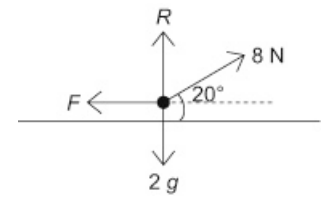


Exercise 7C

- 1 Let the normal reaction be R N, the friction force be F N and the coefficient of friction be μ .

Resolve horizontally to find F , vertically to find R and use $F = \mu R$ to find μ :



$$R(\rightarrow)$$

$$8\cos 20^\circ - F = 0$$

$$\therefore F = 8\cos 20^\circ$$

$$R(\uparrow)$$

$$R + 8\sin 20^\circ - 2g = 0$$

$$\therefore R = 2g - 8\sin 20^\circ$$

As the book is on the point of slipping the friction is limiting:

$$F = \mu R$$

$$\therefore \mu = \frac{F}{R}$$

$$= \frac{8\cos 20^\circ}{2g - 8\sin 20^\circ}$$

$$= \frac{7.518}{16.86}$$

$$= 0.446 \text{ (3 s.f.)}$$

- 2 Let the normal reaction be R N, the friction force be F N and the coefficient of friction be μ .

$$R(\rightarrow): 6\cos 30^\circ - F = 0$$

$$F = 6\cos 30^\circ$$

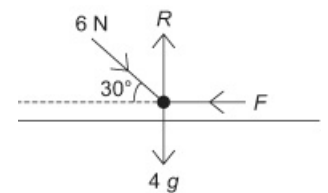
$$= 3\sqrt{3} = 5.20 \text{ (3 s.f.)}$$

$$R(\uparrow): R - 6\sin 30^\circ - 4g = 0$$

$$R = 6\sin 30^\circ + 4g$$

$$= 3 + 4 \times 9.8$$

$$= 42.2$$



As the block is on the point of slipping

$$F = \mu R$$

$$\therefore \mu = \frac{F}{R}$$

$$= 0.123 \text{ (3 s.f.)}$$

3 Let the normal reaction force be R and the friction force be F .

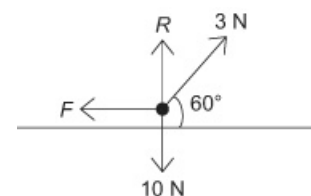
- a Resolve horizontally to find the magnitude of the friction force necessary to maintain equilibrium:

$$R(\rightarrow)$$

$$3 \cos 60^\circ - F = 0$$

$$\therefore F = 3 \cos 60^\circ$$

$$F = 1.5 \text{ N}$$



- b Resolve vertically to calculate R and hence μR :

$$R(\uparrow)$$

$$R + 3 \sin 60^\circ - 10 = 0$$

$$\therefore R = 10 - 3 \sin 60^\circ$$

$$= 10 - \frac{3\sqrt{3}}{2}$$

$$= 7.40 \text{ (3 s.f.)}$$

$$\therefore \mu R = 0.3 \times 7.40$$

$$= 2.22 \text{ (3 s.f.)}$$

Since $F = 1.5 \text{ N} < 2.2 \text{ N} = \mu R$, the friction required to maintain equilibrium is not limiting friction.

- 4 a Let the normal reaction be $R \text{ N}$ and the friction force required to maintain equilibrium be $F \text{ N}$.

Let the mass of the books be $m \text{ kg}$.

$$R(\rightarrow)$$

$$147 - F = 0$$

$$\therefore F = 147 \text{ N}$$

$$R(\uparrow)$$

$$R - 10g - mg = 0$$

$$\therefore R = 10g + mg$$

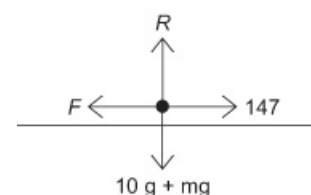
As the equilibrium is limiting, $F = \mu R$

$$147 = 0.3(10g + mg)$$

$$147 = 3g + 0.3mg$$

$$\therefore m = \frac{147 - 3g}{0.3g}$$

$$= 40 \text{ kg}$$



- b The assumption is that the crate and books may be modelled as a particle.

- 5 a Let R be the normal reaction and F be the force of friction when P acts downwards.

$$R(\rightarrow)$$

$$P \cos 45^\circ - F = 0$$

$$\therefore F = P \cos 45^\circ$$

$$R(\uparrow)$$

$$R - P \sin 45^\circ - 2g = 0$$

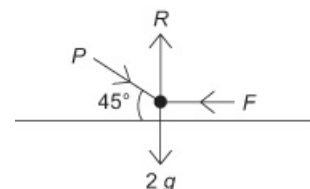
$$\therefore R = P \sin 45^\circ + 2g$$

As the equilibrium is limiting, $F = \mu R$

$$\therefore P \cos 45^\circ = 0.3(P \sin 45^\circ + 2g)$$

$$P(\cos 45^\circ - 0.3 \sin 45^\circ) = 0.6g$$

$$\begin{aligned} \therefore P &= \frac{0.6g}{\cos 45^\circ - 0.3 \sin 45^\circ} \\ &= \frac{6g\sqrt{2}}{7} \\ &= 11.9 \text{ N (3 s.f.)} \end{aligned}$$



Resolve horizontally and vertically to find F and R , then use the condition for limiting friction.

- b Let R' be the normal reaction and F' be the force of friction when P acts upwards.

$$R(\rightarrow)$$

$$P \cos 45^\circ - F' = 0$$

$$\therefore F' = P \cos 45^\circ$$

$$R(\uparrow)$$

$$R' + P \sin 45^\circ - 2g = 0$$

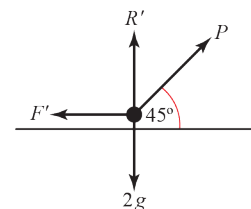
$$\therefore R' = 2g - P \sin 45^\circ$$

As the equilibrium is limiting, $F = \mu R$

$$\therefore P \cos 45^\circ = 0.3(2g - P \sin 45^\circ)$$

$$P(\cos 45^\circ + 0.3 \sin 45^\circ) = 0.6g$$

$$\begin{aligned} \therefore P &= \frac{6g\sqrt{2}}{13} \\ &= 6.40 \text{ N (3 s.f.)} \end{aligned}$$



- 6 Let R be the normal reaction and F be the force of friction required to maintain equilibrium.

Since the particle is on the point of slipping up the plane, the force of friction acts down the slope.

$$R(\nearrow)$$

$$3 - F - 0.3g \sin 30^\circ = 0$$

$$\begin{aligned} \therefore F &= 3 - 0.3g \sin 30^\circ \\ &= 1.53 \text{ N} \end{aligned}$$

$$R(\nwarrow)$$

$$R - 0.3g \cos 30^\circ = 0$$

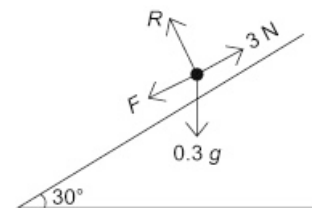
$$\begin{aligned} \therefore R &= 0.3g \cos 30^\circ \\ &= 2.546 \text{ N} \end{aligned}$$

As the particle is on the point of slipping, $F = \mu R$

$$\therefore 1.53 = \mu \times 2.546$$

$$\therefore \mu = \frac{1.53}{2.546}$$

$$= 0.601 \text{ (3 s.f.) (accept 0.6)}$$



- 7 Let R be the normal reaction and F be the force of friction required to maintain equilibrium.

Since the particle is on the point of slipping up the plane, the force of friction acts down the slope.

a $R(\nwarrow)$

$$R - 1.5g \cos 25^\circ = 0$$

$$R - 1.5g \cos 25^\circ = 0$$

$$\begin{aligned} \therefore R &= 1.5g \cos 25^\circ \\ &= 13.3 \text{ N (3 s.f.)} \end{aligned}$$

b $R(\nearrow)$

$$X - F - 1.5g \sin 25^\circ = 0$$

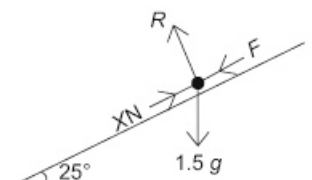
$$X = F + 1.5g \sin 25^\circ \quad (1)$$

The particle is in limiting equilibrium, so $F = \mu R$

$$\begin{aligned} \therefore F &= 0.25 \times 13.3227 \quad (\text{using } R = 13.3 \text{ N from a}) \\ &= 3.3306\dots \end{aligned}$$

Sub $F = 3.33 \text{ N}$ into (1):

$$\begin{aligned} X &= 3.33 + 1.5g \sin 25^\circ \\ &= 9.54 \text{ N (3 s.f.)} \end{aligned}$$



8 Let the normal reaction be R and the friction force be F acting down the plane.

a R (\nearrow)

$$R - 20 \sin 30^\circ - 1.5g \cos 30^\circ = 0$$

$$\begin{aligned} \therefore R &= 20 \sin 30^\circ + 1.5g \cos 30^\circ \\ &= 22.7 \text{ (3 s.f.)} \end{aligned}$$

The normal reaction has magnitude 22.7 N or 23 N (2 s.f.).

b R (\nearrow)

$$20 \cos 30^\circ - F - 1.5g \sin 30^\circ = 0$$

$$\begin{aligned} \therefore F &= 20 \cos 30^\circ - 1.5g \sin 30^\circ \\ &= 9.97 \text{ (3 s.f.)} \end{aligned}$$

The friction force has magnitude 9.97 N and acts down the plane.

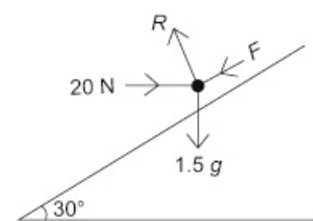
c Minimum possible value of μ occurs when frictional force required to maintain equilibrium is μR :

$$F = \mu R$$

$$9.9705\dots = \mu \times 22.730\dots \quad (\text{using } F \text{ from b and } R \text{ from a})$$

$$\begin{aligned} \mu &= \frac{22.730\dots}{9.9705\dots} \\ &= 0.43863\dots \end{aligned}$$

The coefficient of friction must be at least 0.439 (3 s.f.) to prevent the block sliding.



If you are told the particle is in equilibrium, but not told which way the particle is about to slip, then draw a diagram showing all the forces acting on the particle, with friction acting down the plane.

Resolve forces parallel to the plane. If $F > 0$ then you have chosen the correct direction. If $F < 0$ then you know friction acts up the plane.

9 Let the normal reaction be R and the friction force be F acting down the plane.

a

R (\nearrow)

$$R - X \sin 40^\circ - 3g \cos 40^\circ = 0$$

$$R = X \sin 40^\circ + 3g \cos 40^\circ \quad (1)$$

R (\nearrow)

$$X \cos 40^\circ - F - 3g \sin 40^\circ = 0$$

$$F = X \cos 40^\circ - 3g \sin 40^\circ \quad (2)$$

As the friction is limiting, $F = \mu R$

Using F from (2) and R from (1) gives:

$$X \cos 40^\circ - 3g \sin 40^\circ = 0.3(X \sin 40^\circ + 3g \cos 40^\circ)$$

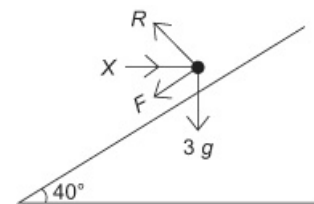
$$X \cos 40^\circ - 0.3X \sin 40^\circ = 0.9g \cos 40^\circ + 3g \sin 40^\circ$$

$$X(\cos 40^\circ - 0.3 \sin 40^\circ) = 0.9g \cos 40^\circ + 3g \sin 40^\circ$$

$$X = \frac{0.9g \cos 40^\circ + 3g \sin 40^\circ}{\cos 40^\circ - 0.3 \sin 40^\circ}$$

$$= \frac{25.65}{0.5732}$$

$$X = 44.8 \text{ N (3 s.f.)}$$



b Substituting $X = 44.8 \text{ N}$ into equation (1) gives

$$R = 44.8 \times \sin 40^\circ + 3g \cos 40^\circ$$

$$= 51.3 \text{ N (3 s.f.)}$$

- 10 Let the normal reaction be R and the friction force be F acting up the plane.

The friction acts up the plane, as the sledge is on the point of slipping down the plane.

$$R(\nearrow)$$

$$T + F - 22g \sin 35^\circ = 0 \quad (1)$$

$$R(\nwarrow)$$

$$R - 22g \cos 35^\circ = 0$$

$$\therefore R = 22g \cos 35^\circ$$

$$R = 176.6 \text{ N}$$

As the friction is limiting, $F = \mu R$

$$\therefore F = 0.125 \times 176.6$$

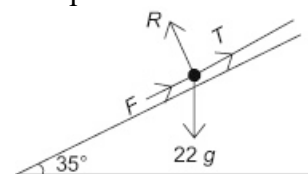
$$= 22.1 \text{ N (3 s.f.)}$$

Substituting $T = 22.1 \text{ N}$ into equation (1) gives:

$$T = 22g \sin 35^\circ - 22.1$$

$$= 101.6$$

$$= 102 \text{ N (3 s.f.)}$$



11 $R(\nwarrow)$

$$R - 0.5g \cos 40^\circ + T \sin 20^\circ = 0$$

$$R = 0.5g \cos 40^\circ - T \sin 20^\circ$$

T_{MAX} occurs when the particle is on the point of moving up the plane.

At this point, limiting friction $F = \mu R$ acts down the plane:

$$R(\nearrow)$$

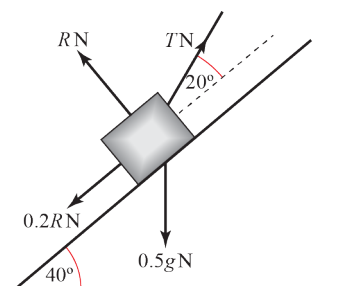
$$T_{\text{MAX}} \cos 20^\circ - 0.5g \sin 40^\circ - F = 0$$

$$T_{\text{MAX}} \cos 20^\circ - 0.5g \sin 40^\circ - \frac{1}{5}(0.5g \cos 40^\circ - T \sin 20^\circ) = 0$$

$$T_{\text{MAX}} \cos 20^\circ + 0.2T_{\text{MAX}} \sin 20^\circ = 0.5g \sin 40^\circ + 0.1g \cos 40^\circ$$

$$T_{\text{MAX}} = \frac{0.5g \sin 40^\circ + 0.1g \cos 40^\circ}{\cos 20^\circ + 0.2 \sin 20^\circ}$$

$$= 3.8690\dots$$



Mechanics 1

Solution Bank

- 11 T_{MIN} occurs when the particle is on the point of moving down the plane.

At this point, limiting friction $F = \mu R$ acts up the plane:

$$R(\nearrow)$$

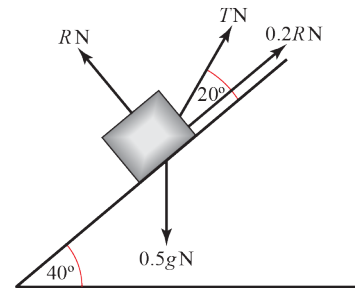
$$T_{\text{MIN}} \cos 20^\circ - 0.5g \sin 40^\circ - F = 0$$

$$T_{\text{MIN}} \cos 20^\circ - 0.5g \sin 40^\circ + \frac{1}{5}(0.5g \cos 40^\circ - T \sin 20^\circ) = 0$$

$$T_{\text{MIN}} \cos 20^\circ - 0.2T_{\text{MIN}} \sin 20^\circ = 0.5g \sin 40^\circ - 0.1g \cos 40^\circ$$

$$\begin{aligned} T_{\text{MAX}} &= \frac{0.5g \sin 40^\circ - 0.1g \cos 40^\circ}{\cos 20^\circ - 0.2 \sin 20^\circ} \\ &= 2.7533\dots \end{aligned}$$

T lies between 2.75 N and 3.87 N (both values to 3 s.f.).



- 12 $R(\nwarrow)$

$$R + 10 \sin 20^\circ - g \cos 40^\circ = 0$$

$$R = g \cos 40^\circ - 10 \sin 20^\circ$$

$$= 4.087$$

$$R(\nearrow)$$

$$10 \cos 20^\circ - F - g \sin 40^\circ = 0$$

$$F = 10 \cos 20^\circ - g \sin 40^\circ$$

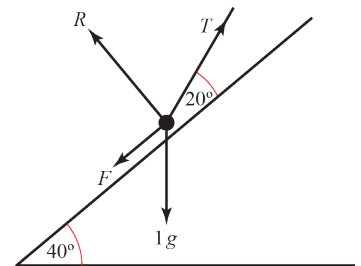
$$= 3.0976\dots$$

As the friction is limiting, $F = \mu R$

$$\therefore \mu = \frac{F}{R}$$

$$= \frac{3.0976}{4.087}$$

$$= 0.758 \quad (3 \text{ s.f.})$$



Mechanics 1

Solution Bank

$$13 \tan \theta = \frac{3}{4} \text{ so } \sin \theta = \frac{3}{5} \text{ and } \cos \theta = \frac{4}{5}$$

Before P is applied, the particle will be on the point of moving down the slope and the limiting frictional force therefore acts up the slope:

$$R(\nwarrow)$$

$$R = 2g \cos \theta$$

$$R(\nearrow)$$

$$2g \sin \theta = F_{\text{Max}}$$

$$2g \sin \theta = \mu R$$

$$2g \sin \theta = \mu \times 2g \cos \theta$$

$$\mu = \tan \theta$$

$$\mu = \frac{3}{4}$$

After applying the max value of P that will allow the particle to remain in equilibrium, particle is on the point of moving up the slope. Therefore, the frictional force acts down the slope.

$$R(\nwarrow)$$

$$R' = 2g \cos \theta + P \sin \theta$$

$$R(\nearrow)$$

$$P \cos \theta = F_{\text{Max}} + 2g \sin \theta$$

$$P \cos \theta = \mu R' + 2g \sin \theta$$

$$P \cos \theta = \mu(2g \cos \theta + P \sin \theta) + 2g \sin \theta$$

$$P(\cos \theta - \mu \sin \theta) = 2g(\mu \cos \theta + \sin \theta)$$

$$P\left(\frac{4}{5} - \frac{3}{4} \times \frac{3}{5}\right) = 2 \times 9.8 \times \left(\frac{3}{4} \times \frac{4}{5} + \frac{3}{5}\right)$$

$$0.35P = 23.52$$

$$P = 67.2 \text{ N}$$

So max. P is 67.2 N

