Practice exam paper

1 a P has speed 0.2 m s^{-1} and Q has speed 0.1 m s^{-1}.
So the distance between P and Q is reducing at a rate of 0.1 m s^{-1}.
As the particles were initially 0.8 m apart it takes
$0.8 \div 0.1 = 8$ s for them to collide.
Alternative method:
Giving distances from initial position of P.
For P: distance = $0.2t$
For Q: distance = $0.8 + 0.1t$
So, P and Q collide when $0.2t = 0.8 + 0.1t$.
$0.1t = 0.8$
$t = 8$ seconds

b By the conservation of momentum,
momentum before = momentum after
$4\times0.2+2\times0.1=4\times0.1+2v$
$2v=0.6$
$v = 0.3$ m s^{-1}

2 a i The train starts to move at t_o then accelerates, with constant acceleration until t_1.

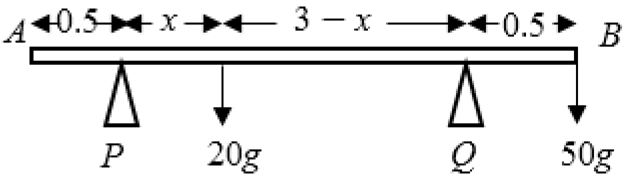
ii The train is travelling with constant velocity.

iii The train decelerates,with constant deceleration, until it is stationary at t_3.

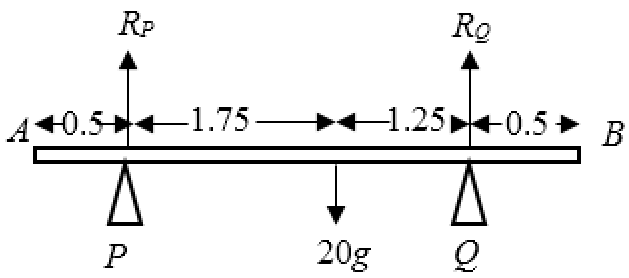
b $80(t_2-t_1)=120$
$t_2-t_1=1.5$
Therefore each unit on the x-axis represents 30 minutes.
So the total length of the journey is 4 hours.

c Total distance travelled d is given by
$d=\frac{1}{2}\times1\times80+120+\frac{1}{2}\times1.5\times80$
= 220 km

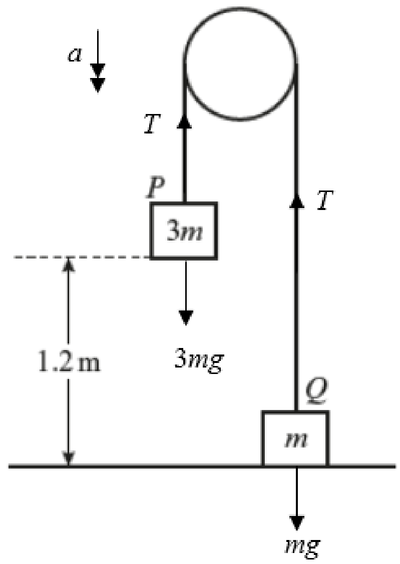
3 a Taking moments about Q
$50g \times 0.5 = 20g \times (3 – x)$
$25 = 20(3 – x)$
$3 – x = 1.25$
$x = 1.75$
Therefore the centre of mass is
$0.5 + 1.75 = 2.25$ m from A



3 b Taking moments about P,
$1.75 \times 20g = 3R_Q$
$R_Q = \frac{35}{3}g$ N
Taking moments about Q,
$1.25 \times 20g = 3R_P$
$R_P = \frac{25}{3}g$ N

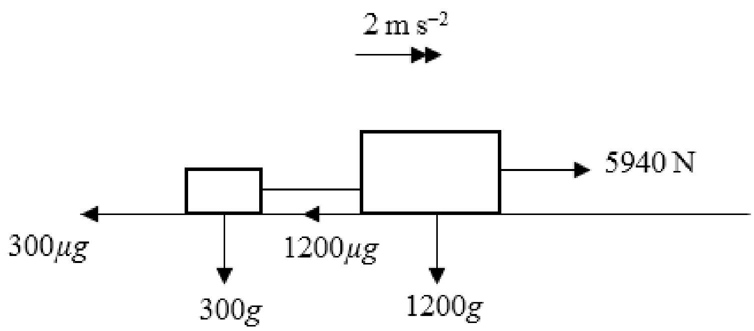


4 a Using Newton's second law, $\mathbf{F} = m\mathbf{a}$
For the $3m$ kg mass,
(↓) $3mg - T = 3ma$ **(1)**
For the 1 kg mass,
(↑) $T - mg = ma$
$T = ma + mg$ **(2)**
Substituting **(2)** into **(1)** gives
$3mg - (ma + mg) = 3ma$
$2mg = 4ma$
$a = 0.5g$ m s^{-2}



b Using $s = ut + \frac{1}{2}at^2$ gives
$1.2 = \frac{1}{2}(0.5g)t^2$
$t^2 = \frac{2.4}{0.5g}$
$t = \frac{2\sqrt{6}}{7}$
$= 0.700$ s (3 s.f.)

5 a Using Newton's second law, $\mathbf{F} = m\mathbf{a}$,
on the system
(→) $5940 - 1500\mu g = 1500 \times 2$
$1500\mu g = 5940 - 3000$
$\mu = \frac{5940 - 3000}{1500g}$
$= 0.2$ as required



b For the smaller sledge
(→) $T - 300\mu g = 300a$
$T = 300a + 300\mu g$
$= 300(2) + 300(0.2)g$
$= 1188$ N

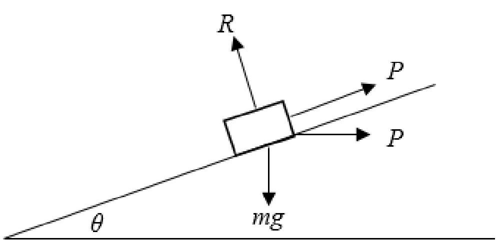
c Constant acceleration/ constant force/ constant friction

d Using $v = u + at$ at point the bar snaps gives
$v = 2(5) = 10\text{ m s}^{-1}$
Using Newton's second law, $\mathbf{F} = m\mathbf{a}$
$(\rightarrow)\ -300\mu g = 300a$
$a = -\mu g$
$\quad = -0.2g\text{ m s}^{-2}$
Using $v^2 = u^2 + 2as$ gives
$0 = 10^2 + 2(-0.2g)s$
$s = \dfrac{100}{0.4g}$
$\quad = 25.5\text{ m (3 s.f.)}$

6 a Between A and B
$|\mathbf{v}| = \sqrt{5^2 + 4^2}$
$\quad = \sqrt{41}\text{ km h}^{-1}$
Between B and C
$|\mathbf{v}| = \sqrt{8^2 + (-2)^2}$
$\quad = \sqrt{68}\text{ km h}^{-1}$
Total distance travelled is
$\left(3\sqrt{41} + 4\sqrt{68}\right)$ km
Since the ship travels this distance in 7 hours, the average speed between A and C is
$\dfrac{\left(3\sqrt{41} + 4\sqrt{68}\right)}{7} = 7.46\text{ km h}^{-1}\text{ (3 s.f.)}$

b Let port A be the origin, then the position vector of port C is
$\mathbf{r} = 3(5\mathbf{i} + 4\mathbf{j}) + 4(8\mathbf{i} - 2\mathbf{j})$
$\quad = 47\mathbf{i} + 4\mathbf{j}$
$|\mathbf{r}| = \sqrt{47^2 + 4^2}$
$\quad = \sqrt{2225}$
$\quad = 5\sqrt{89}$ km
Since the ship is travelling at 10 km h^{-1}, the time taken for the ship to reach A is
$\dfrac{5\sqrt{89}}{10} = 4.72$ hours (3 s.f.)

7 a Using Newton's second law, $\mathbf{F} = m\mathbf{a}$
$(\nearrow)\ P + P\cos\theta = mg\sin\theta$
$P(1 + \cos\theta) = mg\sin\theta$
$P = \dfrac{mg\sin\theta}{1 + \cos\theta}$ as required



7 b Using Newton's second law, $\mathbf{F} = m\mathbf{a}$

(↖) $R = P \sin\theta + mg\cos\theta$

$$R = \frac{mg\sin^2\theta}{1+\cos\theta} + mg\cos\theta$$

$$= \frac{mg\sin^2\theta + mg\cos\theta(1+\cos\theta)}{1+\cos\theta}$$

$$= \frac{mg\sin^2\theta + mg\cos\theta + mg\cos^2\theta}{1+\cos\theta}$$

$$= \frac{mg + mg\cos\theta}{1+\cos\theta}$$

$$= \frac{mg(1+\cos\theta)}{1+\cos\theta}$$

$= mg$ as required

c Using Newton's second law, $\mathbf{F} = m\mathbf{a}$

(↙) $mg\sin 30 - 0.25mg = ma$

$a = 0.5g - 0.25g$

$a = 0.25g$

Therefore initial acceleration is $0.25g\ \text{m s}^{-2}$ down the slope.