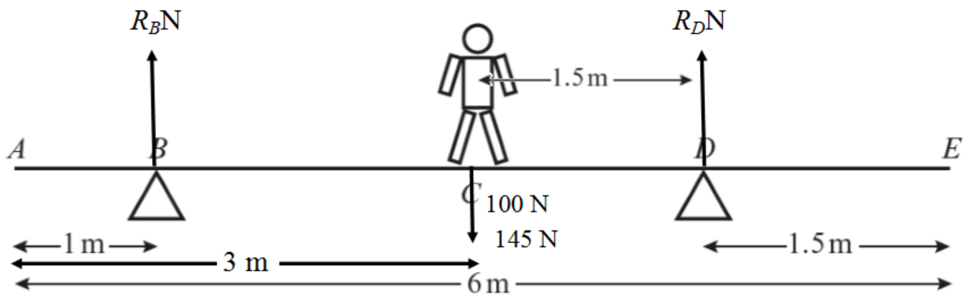
Chapter review 8

1 a



The plank is in equilibrium.
Let the reaction forces at the supports be R_B and R_D.
Considering moments about point D:

$$R_B \times (6-1.5-1) = (100+145)\times(3-1.5)$$
$$3.5R_B = 245\times 1.5$$
$$3.5R_B = 367.5$$
$$R_B = 105$$

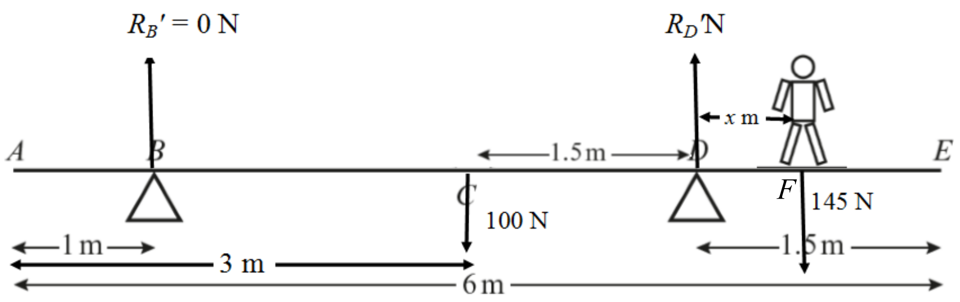
The support at B exerts a force of 105 N on the plank.

b The plank is in equilibrium.
Resolving vertically:

$$R_B + R_D = 100+145$$
$$R_D = 245-105$$
$$R_D = 140$$

The support at D exerts a force of 140 N on the plank.

c

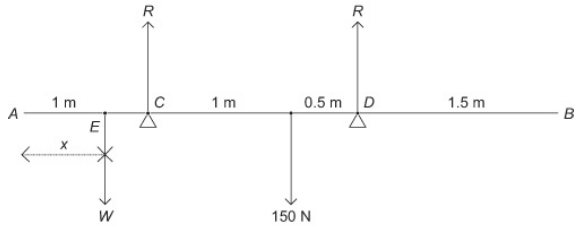


When the plank is on the point of tilting, the new reaction force at support B, $R'_B = 0$ N and plank is again in equilibrium. The child stands a distance x m from support D.
Considering moments about point D:

$$145x = 100\times(3-1.5)$$
$$145x = 150$$
$$x = 1.03$$

The distance DF is 1.03 m.

2



a Since the rod is uniform, the centre of mass is at the mid-point.
Taking moments about A:

$$Wx + 150 \times 2 = R \times 1 + R \times 2.5,$$

$$Wx + 300 = 3.5R \qquad \textbf{(1)}$$

$$R(\uparrow):\ W + 150 = R + R,$$

$$2R = W + 150$$

$$R = \frac{W + 150}{2} \qquad \textbf{(2)}$$

Sub (**2**) into (**1**) gives:

$$Wx + 300 = \frac{7}{2} \times \frac{W + 150}{2}$$

$$4(Wx + 300) = 7W + 7 \times 150$$

$$4Wx + 1200 = 7W + 1050$$

$$1200 - 1050 = 7W - 4Wx$$

$$W(7 - 4x) = 150$$

$$W = \frac{150}{7 - 4x}$$

b The range of values of x are:

$$x \geq 0 \text{ and } \frac{150}{7 - 4x} > 0$$

$$\Rightarrow 7 - 4x > 0$$

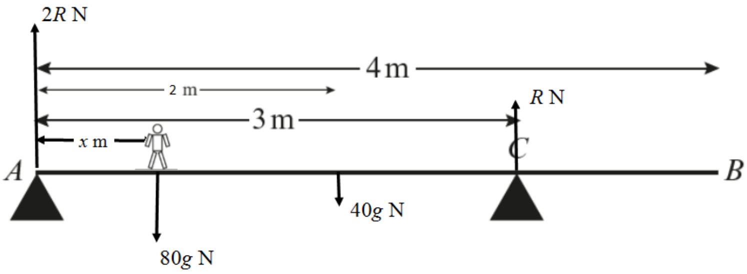
$$4x < 7$$

$$x < \frac{7}{4}$$

$$x < 1.75$$

So $0 \leq x < 1.75$

3 a



The plank is in equilibrium.

Resolving vertically:

$2R + R = 40g + 80g$

$3R = 120 \times 9.8$

$3R = 1176$

$R = 392$

The value of R is 392 N.

b Taking moments about A:

$80gx + (40g \times 2) = 3R$

$80g(x+1) = 3 \times 392$

$x + 1 = \dfrac{1176}{80 \times 9.8}$

$x + 1 = 1.5$

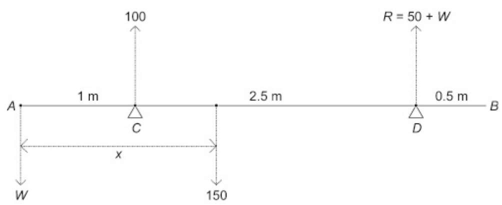
The man stands 0.5 m from A.

c i Assuming the plank is uniform means the weight of the plank acts at its centre of mass: i.e. halfway along the plank.

ii Assuming the plank is a rod means its width can be ignored.

iii Assuming the man is a particle means all his weight acts at the point at which he stands.

4 a



$R(\uparrow)$:

$100 + R = W + 150$

$R = W + 50$

Taking moments about A:

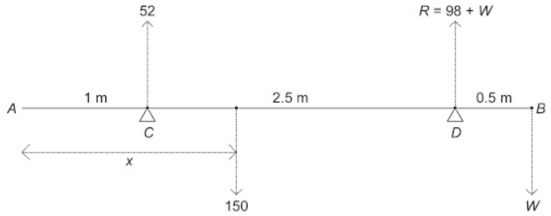
$100 \times 1 + (W + 50) \times 3.5 = 150 \times x$

$100 + 175 + 3.5W = 150x$

$275 + 3.5W = 150x$

$550 + 7W = 300x$

4 b



$R(\uparrow)$

$$52 + R = 150 + W$$
$$R = 150 + W - 52$$
$$= 98 + W$$

Taking moments about B:

$$52 \times 3 + (98 + W) \times 0.5 = 150 \times (4 - x)$$
$$156 + 49 + 0.5W = 600 - 150x$$

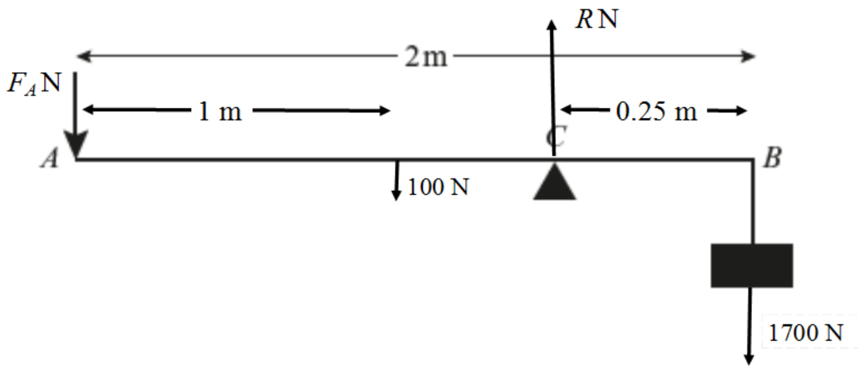
Doubling,

$$410 + W = 1200 - 300x$$
$$W = 790 - 300x$$

c Solving the simultaneous equations obtained in **a** and **b**:

$$\Rightarrow W = 790 - (550 + 7W)$$
$$8W = 790 - 550$$
$$= 240$$
$$\Rightarrow W = 30$$
$$\Rightarrow 410 + 30 = 1200 - 300x$$
$$300x = 760$$
$$x = 2.53 \text{ (3 s.f.)}$$

5 a

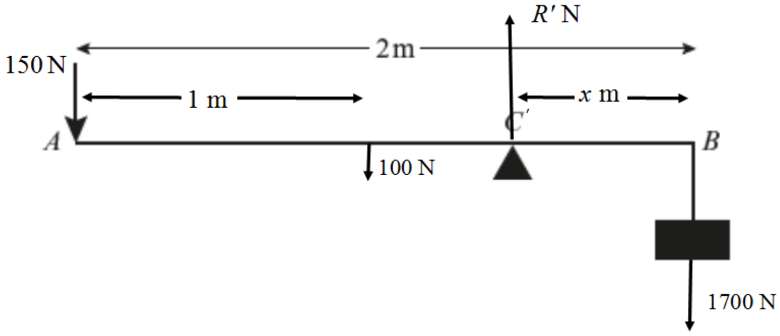


The lever is in equilibrium.
Considering moments about point C:

$$F_A(2 - 0.25) + 100(1 - 0.25) = 1700 \times 0.25$$
$$1.75F_A + 75 = 425$$
$$F_A = \frac{425 - 75}{1.75}$$
$$F_A = 200$$

The force at A is 200 N.

5 b

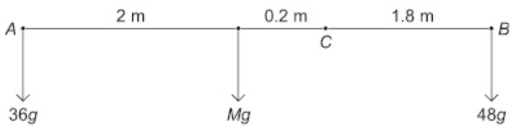


The lever is again in equilibrium. Let x be the distance of the pivot from B.
Considering moments about the new support position C':

$$150(2-x)+100(1-x)=1700x$$
$$300-150x+100-100x=1700x$$
$$400=1700x+250x$$
$$400=1950x$$
$$x=0.205$$

The pivot is now 0.21 m from B (to the nearest cm).

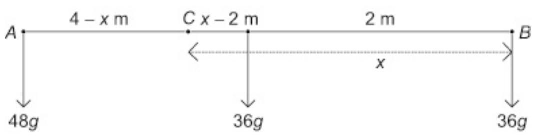
6 a Let the mass of the plank be M. Since the plank is uniform, its centre of mass is at its mid-point.



Taking moments about C:

$$48g\times1.8=Mg\times0.2+36g\times2.2$$
$$86.4g=0.2Mg+79.2g$$
$$86.4=0.2M+79.2$$
$$0.2M=86.4-79.2$$
$$=7.2$$
$$\Rightarrow M=36\text{kg}$$

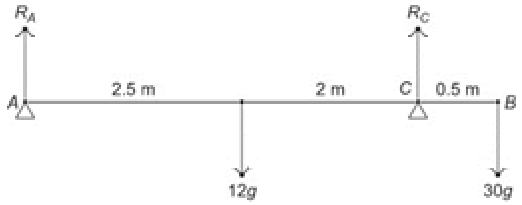
b Let the distance BC be x



Taking moments about C:

$$36gx+36g(x-2)=48g(4-x)$$
$$3x+3(x-2)=4(4-x)$$
$$6x-6=16-4x$$
$$10x=22$$
$$x=2.2\text{ m}$$

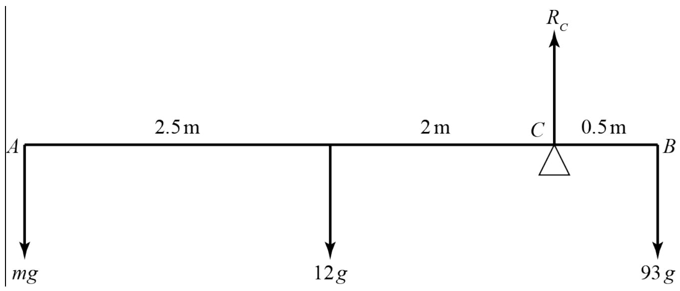
7 a



Taking moments about C:

$$R_A \times 4.5 + 30g \times 0.5 = 12g \times 2$$
$$R_A \times 4.5 = 24g - 15g$$
$$= 9g$$
$$\Rightarrow R_A = 2g$$
$$= 19.6\,\text{N}$$

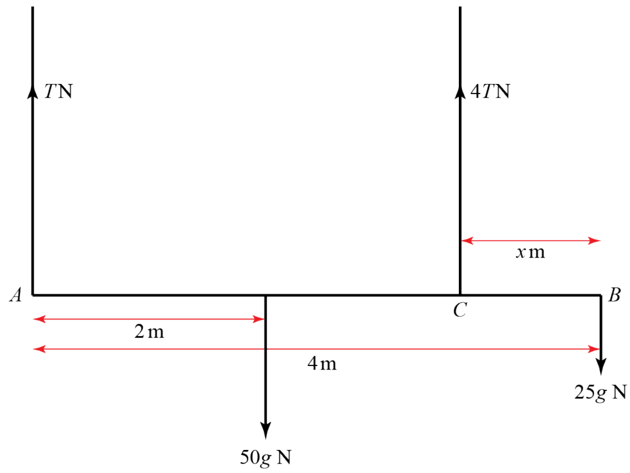
b



The plank is about to tilt about C
$\Rightarrow$reaction at $A = 0$
Taking moments about C:

$$mg \times 4.5 + 12g \times 2 = 93g \times 0.5$$
$$4.5m = 93 \times 0.5 - 24$$
$$= 22.5$$
$$m = 5$$

8 The plank is in equilibrium.



Resolving vertically:

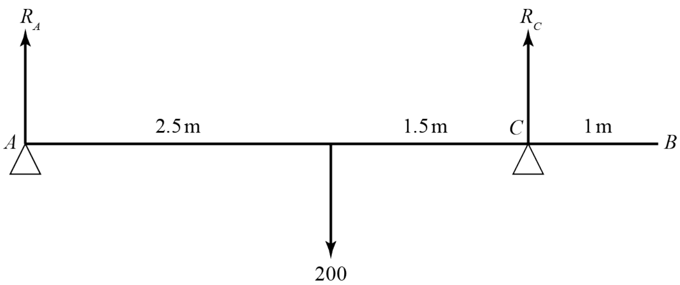
$$T + 4T = 50g + 25g$$
$$5T = 75g$$
$$T = 15g$$
$$4T = 60g$$

Considering moments about B:

$$(50g \times 2) = 60gx + (15g \times 4)$$
$$100g = 60gx + 60g$$
$$100g - 60g = 60gx$$
$$x = \frac{40g}{60g}$$
$$x = 0.666...$$

The distance from B to C is 0.67 m (to the nearest cm).

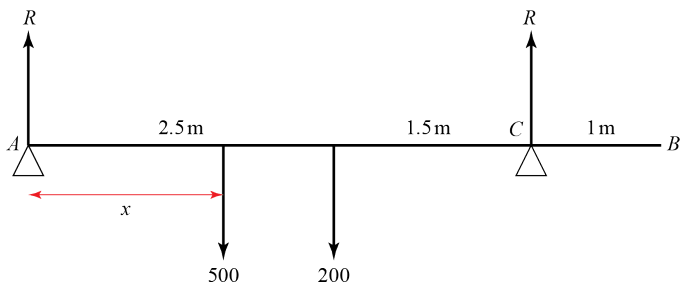
9 **a**



Taking moments about A:

$$200 \times 2.5 = R_C \times 4$$
$$R_C = 125\text{N}$$

9 b



Let the distance AD be x

$R(\uparrow)$

$2R = 500 + 200$

$= 700$

$R = 350\text{ N}$

Taking moments about A:

$R \times 4 = 200 \times 2.5 + 500 \times x$

$1400 = 4R$

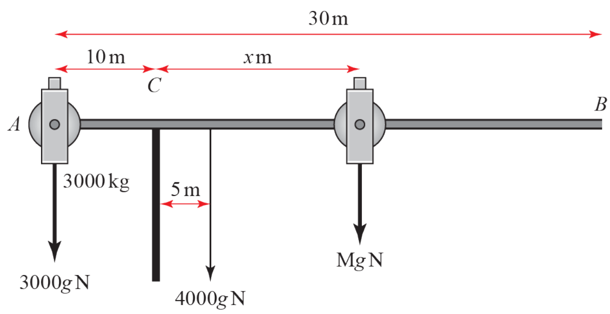
$= 500 + 500x$

$900 = 500x$

$x = 1.8\text{m}$

10 a Centre of mass of beam is 5 m from C.

Taking moments about C:



$3000g \times 10 = (4000g \times 5) + Mgx$

$30\,000 = 20\,000 + Mx$

$M = \dfrac{10\,000}{x}$

b Maximum load is when $x = 5$ m:

$M = \dfrac{10\,000}{5} = 2000\text{ kg}$

Minimum load is when $x = 20$ m:

$M = \dfrac{10\,000}{20} = 500\text{ kg}$

c It is not very accurate to model the beam as a uniform rod. Since the beam may taper at one end, the centre of mass of the beam may not lie in the middle of the beam.

Challenge

1 a When force is a minimum, system is in limiting equilibrium.
Taking moments about P:

$F_A \times (A'B') = 1200 \times PC'$ **(1)**

Finding $A'B'$:

$A'B = 2\cos 20^\circ$

$BB' = 1\sin 20^\circ$

$\therefore A'B' = 2\cos 20^\circ + \sin 20^\circ$

Finding PC':

$PC' = PC\cos(\theta + 20)$

$(PC)^2 = 1^2 + 0.5^2$

$PC = \sqrt{1.25}$

$\tan\theta = \frac{1}{0.5}$

$\theta = 63.434...^\circ$

$PC' = \sqrt{1.25} \times \cos(63.4 + 20)^\circ$

$PC' = \sqrt{1.25} \times \cos 83.434...^\circ$

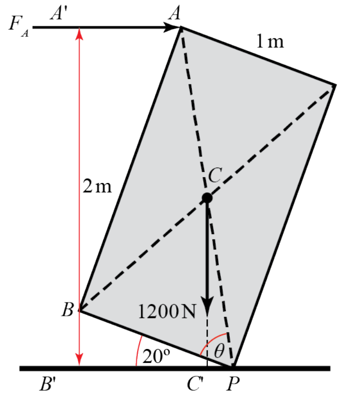
Substituting values for $A'B'$ and PC' into equation **(1)**

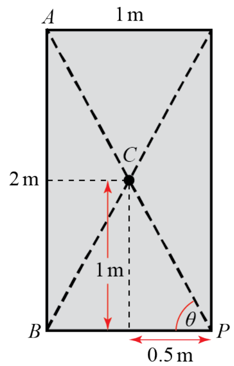
$F_A \times \left(2\cos 20^\circ + \sin 20^\circ\right) = 1200 \times \sqrt{1.25} \times \cos 83.434...^\circ$

$$F_A = \frac{1200 \times \sqrt{1.25} \times \cos 83.434...^\circ}{2\cos 20^\circ + \sin 20^\circ}$$

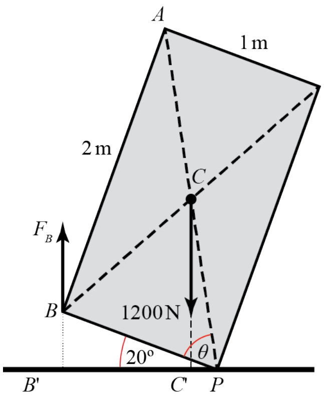
$F_A = 69.051...$

A horizontal force of 69.0 N at A will tip the refrigerator (3 s.f.).





b



When force is a minimum, system is in limiting equilibrium.
Taking moments about P:

$F_B \times (PB') = 1200 \times \sqrt{1.25} \times \cos 83.434...^\circ$

$F_B \times 1\cos 20^\circ = 1200 \times \sqrt{1.25} \times \cos 83.434...^\circ$

$$F_B = \frac{1200 \times \sqrt{1.25} \times \cos 83.434...^\circ}{\cos 20^\circ}$$

$F_B = 163.25...$

A vertical force of 163 N at B will tip the refrigerator (3 s.f.).