Chapter review 7

1 a Finding the components of P along each axis:

$R(\rightarrow): P_x = 12\cos 70^\circ + 10\sin 75^\circ$

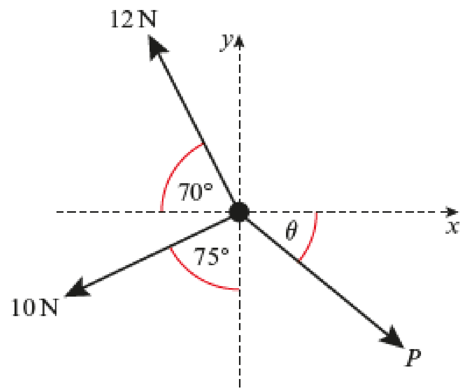
$R(\uparrow): P_y = 12\sin 70^\circ - 10\cos 75^\circ$

$\tan\theta = \frac{P_y}{P_x}$

$\tan\theta = \frac{12\sin 70^\circ - 10\cos 75^\circ}{12\cos 70^\circ + 10\sin 75^\circ} = 0.63124...$

$\theta = 32.261...$

The angle θ is 32.3° (to 3 s.f.).



b Using Pythagoras' theorem:

$P^2 = P_x^2 + P_y^2$

$P^2 = (12\cos 70^\circ + 10\sin 75^\circ)^2 + (12\sin 70^\circ - 10\cos 75^\circ)^2$

$P = \sqrt{264.91...} = 16.276...$

P has a magnitude of 16.3 N (3 s.f.).

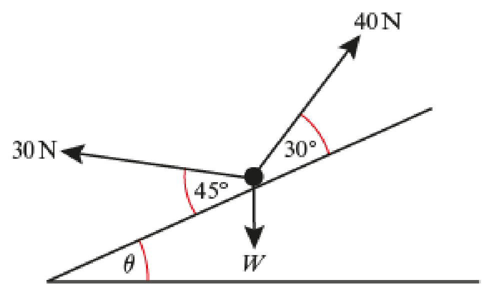
2 a $R(\nwarrow)$:

$W\cos\theta = 40\sin 30^\circ + 30\sin 45^\circ$

$R(\swarrow)$:

$30\cos 45^\circ + W\sin\theta = 40\cos 30^\circ$

$W\sin\theta = 40\cos 30^\circ - 30\cos 45^\circ$



$\frac{W\sin\theta}{W\cos\theta} = \frac{40\cos 30^\circ - 30\cos 45^\circ}{40\sin 30^\circ + 30\sin 45^\circ}$

$\tan\theta = \frac{20\sqrt{3} - 15\sqrt{2}}{20 + 15\sqrt{2}} = 0.35281...$

$\theta = 18.046...$

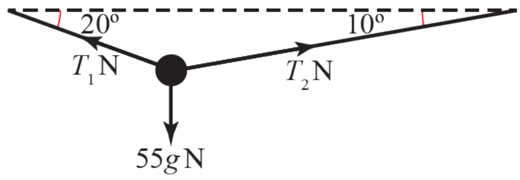
The angle θ is 18.0° (to 3 s.f.).

b Using Pythagoras' theorem, $|W|^2$ is the sum of the squares of the two components.

$W^2 = (20\sqrt{3} - 15\sqrt{2})^2 + (20 + 15\sqrt{2})^2$

$W = \sqrt{1878.8...} = 43.345...$

The weight of the particle is 43.3 N (3 s.f.).



3 Resolving horizontally:

$T_1 \cos 20^\circ = T_2 \cos 10^\circ$

$T_2 = \dfrac{T_1 \cos 20^\circ}{\cos 10^\circ}$ **(1)**

Resolving vertically:

$T_1 \sin 20^\circ + T_2 \sin 10^\circ = 55g$ **(2)**

Substituting $T_2 = \dfrac{T_1 \cos 20^\circ}{\cos 10^\circ}$ from (**1**) into (**2**):

$T_1 \sin 20^\circ + \dfrac{T_1 \cos 20^\circ \times \sin 10^\circ}{\cos 10^\circ} = 55g$

$T_1(\sin 20^\circ + \cos 20^\circ \tan 10^\circ) = 55g$

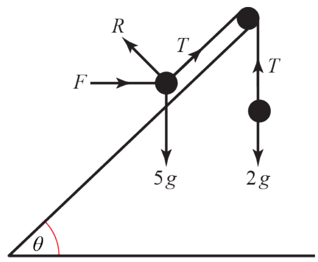
$T_1 = \dfrac{55 \times 9.8}{\sin 20^\circ + \cos 20^\circ \tan 10^\circ} = 1061.6...$

Subsituting this value of T_1 into (**1**):

$T_2 = \dfrac{\cos 20^\circ \times 1061.6...}{\cos 10^\circ} = 1012.9...$

The tensions in the two parts of the rope are 1062 N and 1013 N (nearest whole number).

4 **a** For the 2 kg particle



$R(\uparrow)$

$T - 2g = 0$

$\therefore T = 2g$

For the 5 kg particle:

$R(\nearrow)$

$T + F\cos\theta - 5g\sin\theta = 0$

$\therefore F\cos\theta = 5g\sin\theta - T$

As $T = 2g$, $\cos\theta = \dfrac{4}{5}$ and $\sin\theta = \dfrac{3}{5}$

$F \times \dfrac{4}{5} = 5g \times \dfrac{3}{5} - 2g$

$\therefore F = g \times \dfrac{5}{4}$

$= \dfrac{5g}{4}$

$= 12$ (2 s.f.)

b $R(\nwarrow)$

$R - F\sin\theta - 5g\cos\theta = 0$

$\therefore R = F\sin\theta + 5g\cos\theta$

$= \left(\dfrac{5}{4}g \times \dfrac{3}{5}\right) + \left(5g \times \dfrac{4}{5}\right)$

$= \dfrac{19}{4}g$

$= 47$ (2 s.f.)

4 c F will be smaller

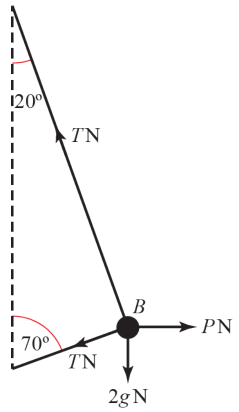
5 a Resolving vertically:

$$T\cos 20^\circ = T\cos 70^\circ + 2g$$

$$T(\cos 20^\circ - \cos 70^\circ) = 2g$$

$$T = \frac{2\times 9.8}{\cos 20^\circ - \cos 70^\circ} = 32.793...$$

The tension in the string, to two significant figures, is 33 N.



b Resolving horizontally:

$$P = T\sin 20^\circ + T\sin 70^\circ$$

$$P = (\sin 20^\circ + \sin 70^\circ)\times 32.793...$$

$$P = 42.032...$$

The value of P is 42 N (2 s.f.).

6 Res$(\nwarrow)$ $R = mg\cos\theta$

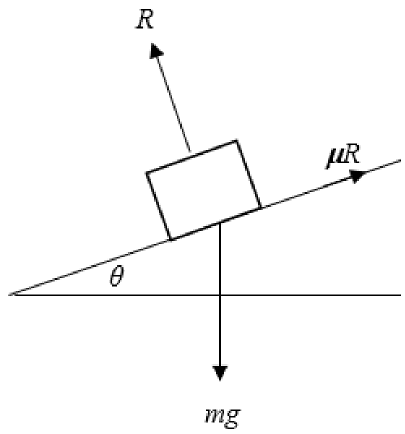
When the box is on the point of slipping

Res$(\nearrow)$ $\mu R = mg\sin\theta$

$$\frac{\mu R}{R} = \frac{mg\sin\theta}{mg\cos\theta}$$

$$\mu = \tan\theta$$

$\theta = \tan^{-1}\mu$ as required



7 Assuming that force is applied so that it acts in a downwards direction:

Res$(\uparrow)$ $R = mg + \frac{1}{5}mg\sin\theta$

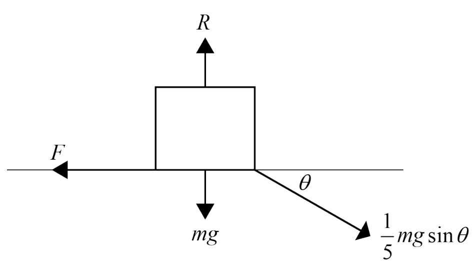
Res$(\rightarrow)$ $\frac{1}{5}mg\cos\theta = F$

Particle remains stationary so $F \le \mu R$

$$\mu R \ge \frac{mg\cos\theta}{5}$$

$$\mu \ge \frac{mg\cos\theta}{5\left(mg + \frac{1}{5}mg\sin\theta\right)}$$

$\mu \ge \dfrac{\cos\theta}{5+\sin\theta}$ as required



8 Assuming that force is applied so that it acts in an upwards direction:

Res$(\uparrow)$ $R + 5\sin\theta = 5g$

$R = 5g - 5\sin\theta$

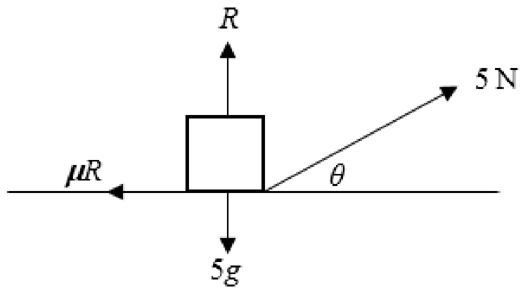
If particle is on the point of slipping

Res$(\rightarrow)$ $5\cos\theta = \mu R$

Therefore,

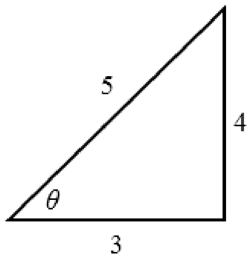
$$\mu = \frac{5\cos\theta}{5g - 5\sin\theta}$$

$$= \frac{\cos\theta}{g - \sin\theta} \quad \textbf{(1)}$$



Since $\theta = \tan^{-1}\left(\frac{4}{3}\right)$

$$\tan\theta = \frac{4}{3}$$



$\sin\theta = \frac{4}{5}$ and $\cos\theta = \frac{3}{5}$

Substituting into (1) gives

$$\mu = \frac{\cos\theta}{g - \sin\theta}$$

$$= \frac{\frac{3}{5}}{g - \frac{4}{5}}$$

$$= \frac{3}{5g - 4}$$

$$= \frac{1}{15} \text{ as required}$$

9 If the system is stationary

For m

$\text{Res}(\uparrow)\ T = mg$

For M

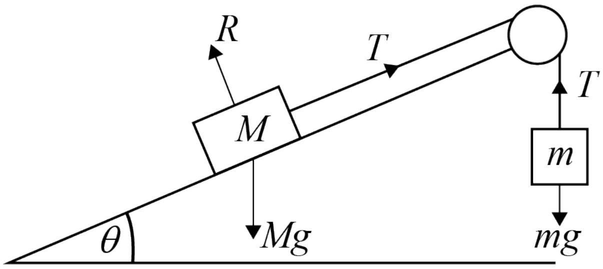
$\text{Res}(\nearrow)\ T = Mg\sin\theta$

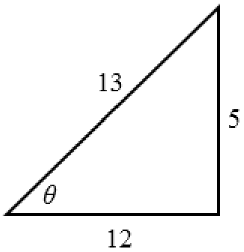
$mg = Mg\sin\theta$

$\frac{m}{M} = \sin\theta$

Since $\theta = \tan^{-1}\left(\frac{5}{12}\right)$

$\tan\theta = \frac{5}{12}$





$\sin\theta = \frac{5}{13}$

Therefore

$\frac{m}{M} = \frac{5}{13}$ as required