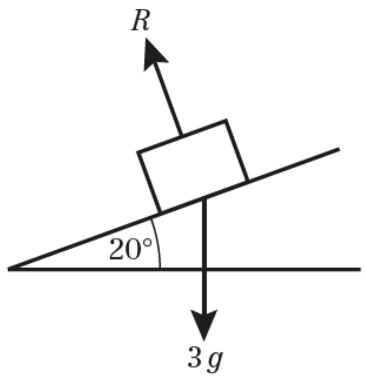
Exercise 5B

1 a



b R(↖):

$R = 3g\cos 20^\circ$

$= 3\times 9.8\cos 20^\circ$

$= 27.626...$

The normal reaction between the particle and the plane is 27.6 N (3 s.f.).

c Using Newton's second law of motion and R(↙):

$F = ma$

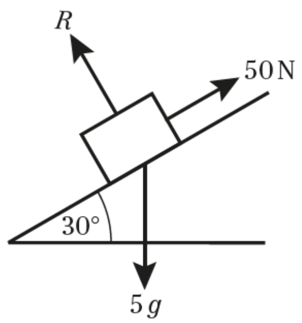
$3g\sin 20^\circ = 3a$

$a = 9.8\times\sin 20^\circ$

$= 3.3517...$

The acceleration of the particle is 3.35 ms^{-2} (3 s.f.).

2 a



b R(↖):

$R = 5g\cos 30^\circ$

$= 5\times 9.8\cos 30^\circ$

$= \dfrac{49\sqrt{3}}{2}$

$= 42.44$

The normal reaction between the particle and the plane is 42.4 N.

2 c Using Newton's second law of motion and R(↙):

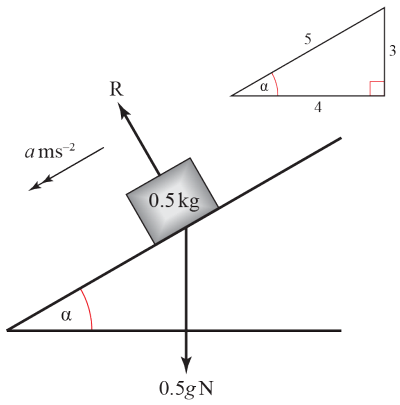
$$F = ma$$

$$50 - 5g\sin 30^\circ = 5a$$

$$a = 10 - \left(9.8 \times \frac{1}{2}\right) = 5.1$$

The acceleration of the particle is 5.1 ms^{-2}

3 $\tan\alpha = \frac{3}{4}$ so $\sin\alpha = \frac{3}{5}$ and $\cos\alpha = \frac{4}{5}$



a R(↖):

$$R = 0.5g\cos\alpha$$

$$= 0.5 \times 9.8 \times \frac{4}{5}$$

$$= 3.92$$

The normal reaction is 3.92 N.

b Using Newton's second law of motion and R(↙):

$$F = ma$$

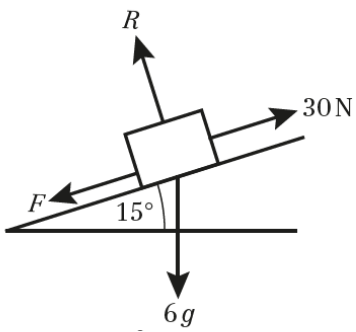
$$0.5g\sin\alpha = 0.5a$$

$$a = 9.8 \times \frac{3}{5}$$

$$= 5.88$$

The acceleration of the particle is 5.88 ms^{-2}

4 a



b Since mass is moving at constant speed, the resultant force parallel to the slope is zero.

R(↗):

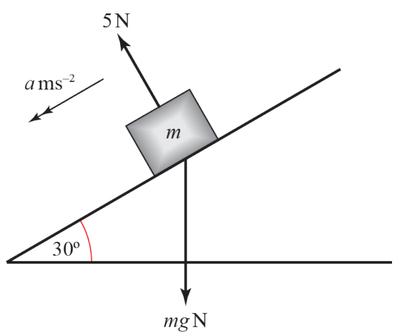
$$30 = F + 6g\sin 15^\circ$$

$$F = 30 - (6 \times 9.8\sin 15^\circ)$$

$$= 14.781...$$

The resistance due to friction is 14.8 N (3 s.f.).

5 a R(↖):



$$5 = mg\cos 30^\circ$$

$$m = \frac{5}{9.8 \times \frac{\sqrt{3}}{2}}$$

$$= 0.58913...$$

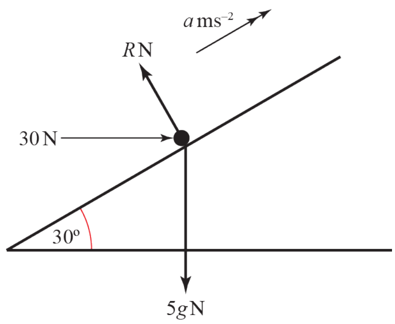
The mass of the particle is 0.589 kg (3 s.f.).

5 b Using Newton's second law of motion and R(↙):

$$F = ma$$
$$mg\sin 30^\circ = ma$$
$$a = 9.8 \times \frac{1}{2} = 4.9$$

The acceleration of the particle is 4.9 ms^{-2}

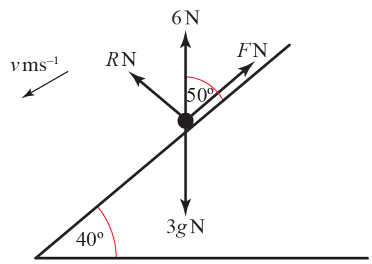
6 Using Newton's second law of motion and R(↗):



$$F = ma$$
$$30\cos 30^\circ - 5g\sin 30^\circ = 5a$$
$$6\cos 30^\circ - g\sin 30^\circ = a$$
$$a = 6\frac{\sqrt{3}}{2} - \left(9.8 \times \frac{1}{2}\right)$$
$$= 0.29615...$$

The acceleration of the particle is 0.296 ms^{-2} (3 s.f.).

7 Since mass is moving at constant speed, the resultant force parallel to the slope is zero.
We are not told whether the mass is moving up or down the slope. However, since the force acting down the slope is greater than the force acting up the slope, the particle must be moving down the slope.
Hence, friction acts up the slope to balance the forces.

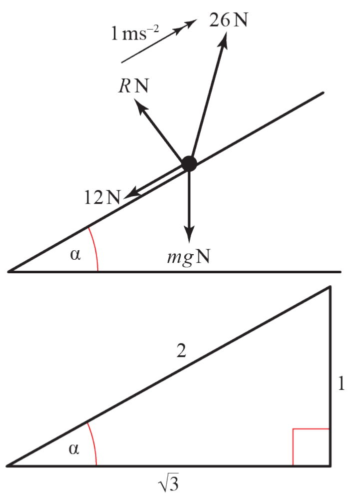


R(↗):

$$F + 6\cos 50^\circ = 3g\sin 40^\circ$$
$$F = (3 \times 9.8\sin 40^\circ) - 6\cos 50^\circ$$
$$F = 15.041...$$

The frictional force is 15.0 N (3s.f.).

8 $\tan\alpha = \frac{1}{\sqrt{3}}$ so $\sin\alpha = \frac{1}{2}$ and $\cos\alpha = \frac{\sqrt{3}}{2}$



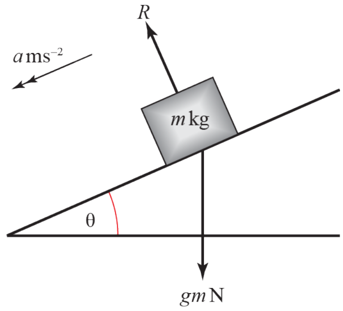
Using Newton's second law of motion and R(↗):

$$F = ma$$
$$26\cos 45^\circ - mg\sin\alpha - 12 = m \times 1$$
$$\frac{26}{\sqrt{2}} - \frac{9.8m}{2} - 12 = m$$
$$13\sqrt{2} - 12 = (4.9 + 1)m$$
$$m = \frac{13\sqrt{2} - 12}{5.9} = 1.0821...$$

The mass of the particle is 1.08 kg (3 s.f.).

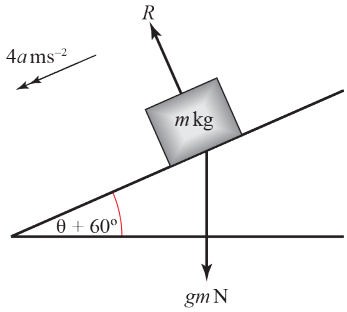
Challenge

a Using $F = ma$ and R(↙) for the plane inclined at θ to the horizontal:



$mg\sin\theta = ma$ **(1)**

Using $F = ma$ and R(↙) for the plane inclined at $(\theta + 60^\circ)$ to the horizontal:



$mg\sin(\theta + 60^\circ) = 4ma$ **(2)**

Substituting **(1)** into **(2)** gives:

$$mg\sin(\theta + 60^\circ) = 4mg\sin\theta$$

$$4\sin\theta = \sin(\theta + 60^\circ)$$

$$4\sin\theta = \sin\theta\cos 60^\circ + \cos\theta\sin 60^\circ$$

$$4\sin\theta = \frac{1}{2}\sin\theta + \frac{\sqrt{3}}{2}\cos\theta$$

$$\frac{7}{2}\sin\theta = \frac{\sqrt{3}}{2}\cos\theta$$

$$\tan\theta = \frac{\sqrt{3}}{7}$$

b $\theta = \tan^{-1}\frac{\sqrt{3}}{7} = 13.9^\circ$