports are $3g$ N and $7g$ N respectively. Find the position of the centre of mass.

Solution:



Let m be the mass of the bar.
R(↑) $m \ g = 3g + 7g$
$\Rightarrow$ the mass of the bar is 10 kg
Let the centre of mass be x m from A:
Taking moments about A:
$m \ g \times x = 7g \times 5$
$\Rightarrow m \ x = 35$, $10x = 35$, $x = 3.5$ m
The centre of mass is 3.5 m from A.

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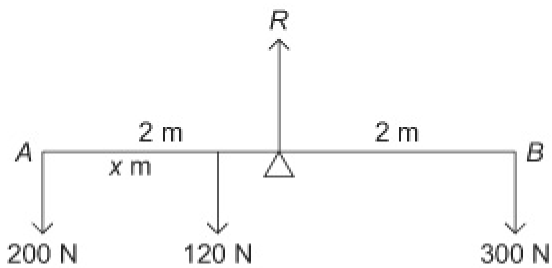
Moments
Exercise D, Question 3

Question:

A non-uniform plank *AB* of length 4 m and weight 120 N is pivoted at its mid-point. The plank is in equilibrium in a horizontal position with a child of weight 200 N sitting at *A* and a child of weight 300 N sitting at *B*. By modelling the plank as a rod and the two children as particles find the distance of the centre of mass of the plank from *A*.

Solution:



Let the centre of mass be x m from *A*.
Taking moments about the mid-point:

$120 \times (2 - x) + 200 \times 2 = 300 \times 2$

$240 - 120x + 400 = 600$

$120x = 40$

$\Rightarrow x = \frac{40}{120} = \frac{1}{3}$

The centre of mass is $\frac{1}{3}$ m from *A*.

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Moments
Exercise D, Question 4

Question:

A non-uniform rod AB of length 5 m and mass 15 kg rests horizontally suspended from the ceiling by two vertical strings attached to C and D, where $AC = 1\text{ m}$ and $AD = 3.5\text{ m}$.

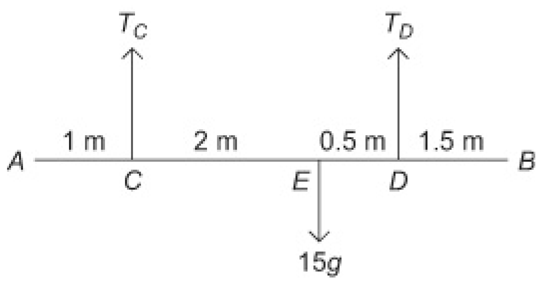
a Given that the centre of mass is at E where $AE = 3\text{ m}$, find the magnitudes of the tensions in the strings.

When a particle of mass 10 kg is attached to the rod at F the rod is just about to rotate about D.

b Find the distance AF.

Solution:

a



Taking moments about C:

$T_D \times 2.5 = 15g \times 2$

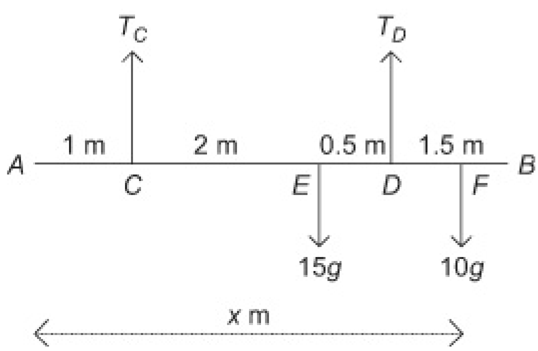
$2.5T_D = 30g$

$\Rightarrow T_D = 12g = 118\text{ N (3 s.f.)}$

R(↑)

$T_C + T_D = 15g$, $T_C = 3g = 29.4\text{ N}$

b



With the particle attached at F, T_C is zero because the rod is about to rotate about D.

Let the distance $AF = x$ m.

Taking moments about D:

$15g \times 0.5 = 10g \times (x - (1 + 2 + 0.5))$

$= 10g \times (x - 3.5)$

$7.5g = 10gx - 35g$

$42.5 = 10x$

$\Rightarrow x = 4.25$

The distance AF is 4.25 m

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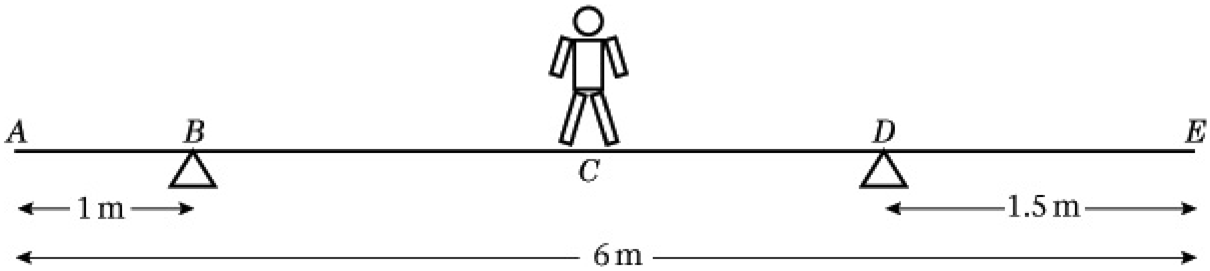
Moments
Exercise E, Question 1

Question:



A plank *AE*, of length 6 m and weight 100 N, rests in a horizontal position on supports at *B* and *D*, where *AB* = 1 m and *DE* = 1.5 m. A child of weight 145 N stands at *C*, the mid-point of *AE*, as shown in the diagram above. The child is modelled as a particle and the plank as a uniform rod. The child and the plank are in equilibrium. Calculate

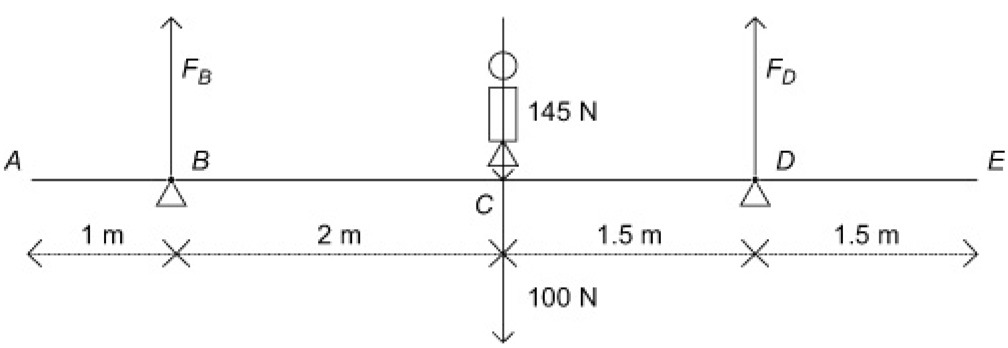
a the magnitude of the force exerted by the support on the plank at *B*,

b the magnitude of the force exerted by the support on the plank at *D*.

The child now stands at a different point *F* on the plank. The plank is in equilibrium and on the point of tilting about *D*.

c Calculate the distance *DF*.

Solution:



a Taking moments about the point *D*:
⇒ since the child and the plank are in equilibrium,
$1.5 \times 100 + 1.5 \times 145 = 3.5 \times F_B$, $150 + 217.5 = 3.5 \times F_B$, ⇒ $F_B = 367.5 \div 3.5 = 105$ N

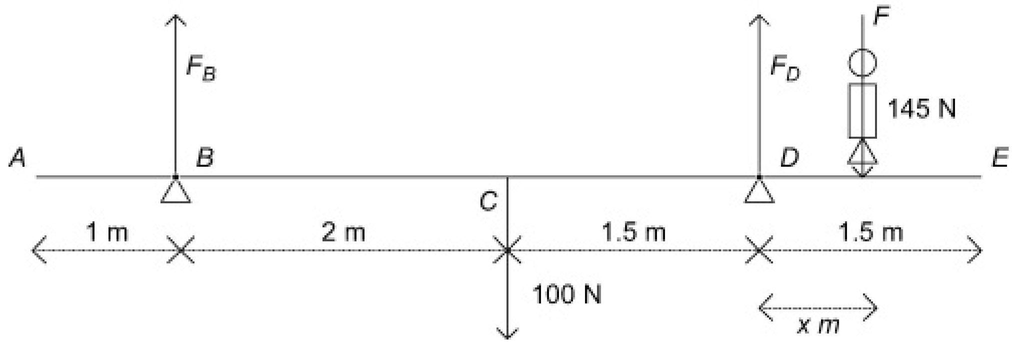
b R(↑), the child and the plank are in equilibrium, so

$100 + 145 = F_B + F_D$,

$245 = 105 + F_D$

⇒ $F_D = 245 - 105 = 140$ N

c



If the plank is about to tilt about D, then $F_B = 0$ and the child must be standing to the right of D.

Let the distance DF be x m. Taking moments about D:

$$\Rightarrow 100 \times 1.5 = 145 \times x, \quad x = \frac{100 \times 1.5}{145} \approx 1.03 \text{ m} \quad \left(= 103 \text{ cm} \right)$$

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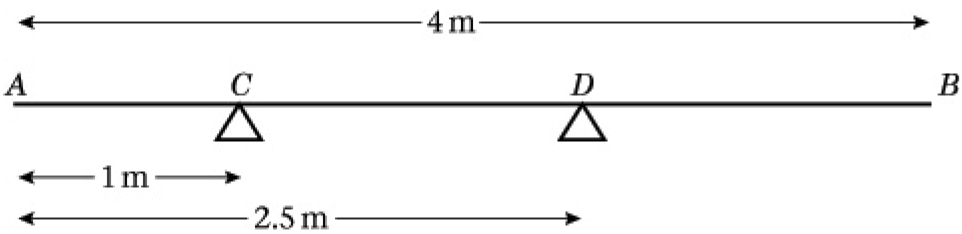
Moments
Exercise E, Question 2

Question:

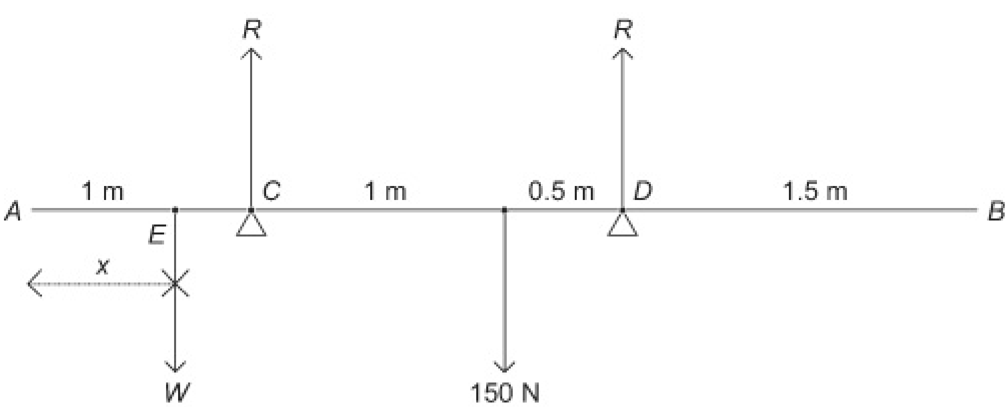


A uniform rod AB has length 4 m and weight 150 N. The rod rests in equilibrium in a horizontal position, smoothly supported at points C and D, where AC = 1 m and AD = 2.5 m as shown in the diagram above. A particle of weight W N is attached to the rod at a point E where $AE = x$ metres. The rod remains in equilibrium and the magnitude of the reaction at C is now equal to the magnitude of the reaction at D.

a Show that $W = \frac{150}{7 - 4x}$

b Hence deduce the range of possible values of x.

Solution:



a Since the rod is uniform, the centre of mass is at the mid-point.
Taking moments about A:

$$Wx + 150 \times 2 = R \times 1 + R \times 2.5 \,,$$

$$Wx + 300 = 3.5R$$

R(↑), equilibrium $\Rightarrow W + 150 = R + R$, $2R = W + 150$

Hence $R = \frac{W + 150}{2}$, and $Wx + 300 = \frac{7}{2} \times \frac{W + 150}{2}$

$$\Rightarrow 4(Wx + 300) = 7W + 7 \times 150 \,,\ 4Wx + 1200 = 7W + 1050$$

$$1200 - 1050 = 7W - 4Wx$$

$$W\left(7 - 4x\right) = 150 \,,\ W = \frac{150}{7 - 4x}$$

b $x \geq 0$ and $\frac{150}{7-4x} > 0$

$\Rightarrow 7 - 4x > 0$

$4x < 7$

$x < \frac{7}{4}$

$x < 1.75$

So $0 \leq x < 1.75$

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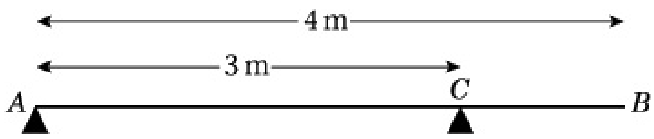
Moments

Exercise E, Question 3

Question:



A uniform plank AB has mass 40 kg and length 4 m. It is supported in a horizontal position by two smooth pivots. One pivot is at the end A and the other is at the point C where $AC = 3$ m, as shown in the diagram above. A man of mass 80 kg stands on the plank which remains in equilibrium. The magnitude of the reaction at A is twice the magnitude of the reaction at C. The magnitude of the reaction at C is RN. The plank is modelled as a rod and the man is modelled as a particle.

a Find the value of R.

b Find the distance of the man from A.

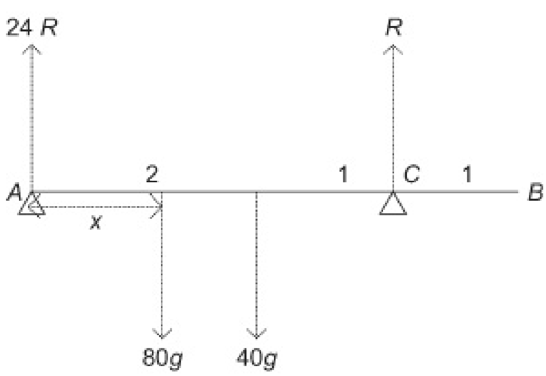
c State how you have used the modelling assumption that

i the plank is uniform,

ii the plank is a rod,

iii the man is a particle.

Solution:



a

$$\text{R} (\uparrow) \quad 3R = 80g + 40g$$
$$R = 40g = 392 \text{ N}$$

b Taking moments about A: $80g \times x + 40g \times 2 = 40g \times 3$

$$80g \times x = 40g, \Rightarrow x = \frac{1}{2} = 0.5\ m$$

c (i) Since the plank is uniform, the weight acts at centre of plank.
(ii) Since the plank is a rod, the plank remains straight.
(iii) Since the man is a particle, his weight acts at a single point.

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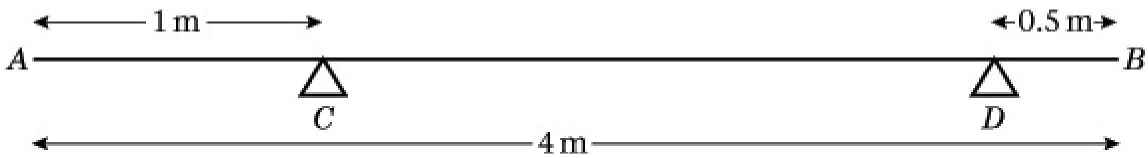
Moments
Exercise E, Question 4

Question:



A non-uniform rod *AB* has length 4 m and weight 150 N. The rod rests horizontally in equilibrium on two smooth supports *C* and *D*, where $AC = 1$ m and $DB = 0.5$ m, as shown in the diagram above. The centre of mass of *AB* is *x* metres from *A*. A particle of weight *W*N is placed on the rod at *A*. The rod remains in equilibrium and the magnitude of the reaction of *C* on the rod is 100 N.

a Show that $550 + 7W = 300x$.

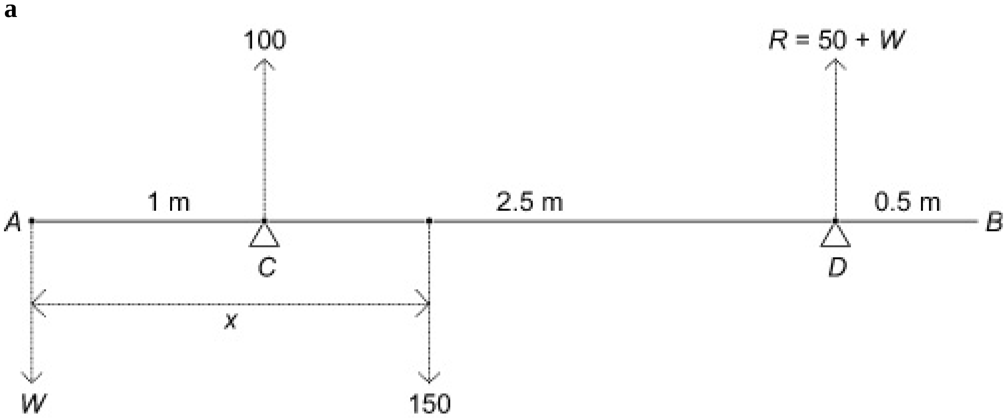
The particle is now removed from *A* and placed on the rod at *B*. The rod remains in equilibrium and the reaction of *C* on the rod now has magnitude 52 N.

b Obtain another equation connecting *W* and *x*.

c Calculate the value of *x* and the value of *W*.

Solution:

a

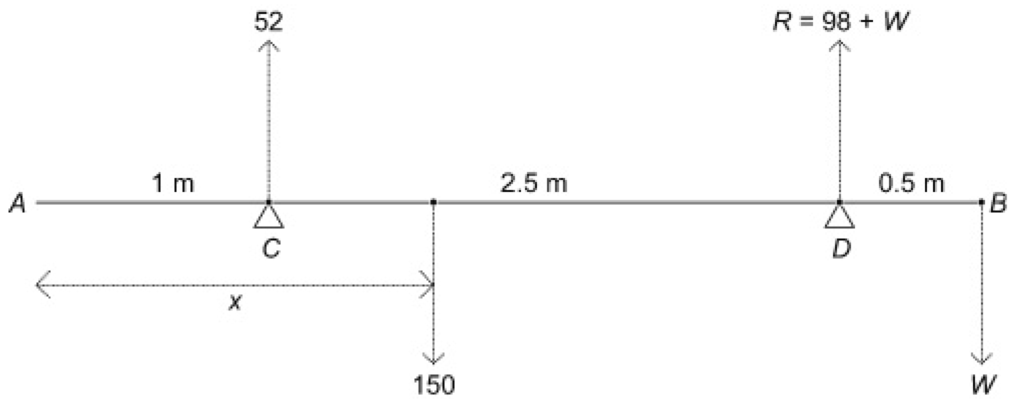


R(↑) $100 + R = W + 150$, $R = W + 50$

Taking moments about *A*,

$$100 \times 1 + (W + 50) \times 3.5 = 150 \times x$$
$$150x = 100 + 175 + 3.5W$$
$$275 + 3.5W = 150x$$
$$550 + 7W = 300x$$

b



R(↑) $52 + R = 150 + W$, $R = 150 + W - 52 = 98 + W$
Taking moments about B: $52 \times 3 + (98 + W) \times 0.5 = 150 \times (4 - x)$
$156 + 49 + 0.5W = 600 - 150x$
doubling, $410 + W = 1200 - 300x$, $W = 790 - 300x$

c Solving the simultaneous equations $\rightarrow W = 790 - (550 + 7W)$,
$8W = 790 - 550 = 240 \Rightarrow W = 30$
$\Rightarrow 410 + 30 = 1200 - 300x$, $300x = 760$, $x = 2.53$ (3 s.f.)

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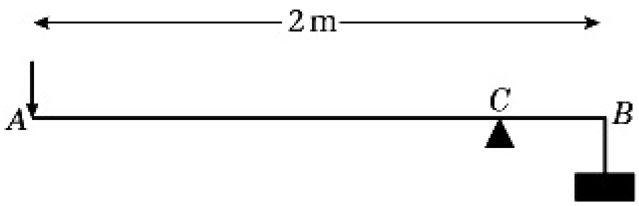
Moments
Exercise E, Question 5

Question:



A lever consists of a uniform steel rod AB of weight 100 N and length 2 m, which rests on a small smooth pivot at a point C. A load of weight 1700 N is suspended from the end B of the rod by a rope. The lever is held in equilibrium in a horizontal position by a vertical force applied at the end A, as shown in the diagram above. The rope is modelled as a light string.

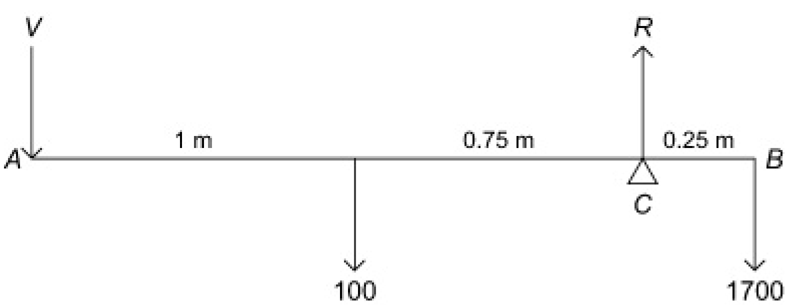
a Given that $BC = 0.25$ m find the magnitude of the force applied at A.

The position of the pivot is changed so that the rod remains in equilibrium when the force at A has magnitude 150 N.

b Find, to the nearest centimetre, the new distance of the pivot from B.

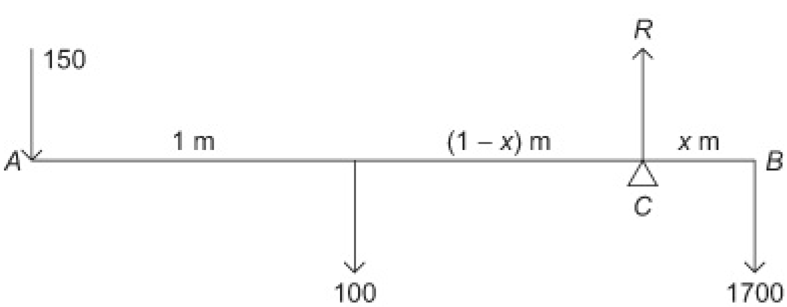
Solution:

a Let the force applied at A be V.



Taking moments about C: $V \times 1.75 + 100 \times 0.75 = 1700 \times 0.25$
$\Rightarrow 1.75V + 75 = 425$, $1.75V = 350$, $V = 200$ N

b If the distance $BC = x$



Taking moments about C: $150\,(1 + 1 - x) + 100\,(1 - x) = 1700x$

$\Rightarrow 300 - 150x + 100 - 100x = 1700x \Rightarrow 400 - 250x = 1700x$

$400 = 1950x$, $x = \frac{400}{1950} \approx 0.21$ m (2 s.f.)

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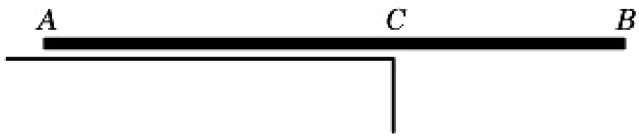
Moments
Exercise E, Question 6

Question:



A plank *AB* has length 4 m. It lies on a horizontal platform, with the end *A* lying on the platform and the end *B* projecting over the edge, as shown above. The edge of the platform is at the point *C*.

Jack and Jill are experimenting with the plank. Jack has mass 48 kg and Jill has mass 36 kg. They discover that if Jack stands at *B* and Jill stands at *A* and $BC = 1.8$ m, the plank is in equilibrium and on the point of tilting about *C*.

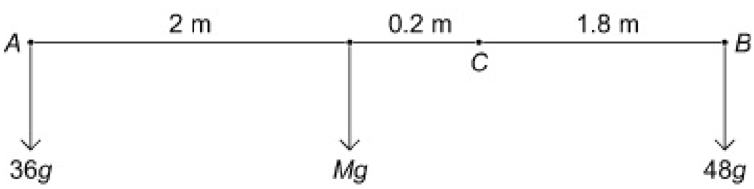
a By modelling the plank as a uniform rod, and Jack and Jill as particles, find the mass of the plank.

They now alter the position of the plank in relation to the platform so that, when Jill stands at *B* and Jack stands at *A*, the plank is again in equilibrium and on the point of tilting about *C*.

b Find the distance *BC* in this position.

Solution:

a Let the mass of the plank be *M*. Since the plank is uniform, its centre of mass is at its mid-point.

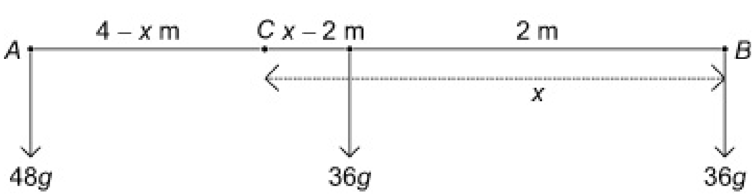


Taking moments about *C*: $48g \times 1.8 = Mg \times 0.2 + 36g \times 2.2$

$$86.4\,g = 0.2Mg + 79.2g\ ,\ 86.4 = 0.2\,M + 79.2$$

$$0.2\,M = 86.4 - 79.2 = 7.2 \Rightarrow M = 36\text{ kg}$$

b Let the distance *BC* be x



Taking moments about *C*: $36gx + 36g\,(\,x - 2\,) = 48g\,(\,4 - x\,)$

$\Rightarrow$ (dividing by the common factor $12g$)

$3x + 3\,(\,x - 2\,) = 4\,(\,4 - x\,)$, $6x - 6 = 16 - 4x$

$\Rightarrow 10x = 22$, $x = 2.2$ m

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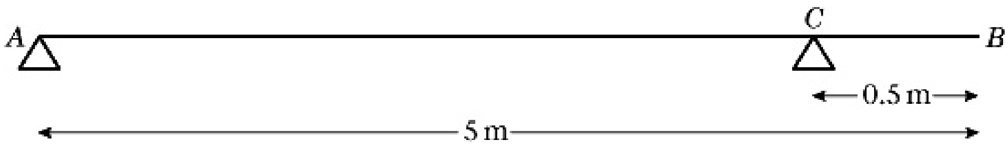
Moments
Exercise E, Question 7

Question:



A plank of wood *AB* has mass 12 kg and length 5 m. It rests in a horizontal position on two smooth supports. One support is at the end *A*. The other is at the point *C*, 0.5 m from *B*, as shown in the diagram above. A girl of mass 30 kg stands at *B* with the plank in equilibrium.

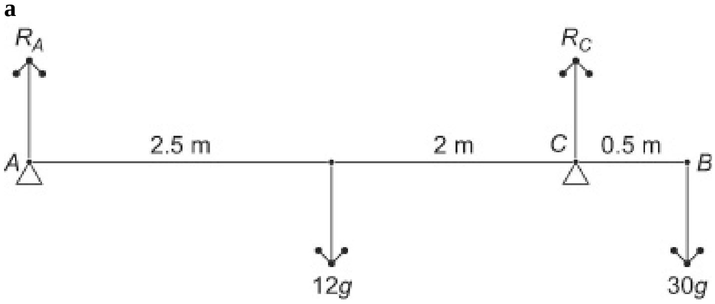
a By modelling the plank as a uniform rod and the girl as a particle, find the reaction on the plank at *A*.

The girl gets off the plank. A boulder of mass *m* kg is placed on the plank at *A* and a man of mass 93 kg stands on the plank at *B*. The plank remains in equilibrium and is on the point of tilting about *C*.

b By modelling the plank again as a uniform rod, and the man and the boulder as particles, find the value of *m*.

Solution:

a

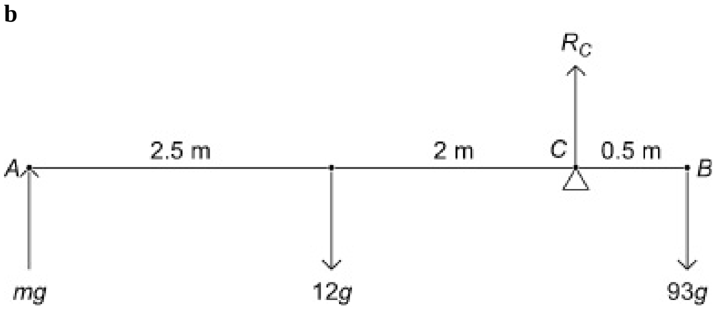


Taking moments about *C*: $R_A \times 4.5 + 30g \times 0.5 = 12g \times 2$

$$R_A \times 4.5 = 24g - 15g = 9g$$

$$\Rightarrow R_A = 2g = 19.6\ N$$

b



The plank is about to tilt about *C* $\Rightarrow$ reaction at $A = 0$

Taking moments about *C*: $mg \times 4.5 + 12g \times 2 = 93g \times 0.5$

$\Rightarrow 4.5m = 93 \times 0.5 - 24 = 22.5$, $m =$ ie 5

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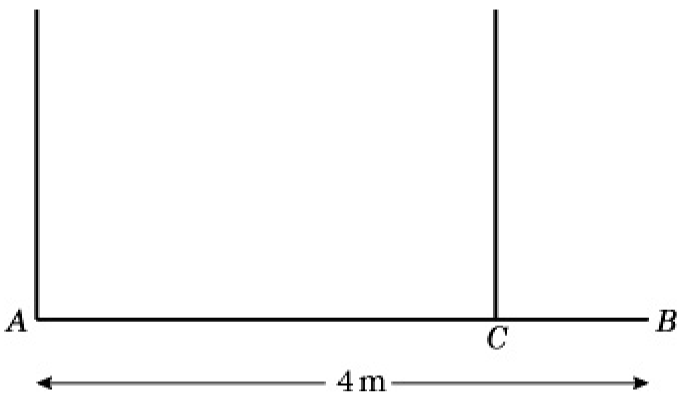
Moments
Exercise E, Question 8

Question:

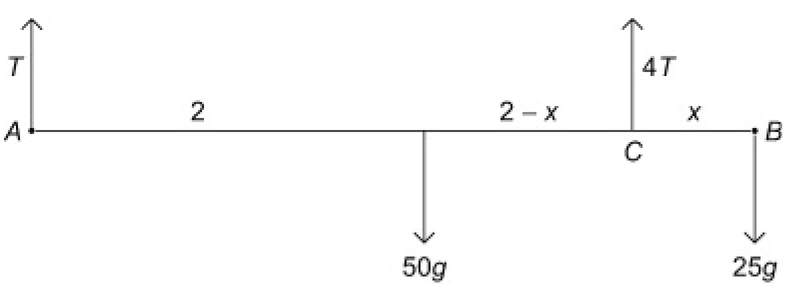


A plank AB has mass 50 kg and length 4 m. A load of mass 25 kg is attached to the plank at B. The loaded plank is held in equilibrium, with AB horizontal, by two vertical ropes attached at A and C, as shown in the diagram. The plank is modelled as a uniform rod and the load as a particle. Given that the tension in the rope at C is four times the tension in the rope at A, calculate

a the tension in the rope at C,

b the distance CB.

Solution:



a Let the tension in the rope at A be T N

R(↑) $T + 4T = 50g + 25g$, $5T = 75g$
$\Rightarrow$ $T = 15g$, so tension at C is $60g$ N = 588 N

b Let the distance BC be x

Taking moments about C: $15g \times (4 - x) + 25g \times x = 50g \times (2 - x)$

$$60 - 15x + 25x = 100 - 50x$$

$$60x = 40 , x = \frac{2}{3} \text{ m}$$

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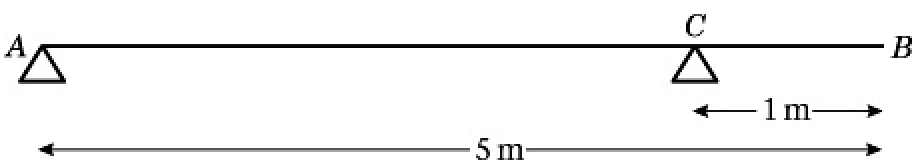
Moments
Exercise E, Question 9

Question:



A uniform beam AB has weight 200 N and length 5 m. The beam rests in equilibrium in a horizontal position on two smooth supports. One support is at end A and the other is at a point C on the beam, where $BC = 1$ m, as shown in the diagram. The beam is modelled as a uniform rod.

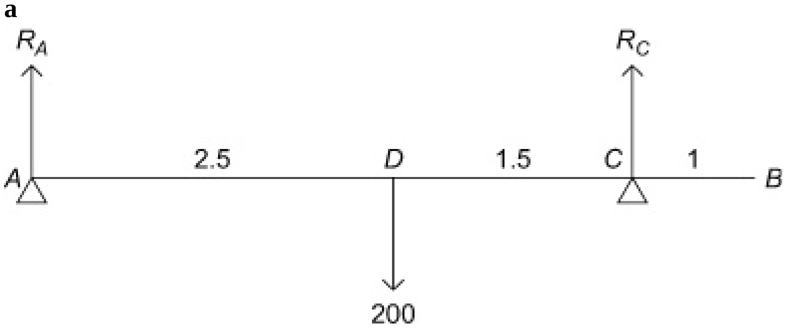
a Find the reaction on the beam at C.

A woman of weight 500 N stands on the beam at the point D. The beam remains in equilibrium. The reactions on the beam at A and C are now equal.

b Find the distance AD.

Solution:

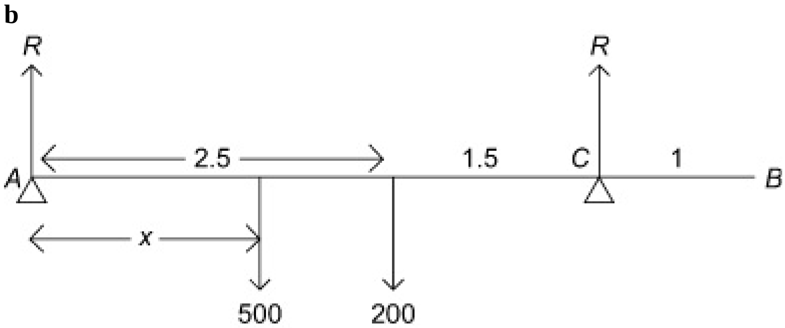
a



Taking moments about A:

$$200 \times 2.5 = R_C \times 4$$
$$R_C = 125 \text{ N}$$

b



Let the distance AD be x

R(↑) $2R = 500 + 200 = 700$

$R = 350$ N

Taking moments about A: $R \times 4 = 200 \times 2.5 + 500 \times x$,

$1400 = 4R = 500 + 500x$, $900 = 500x$, $x = 1.8$ m